JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

A Sustineri Fabricatio Inventory Exemplar Cum Imperfectus Quality Under Preservation Technology and Investment In Quality Improvement

W. Ritha and R. Saarumathi

Department of Mathematics,
Holy Cross College (Autonomous),
Affiliated to Bharathidasan University,
Tiruchirappalli-620 002,
Tamilnadu, India.

ABSTRACT
Production and inventory management for deteriorating items is homogenizing the environmental concerns with great significance. This paper intricates a sustainable economic production quantity model for improvished deteriorating items that pivot on deteriorating items at a constant rate and the control of deteriorating progress, emission of carbon due to production operations and to benchmark by investing in carbon reduction technology and to enchase the quality of unsaleable manufactured product by investing in quality improvement. The objective of this paper is to review the influence of preservation and carbon reduction technologies on the total profit to succour the administrator, mark maximum structured restocking and pricing decisions using Non-Linear Programming technique.

Keywords Sustainable Economic Production Inventory model (SEPQ) · scrapping, carbon reduction investment · preservation technology · Non-Linear programming technique (NLP).

1 Introduction

One of the most extrusive issue on irretrievable damage to the Earth is the environmental pollution due to global warming. Many efficacious techniques are proposed by many countries to attenuate carbon emissions and develop sustainability. To prevent items from being wasted and control deterioration the temperature and humidity of storages are fluctuated. To ease up the deterioration progress, it is imperative to invest on preservation technologies.
The perfect manufacture of products was achieved by production inventory model. However it is rarely satisfied undeniable product results due to long running time, human errors and total control of the manufacturing process.
In this superintendence existed an inventory model for defective items that inspected to identify the rate of imperfection.
To mitigate the imperfect quality items, quality improvement technologies focussed to help manufacturers to keep away from out-of-control situations that lead to manufacturing imperfect quality items.
To overcome the impact of carbon emissions in the imperfect manufacturing of deteriorating items it is mandatory to work on,

- The formulation of a model for defective item
- Control of carbon emission to develop sustainability
- Effect of quality improvement technologies on total profit

The above mentioned challenges can be achieved through controllable carbon emissions and deterioration of imperfect manufacturing.

2 Literature review

In today's business environment, reverse logistics is preordained owing to product returns, incorrect product delivery, damaged products and product exchange programs. According to Bazan, E., Jaber, El Saadany, A.M., [1], the reverse logistic environs pivot to assess the supply chain environmental imputation. In view of Cao, K., Xu, X., Wu, Q., Zhang, Q.,[2] due to rise in environmental awareness among consumers, many manufactures are progressively endorsing carbon emission reduction technology to generate greener products. The data collection of Halat, K., Hafezalkotob, A., [3] indicates that the carbon emission reduction level increases as the carbon trading price increases whereas it is unconstrained of the unit low carbon subsidy. Kim, M.S., Kim, J.S., Sarkar, B., Sarbar, M., Iqbal, M.W., [4] has manifested that the government and the policy makers set synchronizations to reduce greenhouse gases and carbon foot prints. Lu, C.J., Yang, C.J., Yen, H.F., [5] paper advances on the integrated inventory models with imperfect quality and environmental impact. Mishra, U., Wu, J.Z., Sarkar, B., [6] investigates on the influence of emission costs on the replenishment order sizes and the total profit of a buyer in a wrapped supply process. So, it is inevitable to calculate wrapped items in an integrated inventory model with distribution free approach for lead time demand. Paper presented by Mishra, U., Wu, J.Z., Sarkar, B., [7] anchors on extended environmental sustainability EOQ model to prevail over the constraints of the classical EOQ

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

model and develops a new inventory model with carbon tax policy and imperfect quality items where the buyer exerts power over the supplier.

Moreover the work of Wangsa, I.D., Tiwari, S., Wee, H.M., Reong, S., [8] focusses on carbon emissions caused owing to corporate activities that can be checked through specific capital investment in green technology. Yang, Y., Chi, H., Zhou, W., Fan, T., Piramuthu, S., [9] introduced solution technology of carbon tax and carbon cap policies which give prior importance to reduce global warming through carbon emissions. Furthermore, the extended work of Sarkar, B., Sarkar, M., Ganguly, B., Cardenas-Barron, L.E., [10] engrossed about greenhouse firms and modulate carbon emission constraints to promote sustainable supply chain.

