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Solution of One Dimensional Convection Diffusion Equation by Homotopy Perturbation Method

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Abstract- In this research, we have used a technique known as the homotopy perturbation method (HPM) to discover a solution to the one-dimensional convection diffusion problem. Equations of convection and diffusion hold a unique place in the domains of engineering and science, and they serve as a useful model for a great deal of different kinds of systems that are found in a variety of disciplines. The homotopy perturbation technique is one of the most recent analytic methods that can be utilized to determine the exact solution of a number of different classes of PDE. Through the use of the HPM, a complex issue is reduced to a manageable and straightforward issue that may be resolved with relative ease. When contrasted with the results of numerical solution, the results generated by HPM are analyzed.

Keywords-Homotopy perturbation Method, CD Equation, Burgers equation

I. Introduction-

The nature of the governing equation, which comprises both a non-dissipative component and a dissipative component, makes it difficult for numerical methods to solve convection-diffusion issues. As a result, this presents a demanding task for numerical methods. The mix of convective and diffusive processes is at the heart of many different kinds of physical difficulties. They appear in areas where mathematical modelling is significant, such as physics and engineering, and particularly in fluid dynamics and transport issues. They can also be found in phrases like "they occur in domains where mathematical modeling is vital." There is a vast body of written material that has accumulated describing numerical approximations, as well as the various strategies for analysing and overcoming the challenges that are presented by each numerical method. This body of work has been referred to as the numerical approximations literature.

He (1999) created a method for addressing nonlinear beginning and boundary value problems called the homotopy perturbation method by combining the conventional homotopy used in topology with the perturbation approach. Extended one-step time-integration strategies were presented by **Chawla et al. (2000)** for the solution of convection–diffusion equations. Using the Adomian decomposition method, **EL-Wakil and Elhanbaly (2006)** were able to solve the convection-diffusion equation. **Temsah (2009)** developed an El-Gendi method-based steady-state solution for the convection-diffusion equation. The linear and nonlinear reaction–diffusion equations, often known as the Kolmogorov–Petrovsly–Piskunov Equations, were solved by **Kumar and Singh (2010)** using the homotopy perturbation technique (HPM). The sinc-Legendre collocation technique was investigated by **Saadatmandi and colleagues (2012)** for the purpose of solving a class of fractional convection–diffusion equations with variable coefficients. **Appadu (2013)** found a solution to the convection-diffusion equation by employing both conventional and nonconventional finite difference methods. The convection-diffusion equation was solved using a method called variational iteration, which was given by **Olayiwola (2016)**. **Sayevand et al. (2016)** introduced the finite volume element technique for assessing the behaviour of sub-diffusion equations and proved its stability analysis of produced solution. They were able to do this by conducting an analysis of the obtained solution. For the purpose of resolving the time-fractional diffusion problem, **Zhao et al. (2016)** utilized the finite element method. Additionally, they presented an unconditionally stable scheme that was based on the spatial quasi-Wilson nonconforming. The continuation property was utilized by **Mahto et al.**

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(2019) in order to suggest an approximate solution to the sub-diffusion equation through the use of internal control. **Edwan et al. (2021)** provide an alternate method for estimating the space fractional derivative by making use of the fractional Grunwald formula. This method is shown in their work. To solve the convection-diffusion partial differential equations with Neumann's boundary conditions, **Yadav and Kumar (2021)** presented the Haar wavelet collocation method.

II. Solution of one-dimensional convection-diffusion equation-

In order to combine what we've spoken about, we're going to look into two specific situations of the one-dimensional convection diffusion equation, each of which correspond to different physical processes. Let's look at a few different examples and assess whether or not HPM is reliable.

Problem-1: Convection-diffusion equations can be thought of in one-dimensional terms (Burgers equation)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{dx} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0 \qquad (1)$$

With initial condition

$$u(x, 0) = x \qquad (2)$$

Using HPM as a means of resolving the one-dimensional convection diffusion equation (1), we are able to construct the homotopy shown below:

$$\frac{\partial u}{\partial t} - \frac{\partial u_0}{\partial t} = q\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} - u\frac{\partial u}{dx}\right) \qquad (3)$$

as well as some preliminary estimates, which are presented as follows:

$$u_0(x, t) = u(x, 0) = x \qquad (4)$$

Imagine that the answer to equation (3) is given in the following form:

$$u = u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots \qquad (5)$$

It follows that when equation (5) is substituted into equation (3) and the term is equated to the same power of q, it indicates that-

$$\frac{\partial}{\partial t}(u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots) - \frac{\partial u_0}{\partial t} = q\left[\frac{\partial^2}{\partial x^2}(u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots) - \frac{\partial}{\partial t}(u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots) - (u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots)\frac{\partial}{dx}(u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots)\right]$$

$$\frac{\partial}{\partial t}(u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots) - \frac{\partial u_0}{\partial t} = \left[\frac{\partial^2}{\partial x^2}(qu_0 + q^2u_1 + q^3u_2 + q^4u_3 + \cdots) - \frac{\partial}{\partial t}(qu_0 + q^2u_1 + q^3u_2 + q^4u_3 + \cdots) - (qu_0 + q^2u_1 + q^3u_2 + q^4u_3 + \cdots)\frac{\partial}{dx}(u_0 + qu_1 + q^2u_2 + q^3u_3 + \cdots)\right]$$

