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Single - Stage Fuzzy Economic Inventory Models with Backorders and Rework Process for Imperfect Items

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Abstract

The skill of retaining inventory is most essential for business entrepreneurs to diminish the total expenditure. Due to lack of accurateness and the presence of uncertainty among the parameters of the inventory models leads to fuzzify the parameters. In this paper, we studied the economic manufacture quantity (EMQ) model with backorders and rework processes for imperfect items at a single-stage manufacturing system through generalized quadrilateral fuzzy numbers (GQFNs). Fuzziness is involved by permitting the assumptions and respective solutions of the model in the version of GQFNs. A Numerical example is also provided to reveal the reliability of the model by comparing it with the solution of the classical model.

Keywords: Inventory, Generalized quadrilateral fuzzy number, Economic manufacture quantity, Imperfect items, Backorders

1. Introduction

Inventory is the crucial element in the part of the business. The skill of maintaining inventory levels is the major factor for a successful business. In every business scenario, the firm must decide the level of inventories to satisfy their demand. Inventory models are essential tools to effectively handle the problems. A company manufactured a product, erroneously the product has the imperfect quality for some reasons, and then it should be reworked to get perfect quality. For this type of situation, inventory models with the consideration of deteriorating or imperfect quality items are needful. Lin et al.,[9] discussed the integrated model for both production and inventory for the deteriorating items and explained the connectivity between utilization of provider's variable capacity and time of transportation and deterioration in two-stage. Cárdenas-Barrón[3] proposed an EPQ model with poor quality goods with backorders. When the decision variables are vague in nature, the decision-maker needs an inventory model better than the classical one, and this is the place we have to assume the problem into fuzzy nature. The notion of fuzzy sets has been introduced by Zadeh[14]. Further Kaufmann and Gupta [7] and Zimmermann [15] developed the concepts in fuzzy set theory. Fuzzy numbers are playing the main role in the study of fuzzy systems. Dubois and Prade[4] discussed the notion of fuzzy numbers and their arithmetic operations. Stephen Dinagar and Christopar Raj[12] delivered the more generalized version of trapezoidal fuzzy numbers as generalized quadrilateral fuzzy numbers(GQFNs). Stephen Dinagar and Manvizhi[13] proposed the classical equivalent fuzzy mean(CEFM) and new arithmetic operations on GQFNs.

The fuzzy inventory model proposed for the first time is believed to be that of Sommer[11], and it was solved by using a fuzzy dynamic programming approach. Another earlier work has been done in production planning and control using fuzzy set theory by Kacprzyk and Stanieski[6]. Later many authors described fuzzy inventory models in different approaches. Kazemi et al.[8] solved the fully-fuzzified EOQ model involving shortage by triangular and trapezoidal fuzzy numbers. Ehsani et al.[5] and Shekarian et al[10] fuzzified the Cárdenas-Barrón[1,2] model by interchanging the parameters in fuzzy nature.

In this paper, we discussed the fuzzy EMQ model and found the fuzzy optimal solutions using GQFNs. This model involves backorders and reworks processes for imperfect items of the manufacturing system in a single-stage mode. The following gives the structure of the paper: section-2 provides the preliminary definitions. Section-3 describes the basic operations on GQFNs based on CEFM. In section-4, we reviewed the single-stage classical inventory model with backorders and rework process. Section-5 presents the fuzzified version of the inventory model mentioned in section-4. In section-6, an example is provided to illustrate the model numerically. In section-7, the proposed fuzzy EMQ model has been compared with the existing model. Finally, the conclusion of the work is incorporated.

2. Preliminaries

Definition-2.1: Fuzzy set

A fuzzy set A in X is defined as a pair $\left(A, \mu_A\left(x\right)\right)$, where $\mu_A\left(x\right): X \rightarrow \left[0,1\right]$ is a membership function. For each value of X, $\mu_A\left(x\right)$ is mentioned as a membership grade of x in A.

Definition-2.2: Fuzzy number

In R, a fuzzy set $\tilde{A}$ is called as a fuzzy number if

(i). Height of $\tilde{A}$ is unity.
(ii). $\tilde{A}$ is convex
(iii). Supp $\tilde{A}$ is compact.

Definition-2.3: Generalized quadrilateral fuzzy number

A generalized quadrilateral fuzzy number (GQFN) is represented by $\tilde{A}$ = [a_1, a_2, a_3, a_4; l_1, l_2], where $a_1 \le a_2 \le a_3 \le a_4$, and its membership function is given as

$$\mu_A(x) = \begin{cases} l_1 \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{\left(x - a_2\right) l_2 + (a_3 - x) l_1}{a_3 - a_2}, & a_2 \le x \le a_3 \\ l_2 \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

And its pictorial representation is given by,

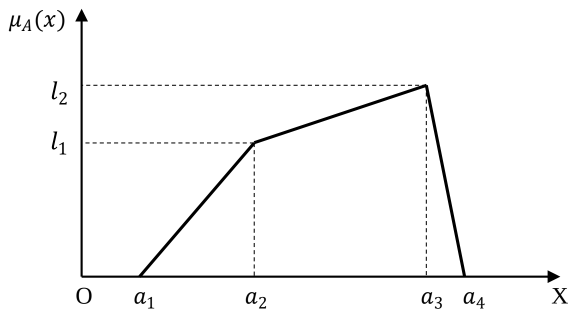

Figure.1- Generalized quadrilateral fuzzy number

If $l_1 = l_2 = 1$, then GQFN will be a trapezoidal fuzzy number. If $a_1 = a_2$ or $a_2 = a_3$ or $a_3 = a_4$, then GQFN will be a generalized triangular fuzzy number.

Note: Classical Equivalent Fuzzy Mean(CEFM)

If F(R) is considered as a set of GQFN's. The classical equivalent fuzzy mean M is defined to provide a real number for each GQFN in F(R).