This paper signifies on the sustainable economic production quantity model for improvised deteriorating items at a steady rate and control the same through preservation technology investment considering carbon reduction investment, without considering carbon reduction investment and taking into account both the preservation investment and carbon reduction investment. This paper aims at analyzing the influence of preservation and carbon reduction technologies on the total profit to accommodate the administrator mark maximum structured restocking and pricing decisions using Non Linear programming technique.

3 Mathematical Model

3.1 Assumptions

A single item model is defined when articles are instantaneously refreshed and a single manufacturer ladles out as a single retailer.

A constant deterioration rate for all articles is attained on ignoring the lead time and shortages.

A demand is defined as a decreasing linear function of selling price that is given by, $D(S) = a - bS$

The fraction of carbon reduction is systematized as $L(G) = \lambda(1 - e^{-mG})$ where m is the productivity of carbon reduction technology. $L(G) \to 0$ when $G = 0$ and when $G \to \infty$, $L(G) \to \lambda$. Here $L(G)$ is a continuously differentiable function with regard to the carbon reduction investment G.

To bring down the fraction of defectiveness the capital investment fraction is $\frac{1}{\psi} In\left(\frac{\mu_0}{\mu}\right) T$ for $0 \le \mu \le \mu_0$ where T is the cycle length and ψ is the percentage of drop in defectiveness. The capital investment cost is ordered as $\frac{\omega}{\psi} In\left(\frac{\mu_0}{\mu}\right) T_1$ where ω is the total opportunity cost.

3.2 Notations

S	- Selling price of the item
T	- Cycle length of the production process during the given unit time
P	- Constant rate of production
R	- Rate of rework in unit
S_P	- Cost of set up per cycle during production process
S_R	- Cost of set up per cycle during rework process
C_P	- Production cost per unit cycle
C_D	- Deterioration cost per unit cycle
C_S	- Cost of scrapping per unit cycle
H_P	- Holding cost per unit cycle in the production process
H_R	- Holding cost per unit cycle in the rework process
t_1	- Manufacturer run time
t_2	- Revamp run time in the cycle length T
G	- Investment in green technology
ξ	- Investment in preservation technology

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

θ - Accessible rigid inventory deterioration rate

μ_0 - Proportion of defective items produced during manufacture prior to quality improvement investment

μ_r - Proportion of defective rework growth

ψ - Percentage of reduction in defectiveness after quality development investment

ω - Comprehensive opportunity cost succeeding investment of quality improvement

e_{S_P} - Carbon emission caused due to the set up process at the time of investment

e_{S_R} - Carbon emission caused by the set up process at the time of rework

e_P - Emission of carbon due to production process

e_R - Carbon emission due to rework process

e_H - Carbon emission on account of stock in the warehouse

e_S - Carbon emission in kg per unit from scrap

e_D - Disintegrated carbon emission

λ - Partial carbon emission reduction after investment in green technology

δ - Tax on carbon per unit cycle

μ - Production of defective items produced during manufacture succeeding quality improvement investment

4 Crisp Model

In this model, a sustainable economic production quantity model is promoted where differentiation of serviceable and reworkable periods takes place. Later, pertinent costs like production, holding, setup, defectiveness, reworking and scrapping costs amalogous to the production inventory are defined. In addition to the above process, emission of carbon will be briefed. Based on the receipt of collected from the perfect item sale, the respective total profit function can be attained.

Based on production, rework, defectiveness and rate of demand the inventory level fluctuations can be expressed as the following equations:

Case (i)

In the absence of preservation technology investment $(\xi = 0)$ and in the presence of carbon reduction investment $(G > 0)$, the total profit function is given by,

$$
\begin{aligned}
TC_1 = {} & S(a-bS)(1-\mu) - \frac{1}{T}\Big\{ S_P + S_R + C_P P t_1 + H_P[I_1+I_2+I_3] + H_R[I_4+I_5] + C_D[Pt_1 - (a-bS)T - \\
& R\mu_r(t_2-t_1)] + C_R R(t_2-t_1) + C_S \mu_r R(t_2-t_1) + \frac{\omega}{\psi} In\left(\frac{\mu_0}{\mu}\right) T \Big\} - \frac{\delta}{T}\Big\{ \Big[e_{S_P} + e_{S_R} + e_P P t_1 + \\
& e_H[I_1+I_2+I_3+I_4+I_5] + e_D[Pt_1 - (a-bS)T - R\mu_r(t_2-t_1)] + e_R R(t_2-t_1) + e_S \mu_r R(t_2-t_1)] \\
& [1-\lambda(1-e^{-mG})] \Big] \Big\} - G
\end{aligned}
$$