Coefficient of q^0: $\frac{\partial u_0}{\partial t} - \frac{\partial u_0}{\partial t} = 0$ (6)

Coefficient of q^1: $\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_0}{\partial x^2} - \frac{\partial u_0}{\partial t} - u_0\frac{\partial u_0}{dx}$ (7)

Coefficient of q^2: $\frac{\partial u_2}{\partial t} = \frac{\partial^2 u_1}{\partial x^2} - \frac{\partial u_1}{\partial t} - u_0\frac{\partial u_1}{dx} - u_1\frac{\partial u_0}{dx}$ (8)

Coefficient of q^3: $\frac{\partial u_3}{\partial t} = \frac{\partial^2 u_2}{\partial x^2} - \frac{\partial u_2}{\partial t} - u_0\frac{\partial u_2}{dx} - u_1\frac{\partial u_1}{dx} - u_2\frac{\partial u_0}{dx}$ (9)

By making the decision that $u_0(x,t) = u(x,0) = x$, and then solving the equations that were presented earlier, we are able to achieve the following approximation:

$$u_1 = -xt \tag{10}$$

$$u_2 = x(t+t^2) \tag{11}$$

$$u_3 = -x(t+2t^2) - xt^3 \tag{12}$$

It is possible to derive the answer to equation (1) by substituting the value 1 for q in the equation (5).

$$u = u_0 + u_1 + u_2 + u_3 + \cdots \tag{13}$$

$$u = x - xt + x(t+t^2) - x(t+2t^2) - xt^3$$

$$u(x,t) = \frac{x(1-2t)}{1-t} \tag{14}$$

Problem-2: Consider the Convection-diffusion equation in one dimension (Burgers equation)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{dx} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$$

With initial condition $u(x,0) = x + x^2$ (15)

$$u_0 = x + x^2$$

$$u_1 = (-2x^3 - 3x^2 - x + 2)t \tag{16}$$

$$u_2 = (10x^4 + 20x^3 + 12x^2 - 14x - 8)\frac{t^2}{2} + (2x^3 + 3x^2 + x - 2)\frac{t^3}{6} \tag{17}$$

$$u_3 = \frac{t^3}{6}(-84x^5 - 210x^4 - 180x^3 + 132x^2 + 188x + 36) + \frac{t^4}{24}(-20x^4 - 40x^3 - 24x^2 + 14x + 16) - (2x^3 + 3x^2 + x - 2)\frac{t^5}{120} \tag{18}$$

$$u = u_0 + u_1 + u_2 + u_3 + \cdots$$

$$u = x(1-t)^{-1} + x^2(1-t)^{-3} - 2tx^3(1+t)^{-5} + 5t^2x^4(1+t)^{-3} \tag{19}$$

Problem-3: Consider the Convection-diffusion equation in one dimension (Burgers equation)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{dx} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$$

With initial condition

$$u(x,0) = \frac{-2sinhx}{coshx-1} \tag{20}$$

In a manner analogous to that of issue number one, the exact solution to equation (15) using the homotopy perturbation approach is provided as