For $\tilde{A} = \left[a_1, a_2, a_3, a_4; l_1, l_2\right] \in \mathrm{F(R)}$, the CEFM is defined as $M\left(\tilde{A}\right) = \dfrac{\left(a_1 + a_2 + a_3 + a_4\right)\left(l_1 + l_2\right)}{8}$.

Also, F(R) is an ordered set with the orders,

$M(\tilde{A}) \geq M(\tilde{B})$ iff $\tilde{A} \underset{M}{\geq} \tilde{B}$,

$M(\tilde{A}) \leq M(\tilde{B})$ iff $\tilde{A} \underset{M}{\leq} \tilde{B}$,

and $M(\tilde{A}) = M(\tilde{B})$ iff $\tilde{A} \underset{M}{=} \tilde{B}$.

3. Arithmetic Operations on Generalized Quadrilateral Fuzzy Numbers Based on CEFM

Let $\tilde{A} = \left[a_1, a_2, a_3, a_4; l_1, l_2\right]$ and $\tilde{B} = \left[b_1, b_2, b_3, b_4; m_1, m_2\right]$ be two GQFN's.

Take $\sigma_l = l_1 + l_2$ and $\sigma_m = m_1 + m_2$.

(i) Addition:

$$\tilde{A} + \tilde{B} = \left[\frac{2(a_1\sigma_l + b_1\sigma_m)}{\sigma_l + \sigma_m}, \frac{2(a_2\sigma_l + b_2\sigma_m)}{\sigma_l + \sigma_m}, \frac{2(a_3\sigma_l + b_3\sigma_m)}{\sigma_l + \sigma_m}, \frac{2(a_4\sigma_l + b_4\sigma_m)}{\sigma_l + \sigma_m}; \frac{l_1 + m_1}{2}, \frac{l_2 + m_2}{2}\right]$$

(ii) Subtraction:

$$\tilde{A} - \tilde{B} = \left[\frac{2(a_1\sigma_l - b_4\sigma_m)}{\sigma_l + \sigma_m}, \frac{2(a_2\sigma_l - b_3\sigma_m)}{\sigma_l + \sigma_m}, \frac{2(a_3\sigma_l - b_2\sigma_m)}{\sigma_l + \sigma_m}, \frac{2(a_4\sigma_l - b_1\sigma_m)}{\sigma_l + \sigma_m}; \frac{l_1 + m_1}{2}, \frac{l_2 + m_2}{2}\right]$$

(iii) Scalar multiplication:

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If $k > 0$, $k\tilde{A} = \left[ka_1, ka_2, ka_3, ka_4; l_1, l_2\right]$

If $k < 0$, $k\tilde{A} = \left[ka_4, ka_3, ka_2, ka_1; l_1, l_2\right]$

(iv) Multiplication:

If $M(\tilde{B}) > 0$,

$$\tilde{A}.\tilde{B} = \left[\frac{2a_1\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}), \frac{2a_2\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}), \frac{2a_3\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}), \frac{2a_4\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}); \frac{l_1 + m_1}{2}, \frac{l_2 + m_2}{2}\right]$$

If $M(\tilde{B}) < 0$,

$$\tilde{A}.\tilde{B} = \left[\frac{2a_4\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}), \frac{2a_3\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}), \frac{2a_2\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}), \frac{2a_1\sigma_l}{\sigma_l + \sigma_m} M(\tilde{B}); \frac{l_1 + m_1}{2}, \frac{l_2 + m_2}{2}\right]$$

(v) Division:

If $M(\tilde{B}) > 0$,

$$\frac{\tilde{A}}{\tilde{B}} = \left[\frac{2a_1\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}, \frac{2a_2\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}, \frac{2a_3\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}, \frac{2a_4\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}; \frac{l_1 + m_1}{2}, \frac{l_2 + m_2}{2}\right]$$

If $M(\tilde{B}) < 0$,

$$\frac{\tilde{A}}{\tilde{B}} = \left[\frac{2a_4\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}, \frac{2a_3\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}, \frac{2a_2\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}, \frac{2a_1\sigma_l}{(\sigma_l + \sigma_m)M(\tilde{B})}; \frac{l_1 + m_1}{2}, \frac{l_2 + m_2}{2}\right]$$

(vi) Square :

$$\tilde{A}^2 = \left[a_1 M(\tilde{A}), a_2 M(\tilde{A}), a_3 M(\tilde{A}), a_4 M(\tilde{A}); l_1, l_2\right]$$

(vi) Square root :

If $M(\tilde{B}) > 0$,

$$\sqrt{\tilde{A}} = \left[\frac{a_1}{\sqrt{M(\tilde{A})}}, \frac{a_2}{\sqrt{M(\tilde{A})}}, \frac{a_3}{\sqrt{M(\tilde{A})}}, \frac{a_4}{\sqrt{M(\tilde{A})}}; l_1, l_2\right]$$

4. The Single-Stage Classical EMQ model with Backorders and Rework Process

The inventory models are the essential tools to retain the level of inventories and balance the expenditure for a successful business. In manufacturing industries, some factors like machinery malfunction, inexperienced machinists, and workers, substandard resources can cause imperfect items. Reworking imperfect items allow some additional allowance of expenditure. In this direction, Cárdenas-Barrón[3] has derived an EPQ model with shortages and repairing processes for imperfect items in a single-stage production system.

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The assumptions of this model are: D - demand rate, units per time, S – backorder quantity, h – holding cost per product per unit of time, c – manufacturing cost of a product per unit, a_F - fixed backorder cost per product, a_L - linear backorder cost per product per unit of time, Q_P - manufacturing quantity, M – a rate of manufacturing product, units per unit of time (M > D), P – the proportion of the imperfect products in each cycle (0 < P < 1), $F(Q_P, S)$ - total cost per unit of time. In this inventory model, as per the reference of the Cárdenas-Barrón[3] model, the total cost and optimal solutions are obtained. The total cost function is:

$$F(Q_P,S)=\frac{bD}{Q_P}+\left(\frac{h}{2}-(H+HP+HP^2)\right)Q_P+\frac{1}{2}\left(\frac{h+a_L}{J}\right)I\frac{S^2}{Q_P}-hS+a_LD\frac{S}{Q_P}+CD\left(1+P\right) \quad (1)$$

The total cost function $F(Q_P, S)$ is convex in Q_P and S iff

$$2bDI\left(h+a_L\right)-J\left(a_FD\right)^2>0 \quad (2)$$

where, $H=\frac{h}{2}\frac{D}{M}, I=1-P, J=1-P-\frac{D}{M}$.