Case (ii)

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

Considering preservation investment $(\xi > 0)$ and without considering carbon reduction investment $(G = 0)$ the total profit function can be expressed as,

$$TC_2 = S(a-bS)(1-\mu) - \frac{1}{T}\Big\{S_P + S_R + C_P P t_1 + H_P[I_1+I_2+I_3] + H_R[I_4+I_5] + C_D[Pt_1 - (a-bS)T -$$
$$R\mu_r(t_2-t_1)] + C_R R(t_2-t_1) + C_S\mu_r R(t_2-t_1) + \frac{\omega}{\psi} In\left(\frac{\mu_0}{\mu}\right)T\Big\} - \frac{\delta}{T}\Big\{\Big[e_{S_P} + e_{S_R} + e_P P t_1 +$$
$$e_H[I_1+I_2+I_3+I_4+I_5] + e_D[Pt_1 - (a-bS)T - R\mu_r(t_2-t_1)] + e_R R(t_2-t_1) + e_S\mu_r R(t_2-t_1)\Big]\Big\} - \xi$$

Case (iii)

Taking into consideration both the preservation investment $(\xi > 0)$ and carbon reduction investment $(G > 0)$ the developed profit function can be expressed as,

$$TC_3 = S(a-bS)(1-\mu) - \frac{1}{T}\Big\{S_P + S_R + C_P P t_1 + H_P[I_1+I_2+I_3] + H_R[I_4+I_5] + C_D[Pt_1 - (a-bS)T -$$
$$R\mu_r(t_2-t_1)] + C_R R(t_2-t_1) + C_S\mu_r R(t_2-t_1) + \frac{\omega}{\psi} In\left(\frac{\mu_0}{\mu}\right)T\Big\} - \frac{\delta}{T}\Big\{\Big[e_{S_P} + e_{S_R} + e_P P t_1 +$$
$$e_H[I_1+I_2+I_3+I_4+I_5] + e_D[Pt_1 - (a-bS)T - R\mu_r(t_2-t_1)] + e_R R(t_2-t_1) + e_S\mu_r R(t_2-t_1)]$$
$$[1-\lambda(1-e^{-mG})]\Big]\Big\} - G - \xi$$

5 Mathematical Analysis of Fuzzy Non-Linear Programming

The objective of a fuzzy non-linear programming problem with fuzzy resources can be articulated as,

$$Min\, g_0(X)$$
$$s.t\; g_i(x) \le \tilde{a}_i;\; i = 1,2,\ldots m$$
$$g_i(x) \le \tilde{u}_j;\; j = 1,2,\ldots n$$

Linear or Non-linear membership functions, a consequence of fuzzy objective and fuzzy resources evolve μ_0 and $\mu_i; i = 1,2,\ldots.. m$ assumed to be non-increasing continuous linear membership functions can be expressed as,

$$\mu_i(g_i(x)) = \begin{cases} 1, & if\; g_i(x) \le a_i \\ 1 - \dfrac{g_i(x) - a_i}{P_i}, & if\; a_i \le g_i(x) \le a_i + P_i \\ 0, & if\; g_i > a_i + P_i \end{cases} \quad \text{where } i = 1,2,\ldots. m$$

$$\mu_j(g_j(x)) = \begin{cases} 1, & if\; g_j(x) < u_i \\ 1 - \dfrac{g_j(x) - a_j}{P_j}, & if\; u_j \le g_j(x) \le u_j + P_j \\ 0, & if\; g_j > a_j + P_j \end{cases}$$

where $j = 0,1,2\ldots. m$

The concept of max min operator of Bellman and Zimmerman's approach is applied to arrive at the solution. The membership function $\mu_D(X)$ is

$$\mu_D(X) = \min\{\mu_0(X), \mu_1(X), \ldots.\mu_m(X)\}\; \forall x \in X$$

To represent the intersection of the fuzzy sets of objectives and constraints, the min operator is considered as the decision maker expects to have a crisp decision proposal, the maximizing decision relative to the value $X, X_{\max}$ which will have the highest degree of the membership in the decision set,

$$\mu_D(X_{\max}) = \max_{x \ge 0}\{\min\{\mu_0(X), \mu_1(X), \ldots..\mu_m(X)\}\}$$

The above equation is in co-ordinance with the following crisp Non-Linear programming solution.

$$Max\, \alpha$$
$$s.t\; \mu_0(X) \ge \alpha$$
$$\mu_i(x) \ge \alpha;\; i = 1,2,\ldots\ldots m \forall x \ge 0; \alpha \in (0,1)$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