$$u(x,t) = \frac{-2sinhx}{coshx-e^{-t}} \tag{21}$$

III. Numerical Illustrations-

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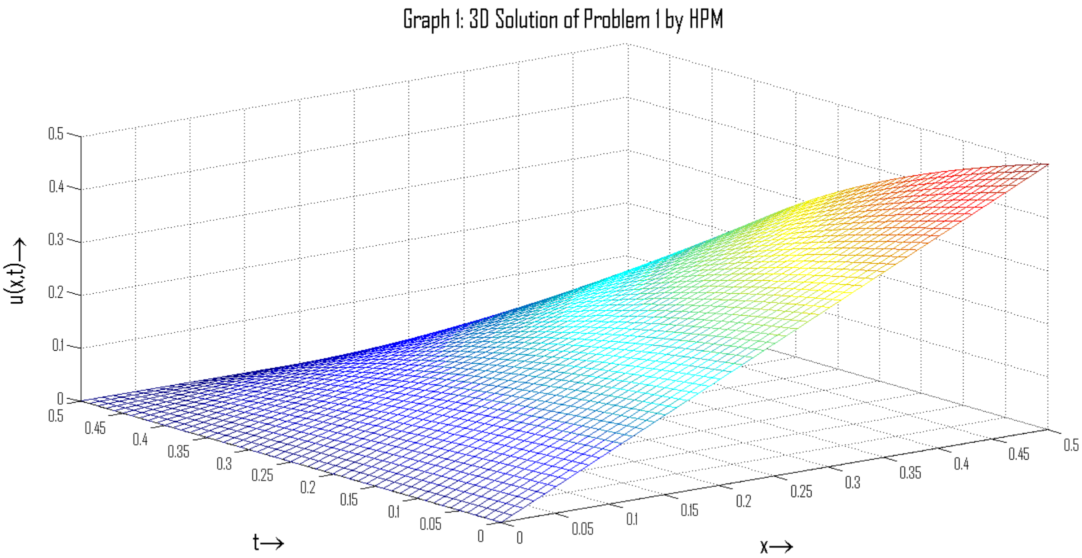