The optimal solution is,

$$Q_P^*=\left(\frac{bD\left(\frac{h+a_L}{J}\right)I-\frac{1}{2}\left(a_FD\right)^2}{\left(\frac{h+a_L}{J}\right)\left(\frac{h}{2}-(H+HP+HP^2)\right)-\frac{1}{2}b^2}\right)^{1/2} \quad (3)$$

and $$S^*=\frac{hQ_P-a_FD}{\left(\frac{h+a_L}{J}\right)I} \quad (4)$$

5. The Proposed Fuzzy EMQ Model

The elements that influence product quality are imprecise and vague. The way insisted by classical inventory models can be used to represent the vague parameters, but the proposed value cannot represent the originality of the parameters. This section proposed a modified EPQ model presented in the previous section by a fully fuzzified nature. In this model, all the input parameters are considered as GQFN's as follows:

$\tilde{D}=\left(d_1,d_2,d_3,d_4;\alpha_1,\alpha_2\right)$, $\tilde{S}=\left(s_1,s_2,s_3,s_4;t_1,t_2\right)$, $\tilde{h}=\left(h_1,h_2,h_3,h_4;\beta_1,\beta_2\right)$, $\tilde{b}=\left(b_1,b_2,b_3,b_4;\theta_1,\theta_2\right)$, $\tilde{a}_F=\left(a_{F1},a_{F2},a_{F3},a_{F4};\gamma_{F1},\gamma_{F2}\right)$, $\tilde{a}_L=\left(a_{L1},a_{L2},a_{L3},a_{L4};\gamma_{L1},\gamma_{L2}\right)$, $\tilde{c}=\left(c_1,c_2,c_3,c_4;\eta_1,\eta_2\right)$, $\tilde{Q}_P=\left(q_{P1},q_{P2},q_{P3},q_{P4};\omega_{P1},\omega_{P2}\right)$, $\tilde{M}=\left(m_1,m_2,m_3,m_4;\delta_1,\delta_2\right)$, $\tilde{P}=\left(p_1,p_2,p_3,p_4;\rho_1,\rho_2\right)$.

The fuzzy total cost $\tilde{F}(\tilde{Q}_P,\tilde{S})$ obtained from Eq. (1) by fuzzifying all the parameters is:

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$$\tilde{F}(\tilde{Q}_P,\tilde{S})=\frac{\tilde{b}\tilde{D}}{\tilde{Q}_P}+\left(\frac{\tilde{h}}{2}-(\tilde{H}+\tilde{H}\tilde{P}+\tilde{H}\tilde{P}^2)\right)\tilde{Q}_P+\frac{1}{2}\left(\frac{\tilde{h}+\tilde{a}_L}{\tilde{J}}\right)\tilde{I}\,\frac{\tilde{S}^2}{\tilde{Q}_P}-\tilde{h}\tilde{S}+\tilde{a}_L\tilde{D}\frac{\tilde{S}}{\tilde{Q}_P}+\tilde{C}\tilde{D}\left(\tilde{1}+\tilde{P}\right) \quad (5)$$

where, $\tilde{H}=\frac{\tilde{h}}{2}\frac{\tilde{D}}{\tilde{M}}$, $\tilde{I}=\tilde{1}-\tilde{P}$, $\tilde{J}=\tilde{1}-\tilde{P}-\frac{\tilde{D}}{\tilde{M}}$. Here $\tilde{1}$ represents a GQFN with CEFM value 1, and it is generally assumed to be a GQFN having all its entries and weights are 1's.

The goal is to obtain the fuzzy economic manufacture quantity $\tilde{Q}_P$ and the fuzzy backorder quantity $\tilde{S}$, which minimizes the fuzzy total cost. The fuzzy parameters $\tilde{Q}_P$ and $\tilde{S}$ are assumed to be continuous depending on their respective CEFM values. The partial derivatives of Eq.(5) concerning and are expressed by Eqs. (6) and (7), respectively.

$$\frac{\partial\tilde{F}(\tilde{Q}_P,\tilde{S})}{\partial\tilde{Q}_P}=\left(\begin{array}{l}-\dfrac{\left(\frac{2b_1\sigma_\theta M(\tilde{D})}{\sigma_\theta+\sigma_\alpha},\frac{2b_2\sigma_\theta M(\tilde{D})}{\sigma_\theta+\sigma_\alpha},\frac{2b_3\sigma_\theta M(\tilde{D})}{\sigma_\theta+\sigma_\alpha},\frac{2b_4\sigma_\theta M(\tilde{D})}{\sigma_\theta+\sigma_\alpha};\frac{\theta_1+\alpha_1}{2},\frac{\theta_2+\alpha_2}{2}\right)}{\tilde{Q}_P^{\,2}}\\[2ex] +\left(\begin{array}{l}\dfrac{32\left(h_1\sigma_\beta-\frac{h_4\sigma_\beta M(\tilde{D})}{M(\tilde{M})}\left[1+M(\tilde{P})(1+M(\tilde{P}))\right]\right)}{44\sigma_\beta+6\sigma_\alpha+6\sigma_\delta+8\sigma_\rho},\dfrac{32\left(h_2\sigma_\beta-\frac{h_3\sigma_\beta M(\tilde{D})}{M(\tilde{M})}\left[1+M(\tilde{P})(1+M(\tilde{P}))\right]\right)}{44\sigma_\beta+6\sigma_\alpha+6\sigma_\delta+8\sigma_\rho},\\ \dfrac{32\left(h_3\sigma_\beta-\frac{h_2\sigma_\beta M(\tilde{D})}{M(\tilde{M})}\left[1+M(\tilde{P})(1+M(\tilde{P}))\right]\right)}{44\sigma_\beta+6\sigma_\alpha+6\sigma_\delta+8\sigma_\rho},\dfrac{32\left(h_4\sigma_\beta-\frac{h_1\sigma_\beta M(\tilde{D})}{M(\tilde{M})}\left[1+M(\tilde{P})(1+M(\tilde{P}))\right]\right)}{44\sigma_\beta+6\sigma_\alpha+6\sigma_\delta+8\sigma_\rho};\\ \dfrac{44\beta_1+6\alpha_1+6\delta_1+8\rho_1}{64},\dfrac{44\beta_2+6\alpha_2+6\delta_2+8\rho_2}{64}\end{array}\right)\\[2ex] -\left(\begin{array}{l}\dfrac{8(h_1\sigma_\beta+a_{L1}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\dfrac{8(h_2\sigma_\beta+a_{L2}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\\ \dfrac{8(h_3\sigma_\beta+a_{L3}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\dfrac{8(h_4\sigma_\beta+a_{L4}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})};\\ \dfrac{4+2\beta_1+4\rho_1+2\alpha_1+2\delta_1+\gamma_{L1}}{16},\dfrac{4+2\beta_2+4\rho_2+2\alpha_2+2\delta_2+\gamma_{L2}}{16}\end{array}\right)\dfrac{\tilde{S}^2}{\tilde{Q}_P^{\,2}}\\[2ex] -\left(\dfrac{2a_{F_1}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha},\dfrac{2a_{F_2}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha},\dfrac{2a_{F_3}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha},\dfrac{2a_{F_4}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha};\dfrac{\gamma_{F_1}+\alpha_1}{2},\dfrac{\gamma_{F_2}+\alpha_2}{2}\right)\dfrac{\tilde{S}}{\tilde{Q}_P^{\,2}}\end{array}\right) \quad (6)$$

$$\frac{\partial\tilde{F}(\tilde{Q}_P,\tilde{S})}{\partial\tilde{S}}=\left(\begin{array}{l}\left(\begin{array}{l}\dfrac{16(h_1\sigma_\beta+a_{L1}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\dfrac{16(h_2\sigma_\beta+a_{L2}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\\ \dfrac{16(h_3\sigma_\beta+a_{L3}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\dfrac{16(h_4\sigma_\beta+a_{L4}\sigma_{\gamma_L})M(\tilde{I})}{M(\tilde{J})(8+2\sigma_\beta+4\sigma_\rho+2\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})};\\ \dfrac{4+2\beta_1+4\rho_1+2\alpha_1+2\delta_1+\gamma_{L1}}{16},\dfrac{4+2\beta_2+4\rho_2+2\alpha_2+2\delta_2+\gamma_{L2}}{16}\end{array}\right)\dfrac{\tilde{S}}{\tilde{Q}_P}-\left(h_1,h_2,h_3,h_4;\beta_1,\beta_2\right)\\[2ex] +\dfrac{\left(\frac{2a_{F_1}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha},\frac{2a_{F_2}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha},\frac{2a_{F_3}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha},\frac{2a_{F_4}\sigma_{\gamma_F}M(\tilde{D})}{\sigma_{\gamma_F}+\sigma_\alpha};\frac{\gamma_{F_1}+\alpha_1}{2},\frac{\gamma_{F_2}+\alpha_2}{2}\right)}{\tilde{Q}_P}\end{array}\right) \quad (7)$$

Eq.(5) is convex in $\tilde{Q}_P$ and $\tilde{S}$ based on its CEFM value, iff $\frac{\partial^2\tilde{F}(\tilde{Q}_P,\tilde{S})}{\partial\tilde{S}^2}>\tilde{0},\frac{\partial^2\tilde{F}(\tilde{Q}_P,\tilde{S})}{\partial\tilde{Q}_P^{\,2}}>\tilde{0}$, (8)

and $\left(\frac{\partial^2\tilde{F}(\tilde{Q}_P,\tilde{S})}{\partial\tilde{S}^2}\right)\left(\frac{\partial^2\tilde{F}(\tilde{Q}_P,\tilde{S})}{\partial\tilde{Q}_P^{\,2}}\right)-\left(\frac{\partial^2\tilde{F}(\tilde{Q}_P,\tilde{S})}{\partial\tilde{Q}_P\partial\tilde{S}}\right)^2>\tilde{0}$. (9)

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By taking the second partial derivatives of Eqs. (7) and (8) with respect to $\tilde{Q}_P$ and $\tilde{S}$ can easily prove that condition (8) is held. After simplification, Eq.(9) can be written as $\tilde{X} - \tilde{Y} > \tilde{0}$, (10)

where $$\tilde{X} = \left(\begin{array}{l} \frac{16 b_1 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{2+2\sigma_\theta+2\sigma_\alpha+\sigma_\rho+\sigma_\beta+\sigma_{\gamma_L}}, \frac{2 b_2 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{2+2\sigma_\theta+2\sigma_\alpha+\sigma_\rho+\sigma_\beta+\sigma_{\gamma_L}}, \frac{2 b_3 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{2+2\sigma_\theta+2\sigma_\alpha+\sigma_\rho+\sigma_\beta+\sigma_{\gamma_L}}, \\ \frac{2 b_4 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{2+2\sigma_\theta+2\sigma_\alpha+\sigma_\rho+\sigma_\beta+\sigma_{\gamma_L}}; \frac{1+2\theta_1+2\alpha_1+\rho_1+\beta_1+\gamma_{L1}}{8}, \frac{1+2\theta_2+2\alpha_2+\rho_2+\beta_2+\gamma_{L2}}{8} \end{array} \right)$$