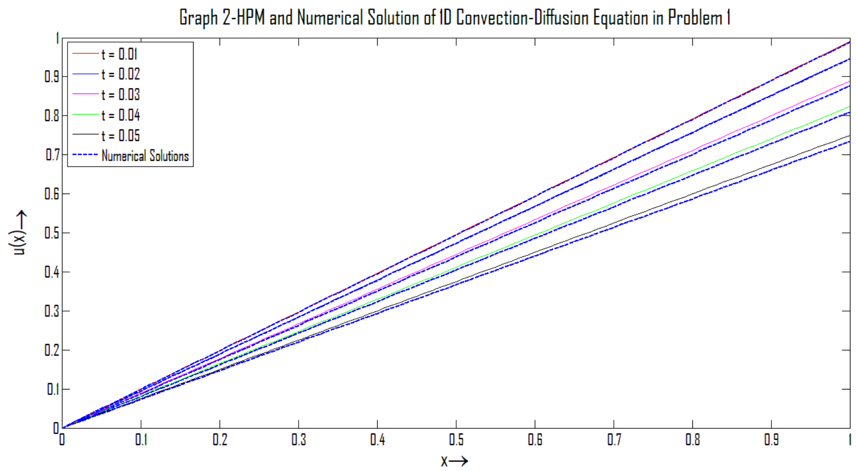

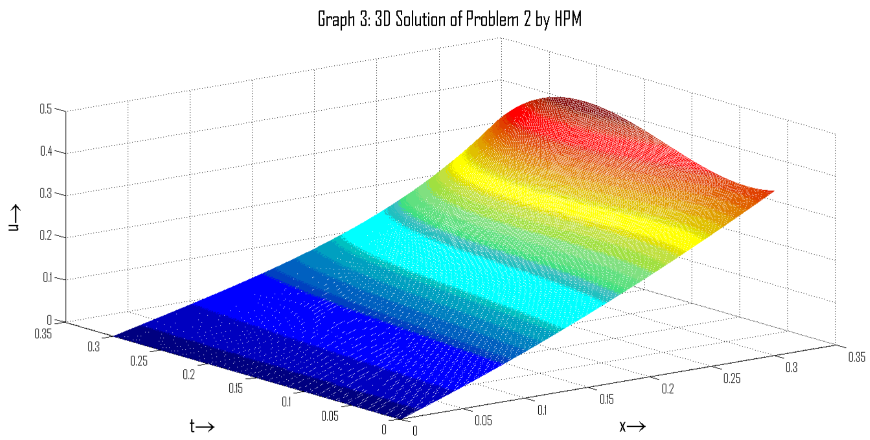

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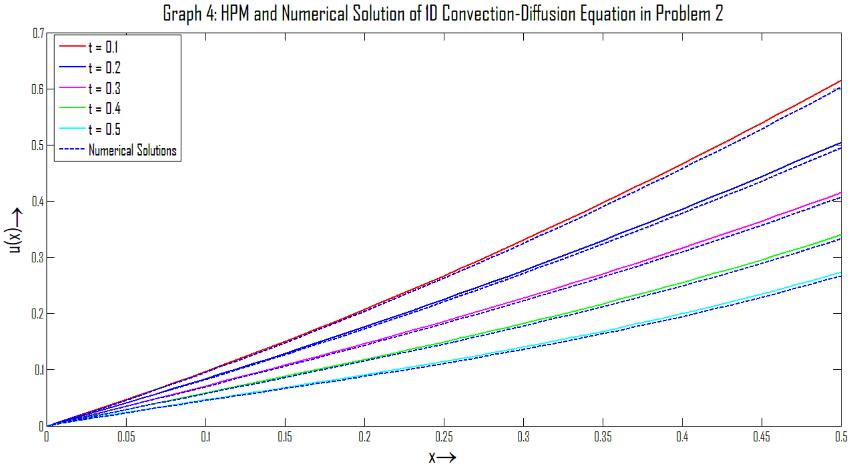
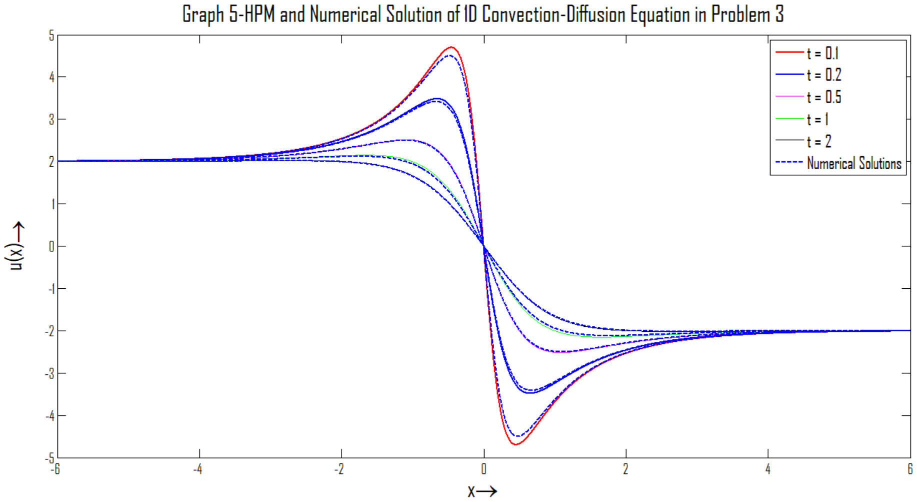
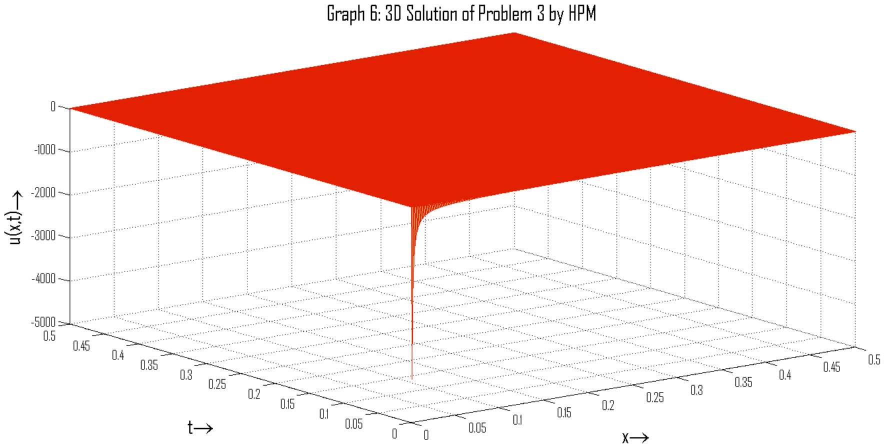
Graph 4: HPM and Numerical Solution of 1D Convection-Diffusion Equation in Problem 2



Graph 5-HPM and Numerical Solution of 1D Convection-Diffusion Equation in Problem 3



Graph 6: 3D Solution of Problem 3 by HPM



IV. Concluding Remarks-

The utilization of HPM in order to find answers to a wide variety of issues has recently attracted a significant amount of attention and interest. It has been demonstrated that the method is effective at solving a wide range of nonlinear problems, including convection–diffusion equations, ordinary differential equations, and partial differential equations, all of which are examples of differential equations. In the fields of physics, chemistry, ecology, biology, and engineering, CD equations can be used to describe a broad variety of nonlinear systems. In this study, the one-dimensional convection diffusion equation was solved using the Homotopy Perturbation Method (HPM), which was a successful approach. All of the examples demonstrate that the outcomes of utilizing this method are extremely congruent with the outcomes of utilising the HPM, and the generated answers are also displayed visually. In this article, the numerical illustrations were done in MATLAB, and the HPM programme was used to generate the analytical results. Every one of the instances was solved using MATLAB and HPM, and the results were compared. In addition, it was discovered that there was no room for error in the process of obtaining the precise solutions by utilizing HPM for any and all of the cases that were presented. It is therefore possible to draw the conclusion that this method is a potent and effective strategy for locating the precise solutions to a large variety of problem classes. This work further demonstrated the validity of the HPM as well as its enormous potential for use in the resolution of non-linear issues arising in the fields of science and engineering. In this context, it is also important to point out that the benefit of using this method is the rapid convergence of the solutions through the utilization of the auxiliary parameter. Furthermore, using this method is simple, and the calculation of successive approximations can be done in a direct and uncomplicated manner.

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