$$\tilde{Y} = \left(\begin{array}{l} \frac{8\left(2-P_4\sigma_\rho-\frac{d_4\sigma_\alpha}{M(\tilde{M})}\right) M(\tilde{a}_F)^2 M(\tilde{D})^2}{2+\sigma_\rho+3\sigma_\alpha+\sigma_\delta+2\sigma_{\gamma_F}}, \frac{8\left(2-P_3\sigma_\rho-\frac{d_3\sigma_\alpha}{M(\tilde{M})}\right) M(\tilde{a}_F)^2 M(\tilde{D})^2}{2+\sigma_\rho+3\sigma_\alpha+\sigma_\delta+2\sigma_{\gamma_F}}, \frac{8\left(2-P_2\sigma_\rho-\frac{d_2\sigma_\alpha}{M(\tilde{M})}\right) M(\tilde{a}_F)^2 M(\tilde{D})^2}{2+\sigma_\rho+3\sigma_\alpha+\sigma_\delta+2\sigma_{\gamma_F}}, \\ \frac{8\left(2-P_1\sigma_\rho-\frac{d_1\sigma_\alpha}{M(\tilde{M})}\right) M(\tilde{a}_F)^2 M(\tilde{D})^2}{2+\sigma_\rho+3\sigma_\alpha+\sigma_\delta+2\sigma_{\gamma_F}}; \frac{1+\rho_1+3\alpha_1+\delta_1+2\gamma_{F1}}{8}, \frac{1+\rho_2+3\alpha_2+\delta_2+2\gamma_{F2}}{8} \end{array} \right)$$

Finally, we can conclude that Eq.(5) is convex in $\tilde{Q}_P$ and $\tilde{S}$ based on CEFM value iff the condition (10) holds. Therefore by solving Eq.(6) and (7), by setting both equal to zero, obtain the optimum values of $\tilde{Q}_P$ and $\tilde{S}$, we get,

$$\tilde{Q}_P^{\ *} = \left(\frac{\tilde{A}}{\tilde{B}} \right)^{1/2}, \tag{11}$$

where $$\tilde{A} = \left\{ \begin{array}{l} 32\left(\frac{\frac{2 b_1 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{M(\tilde{J})} - a_{F4} M(\tilde{a}_F) M(\tilde{D})^2 \sigma_{\gamma_F}}{8+8\sigma_\theta+26\sigma_\alpha+4\sigma_\rho+2\sigma_\beta+2\sigma_\delta+\sigma_{\gamma_L}+16\sigma_{\gamma_F}} \right), 32\left(\frac{\frac{2 b_2 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{M(\tilde{J})} - a_{F3} M(\tilde{a}_F) M(\tilde{D})^2 \sigma_{\gamma_F}}{8+8\sigma_\theta+26\sigma_\alpha+4\sigma_\rho+2\sigma_\beta+2\sigma_\delta+\sigma_{\gamma_L}+16\sigma_{\gamma_F}} \right), \\ 32\left(\frac{\frac{2 b_3 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{M(\tilde{J})} - a_{F2} M(\tilde{a}_F) M(\tilde{D})^2 \sigma_{\gamma_F}}{8+8\sigma_\theta+26\sigma_\alpha+4\sigma_\rho+2\sigma_\beta+2\sigma_\delta+\sigma_{\gamma_L}+16\sigma_{\gamma_F}} \right), 32\left(\frac{\frac{2 b_4 \sigma_\theta M(\tilde{D}) M(\tilde{I}) M(\tilde{h}+\tilde{a}_L)}{M(\tilde{J})} - a_{F1} M(\tilde{a}_F) M(\tilde{D})^2 \sigma_{\gamma_F}}{8+8\sigma_\theta+26\sigma_\alpha+4\sigma_\rho+2\sigma_\beta+2\sigma_\delta+\sigma_{\gamma_L}+16\sigma_{\gamma_F}} \right); \\ \frac{4+8\theta_1+26\alpha_1+4\rho_1+2\beta_1+4\rho_1+2\delta_1+\gamma_{L1}+16\gamma_{F1}}{64}, \frac{4+8\theta_2+26\alpha_2+4\rho_2+2\beta_2+4\rho_2+2\delta_2+\gamma_{L2}+16\gamma_{F2}}{64} \end{array} \right\}$$

$$\tilde{B} = \left(\begin{array}{l} \frac{128\left((h_1\sigma_\beta+a_{L1}\sigma_{\gamma_L}) M(\tilde{I}) M(\tilde{h}) \left\{ 1-\left[1+M(\tilde{P})(1+M(\tilde{P}))\right] \frac{M(\tilde{D})}{M(\tilde{M})} \right\} - h_4 M(\tilde{h}) \sigma_\beta \right)}{M(\tilde{J})(32+180\sigma_\beta+24\sigma_\rho+14\sigma_\alpha+14\sigma_\delta+\sigma_{\gamma_L})}, \\ \frac{128\left((h_2\sigma_\beta+a_{L2}\sigma_{\gamma_L}) M(\tilde{I}) M(\tilde{h}) \left\{ 1-\left[1+M(\tilde{P})(1+M(\tilde{P}))\right] \frac{M(\tilde{D})}{M(\tilde{M})} \right\} - h_4 M(\tilde{h}) \sigma_\beta \right)}{M(\tilde{J})(32+180\sigma_\beta+24\sigma_\rho+14\sigma_\alpha+14\sigma_\delta+\sigma_{\gamma_L})}, \\ \frac{128\left((h_3\sigma_\beta+a_{L3}\sigma_{\gamma_L}) M(\tilde{I}) M(\tilde{h}) \left\{ 1-\left[1+M(\tilde{P})(1+M(\tilde{P}))\right] \frac{M(\tilde{D})}{M(\tilde{M})} \right\} - h_4 M(\tilde{h}) \sigma_\beta \right)}{M(\tilde{J})(32+180\sigma_\beta+24\sigma_\rho+14\sigma_\alpha+14\sigma_\delta+\sigma_{\gamma_L})}, \\ \frac{128\left((h_4\sigma_\beta+a_{L4}\sigma_{\gamma_L}) M(\tilde{I}) M(\tilde{h}) \left\{ 1-\left[1+M(\tilde{P})(1+M(\tilde{P}))\right] \frac{M(\tilde{D})}{M(\tilde{M})} \right\} - h_4 M(\tilde{h}) \sigma_\beta \right)}{M(\tilde{J})(32+180\sigma_\beta+24\sigma_\rho+14\sigma_\alpha+14\sigma_\delta+\sigma_{\gamma_L})}; \\ \frac{16+180\beta_1+24\rho_1+14\alpha_1+14\delta_1+\gamma_{L1}}{16}, \frac{16+180\beta_2+24\rho_2+14\alpha_2+14\delta_2+\gamma_{L2}}{16} \end{array} \right)$$

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$$\tilde{S}^{*}=\left(\begin{array}{l}\dfrac{32\left[h_1\sigma_\beta M(\tilde{Q}_P)-a_F 4\sigma_{\gamma_F} M(\tilde{D})\right]M(\tilde{J})}{M(\tilde{h}+\tilde{a}_L)M(\tilde{I})(8+6\sigma_\beta+4\sigma_{\omega_P}+4\sigma_{\gamma_F}+4\sigma_\rho+6\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\dfrac{32\left[h_1\sigma_\beta M(\tilde{Q}_P)-a_F 4\sigma_{\gamma_F} M(\tilde{D})\right]M(\tilde{J})}{M(\tilde{h}+\tilde{a}_L)M(\tilde{I})(8+6\sigma_\beta+4\sigma_{\omega_P}+4\sigma_{\gamma_F}+4\sigma_\rho+6\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\\ \dfrac{32\left[h_1\sigma_\beta M(\tilde{Q}_P)-a_F 4\sigma_{\gamma_F} M(\tilde{D})\right]M(\tilde{J})}{M(\tilde{h}+\tilde{a}_L)M(\tilde{I})(8+6\sigma_\beta+4\sigma_{\omega_P}+4\sigma_{\gamma_F}+4\sigma_\rho+6\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})},\dfrac{32\left[h_1\sigma_\beta M(\tilde{Q}_P)-a_F 4\sigma_{\gamma_F} M(\tilde{D})\right]M(\tilde{J})}{M(\tilde{h}+\tilde{a}_L)M(\tilde{I})(8+6\sigma_\beta+4\sigma_{\omega_P}+4\sigma_{\gamma_F}+4\sigma_\rho+6\sigma_\alpha+2\sigma_\delta+\sigma_{\gamma_L})};\\ \dfrac{4+6\beta_1+4\omega_{P1}+4\gamma_{F1}+4\rho_1+6\alpha_1+2\delta_1+\gamma_{L1}}{32},\dfrac{4+6\beta_2+4\omega_{P2}+4\gamma_{F2}+4\rho_2+6\alpha_2+2\delta_2+\gamma_{L2}}{32}\end{array}\right) \quad (12)$$

The optimum fuzzy total cost $\tilde{F}(\tilde{Q}_P{}^{*},\tilde{S}^{*})$ can be obtained by substituting the values of $\tilde{Q}_P{}^{*}$ and $\tilde{S}^{*}$ in Eq.(5).

6. Numerical example

To demonstrate the utility of the proposed model, let us find the optimal solution of the example referred to from Cárdenas-Barrón[3], which has been solved for different values of the proportion of imperfect items. From the result of the problem in the classical sense, we observed that the value of the proportion of imperfect items P = 0.25 provides the best optimal solution of the model. So here we can choose the fixed value for P as 0.25. The further details for the example are given as follows: D = 300 units/year, M = 550 units/year, b = \$50/lot size, h = \$50 per unit/year, a_F = \$1/unit short, a_L = \$10/unit short per year and c = \$7 per unit. For the above problem the optimal solution in the classical sense is, Q_P = 102.0588066, S = 21.8315469, $F(Q_P, S)$ = 2983.1216096. We will solve the problem in a fuzzy environment by the proposed method and the required fuzzy parameters with different levels of fuzziness are given in table-1. The optimal solutions are obtained by using Eqs. (11) and (12) after verifying the condition (10). Later the minimum fuzzy total cost has been obtained by substituting the values of $\tilde{Q}_P{}^{*}$ and $\tilde{S}^{*}$ in Eq.(5) in different levels of fuzziness, and its change percentage is also calculated by comparing the CEFM values of the fuzzy optimal solutions with the optimal solutions in the classical sense.

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Table-1: Fuzzy parameters in the form of generalized quadrilateral fuzzy numbers.

LOF	$\tilde{D}$	M	$\tilde{c}$	M	$\tilde{b}$	M	$\tilde{h}$	M
-40 %	(160,380,400,500;0.6,0.4)	180	(3,5,6,7;0.9,0.7)	4.2	(30,60,100,110;0.7,0.1)	30	(30,70,90,110;0.7,0.1)	30
-20 %	(300,500,520,600;0.5,0.5)	240	(6,6.5,7,8.5;0.8,0.8)	5.6	(60,80,120,140;0.6,0.2)	40	(60,90,110,140;0.6,0.2)	40
-10 %	(420,510,550,680;0.4,0.6)	270	(6.5,7,8,10;0.7,0.9)	6.3	(70,90,130,160;0.5,0.3)	45	(60,90,140,160;0.5,0.3)	45
0%	(450,600,650,700;0.3,0.7)	300	(7,8,9,11;0.8,0.8)	7	(80,100,150,170;0.4,0.4)	50	(70,90,160,180;0.4,0.4)	50
10%	(510,650,700,780;0.8,0.2)	330	(7.5,9,10,12;0.9,0.7)	7.7	(90,120,160,180;0.3,0.5)	55	(100,130,150,170;0.3,0.5)	55
20%	(570,700,770,840;0.7,0.3)	360	(8,10,11,13;0.8,0.8)	8.4	(100,130,180,190;0.2,0.6)	60	(100,140,170,190;0.2,0.6)	60
40%	(700,810,900,950;0.2,0.8)	420	(10,11,13,15;0.7,0.9)	9.8	(120,150,200,230;0.1,0.7)	70	(120,160,200,22;0.1,0.7)	70
LOF	$\tilde{M}$	**M**	$\tilde{a}_F$	**M**	$\tilde{a}_L$	**M**	$\tilde{P}$	**M**
-40 %	(220,370,460,600;0.7,0.9)	330	(1,2,4,5;0.1,0.3)	0.6	(2,5,16,25;0.5,0.5)	6	(0,0.5,1,1.5;0.2,0.2)	0.15
-20 %	(360,500,600,740;0.8,0.8)	440	(2,3,5,6;0.2,0.2)	0.8	(5,10,21,28;0.7,0.3)	8	(0,0.5,1,2.5;0.1,0.3)	0.2
-10 %	(430,555,680,810;0.9,0.7)	495	(2,4,5,7;0.3,0.1)	0.9	(8,11,24,29;0.6,0.4)	9	(0,0.5,1.5,2.5;0.3,0.1)	0.225
0%	500,625,750,875;0.7,0.9)	550	(3,4,6,7;0.2,0.2)	1	(0,10,30,40;0.4,0.6)	10	(0,1,1.5,2.5;0.2,0.2)	0.25
10%	(570,690,820,945;0.8,0.8)	605	(4,5,6,7;0.1,0.3)	1.1	(2,10,32,44;0.5,0.5)	11	(0.5,1,1.5,2.5;0.1,0.3)	0.275
20%	(640,760,900,1000;0.9,0.7)	660	(4,5,7,8;0.3,0.1)	1.2	(5,20,26,45;0.6,0.4)	12	(0,1.5,2,2.5;0.3,0.1)	0.3
	(780,800,1040,1130;0.8,0.8)	770	(5,6,8,9;0.2,0.2)	1.4	(9,24,30,49;0.3,0.7)	14	(0.5,1.5,2,3;0.2,0.2)	0.35

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Table-2: Fuzzy optimal solutions in the form of generalized quadrilateral fuzzy numbers.

LOF	$\tilde{Q}_P^*$	M	$\tilde{S}^*$	M	$\tilde{F}(\tilde{Q}_P^*,\tilde{S}^*)$	M
-40%	(48.1153,98.9377,167.00163, 184.2432;0.564648,0.294336)	53.50374962	(7.61049,22.15337,30.33393, 37.92553;0.53621,0.38992)	11.34770697	(-716.83436,1491.83493,2955.07807, 4783.95696;0.60030,0.44762)	1115.251329
-20%	(149.5844,188.7873,286.4661, 325.669; 0.50957,0.349414)	60.68709973	(14.43493,24.02426,31.20777, 40.7971;0.504322,0.42180)	12.78791362	(919.31331,2850.08141,4355.99396, 6754.24206;0.54243,0.50550)	1949.078996
-10%	(150.0243,96.57499,146.7547, 171.8446; 0.475,0.383984)	79.16979372	(16.71371,28.33189,46.31193, 54.45269;0.50313,0.42299)	16.87977645	(218.13109,2338.82443,6551.00494, 9475.92759;0.53096,0.51696)	2434.298705
0%	(149.9668,137.8388,200.8154, 248.713; 0.378125,0.480859)	102.0588066	(24.09074,32.75271,62.18016, 70.84213;0.40977,0.51636)	21.97987956	(-588.27102,1941.68676,9159.02769, 12261.54939;0.46998,0.57794)	2983.159449
10%	(148.4582,275.0732,369.1492, 416.5848; 0.435352,0.423633)	129.842504	(42.51126,57.10283,67.02821, 76.95358;0.47629,0.44983)	28.19996927	(1150.81762,5151.07391,8424.49385, 12751.87871;0.57094,0.47698)	3599.370731
20%	(146.3377,354.1667,494.8242, 523.5234; 0.415234,0.44375)	163.0837627	(48.98033,71.06292,88.36608, 99.70386;0.49565,0.43047)	35.66884131	(-470.68582,5551.51419,10813.82188, 16829.44112;0.57059,0.47733)	4286.520294
40%	(142.0666,556.3224,747.7729, 862.3856; 0.252539,0.606445)	247.8757789	(78.65313,107.2427,136.4253, 151.0165;0.34094,0.58518)	54.79610838	(-2177.65907,5670.28784,16381.7821, 25076.05683;0.40892,0.63900)	5888.05018

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7. Analogues to earlier work

Shekarian et al[10], proposed the fuzzified version of the model presented in this paper by fuzzifying the input parameters except for the manufacture quantity and backorder quantity in the form of triangular(TFN) and trapezoidal fuzzy numbers(TRFN). The example has been chosen as same as in Cárdenas-Barrón[3], table-2 analyses the results from Shekarian et al[10], and the results presented in the paper. This analog shows the distinctions in the optimum values of the fuzzy decision variables and the fuzzy total cost.

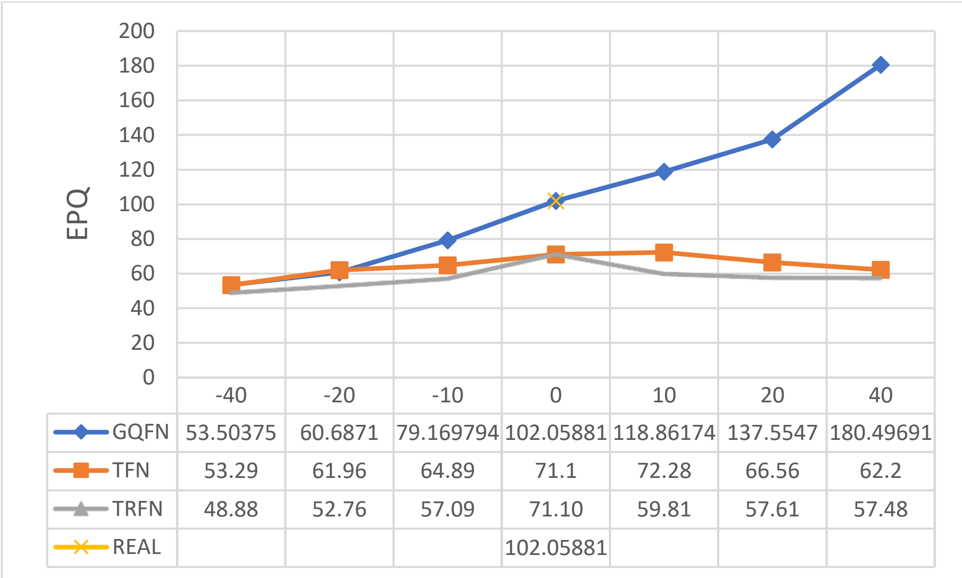


Figure.2-Analogues on EPQ

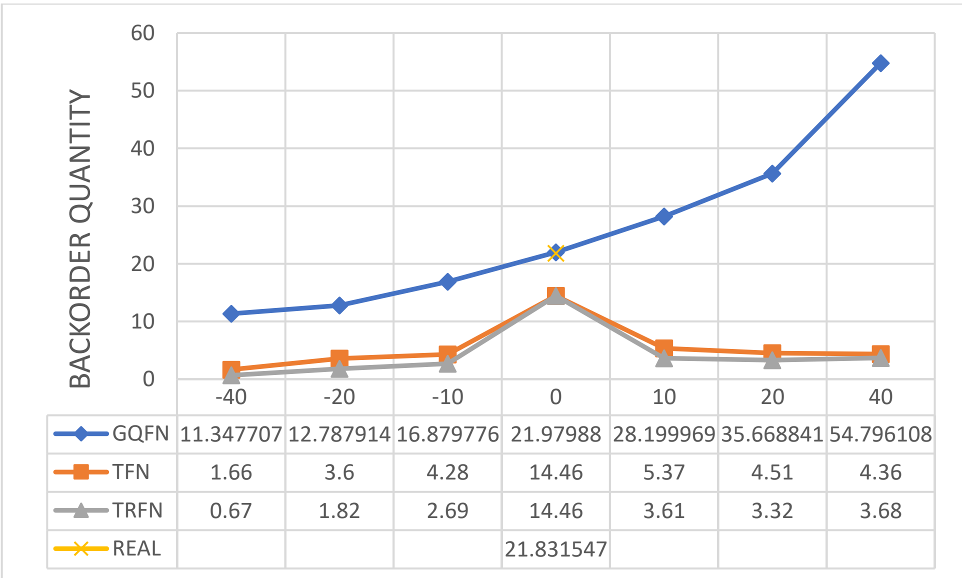
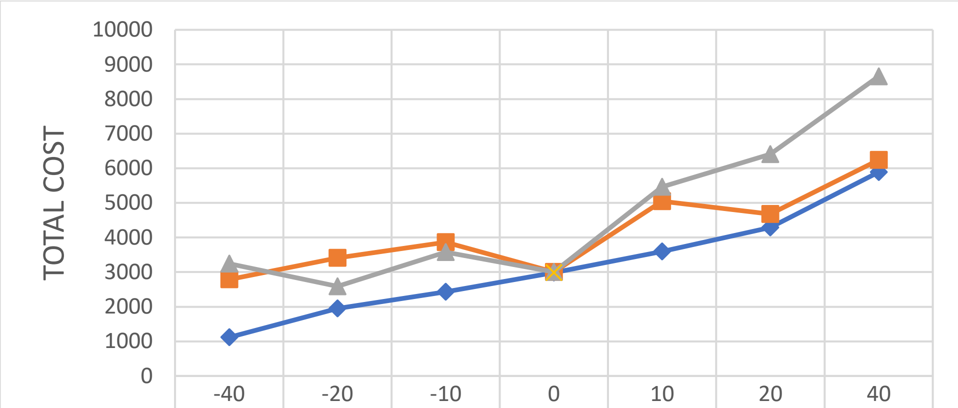


Figure.3-Analogues on backorder quantity



	-40	-20	-10	0	10	20	40
GQFN	1115.2513	1949.079	2434.2987	2983.1594	3599.3707	4286.5203	5888.0502
TFN	2,786.60	3,411.75	3,858.89	3,002.45	5,047.21	4,679.10	6,239.52
TRFN	3,245.61	2,588.71	3,577.23	3,002.45	5,466.79	6,409.17	8,654.97
REAL				2983.1216			

Figure.4-Analogues on the total cost

Figure.2-4 indicates the variations of the fuzzy decision variables and fuzzy total cost as given in the table-2. From these figures, we observed that the curve of the triangular and trapezoidal fuzzy values never reached the values of the crisp solutions. However, the curve of generalized quadrilateral fuzzy values is very close to the crisp solutions when the zero levels of fuzziness. Also, the values are linearly increased as the level of fuzziness increases. Hence the proposed model with the GQFNs provides the optimal solutions better than the previous fuzzy models.

8. Conclusion

This paper developed the fully fuzzified EMQ model with backorders and poor quality items that are repaired in the same single-stage manufacturing system, through the generalized quadrilateral fuzzy numbers. The classical equivalent fuzzy mean method of defuzzification of GQFNs possesses the leading role in this model to provide the optimal solutions. A numerical example was solved to show the reliability of the proposed model and to analyze the outcomes in this paper to those of Shekarian et al[17]. The analysis shows that the optimal solutions of the model may vary based on the use of alternate fuzzy numbers and methods of defuzzification. To conclude, GQFNs along with CEFM provide the best optimal solution of the proposed model than the previous models in a fuzzy environment. Some of the inventory models with vague situations are yet solved by classical methods, and this remains the motivating task for further research.

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