The Lagrangian function $L(a, X, \lambda)$, a new function is formed by introducing $(m+1)$ Lagrangian multipliers $\lambda = (\lambda_0, \lambda_1, ...\lambda_m)$. The required constraint of Kuhn et al. for the optimal solution to this problem alluded that optimal values $x_1^*, x_2^*, ... x_m^*$ and $\lambda_1^*, \lambda_2^*, ... \lambda_n^*$ should satisfy

$$\frac{\partial L}{\partial X_j} = 0;\ j = 1, 2, ...n$$

$$\frac{\partial L}{\partial a} = 0$$

$$\lambda_i (g_i(X) - a_i - (1-a)P_i) = 0$$

$$g_i(X) \le a_i + (1-a)P_i$$

$$\lambda_i \le 0;\ i = 1, 2, ... m$$

Besides, Kuhn Tucker's adequate constraint claims that the objective function for maximization and the constraint should be respectively colane and convex. This articulation focusses that both the objective function and the constraints satisfy the adequate condition.

6 Solution of the Proposed Inventory Model

6.1 In the absence of preservation technology investment $(\xi = 0)$ and in the presence of carbon reduction investment $(G > 0)$

The proposed inventory model is,

$$\begin{aligned}
\tilde{M}in\, TC_1 = {} & S(a-bS)(1-\mu) - \frac{1}{T}\{ S_P + S_R + C_P P t_1 + H_P[I_1 + I_2 + I_3] + H_R[I_4 + I_5] + C_D[Pt_1 - (a-bS)T - \\
& R\mu_r(t_2 - t_1)] + C_R R(t_2 - t_1) + C_S \mu_r R(t_2 - t_1) + \frac{\omega}{\psi} In\left(\frac{\mu_0}{\mu}\right) T \} - \frac{\delta}{T}\{ [\, e_{S_P} + e_{S_R} + e_P P t_1 + \\
& e_H[I_1 + I_2 + I_3 + I_4 + I_5] + e_D[Pt_1 - (a-bS)T - R\mu_r(t_2 - t_1)] + e_R R(t_2 - t_1) + e_S \mu_r R(t_2 - t_1)] \\
& [1 - \lambda(1 - e^{-mG})]\,] \} - G
\end{aligned}$$

$$\begin{aligned}
s.t \quad & \mu \tilde{R} \le \tilde{P} \\
& \tilde{S} \lambda \le T \\
& \forall\, S, P > 0
\end{aligned}$$

which reduces to the following equation

$$Max\ a$$

$$\begin{aligned}
s.t \quad & S(a-bS)(1-\mu) - \frac{1}{T}\{ S_P + S_R + C_P P t_1 + H_P[I_1 + I_2 + I_3] + H_R[I_4 + I_5] + C_D[Pt_1 - (a-bS)T - \\
& R\mu_r(t_2 - t_1)] + C_R R(t_2 - t_1) + C_S \mu_r R(t_2 - t_1) + \frac{\omega}{\psi} In\left(\frac{\mu_0}{\mu}\right) T \} - \frac{\delta}{T}\{ [\, e_{S_P} + e_{S_R} + e_P P t_1 + \\
& e_H[I_1 + I_2 + I_3 + I_4 + I_5] + e_D[Pt_1 - (a-bS)T - R\mu_r(t_2 - t_1)] + e_R R(t_2 - t_1) + e_S \mu_r R(t_2 - t_1)] \\
& [1 - \lambda(1 - e^{-mG})]\,] \} - G \le C_P + (1-\psi) S_P \\
& \mu R \le P + (1-\psi) H_P \\
& S\omega \le T + (1-\psi) e_P \\
& \forall\, S, P > 0\ \&\ \psi \in [0,1]
\end{aligned}$$

The corresponding Lagrangian function is,

$$\begin{aligned}
& L(\psi, S, P, S_P, S_R, C_D, C_R, C_S, H_P, H_R, \lambda_1, \lambda_2, \lambda_3) = \psi - \lambda_1 \{ S(a-bS)(1-\mu) - \frac{1}{T}\{ S_P + S_R + C_P P t_1 + \\
& H_P[I_1 + I_2 + I_3] + H_R[I_4 + I_5] + C_D[Pt_1 - (a-bS)T - R\mu_r(t_2 - t_1)] + C_R R(t_2 - t_1) + C_S \mu_r R(t_2 - t_1) + \\
& \frac{\omega}{\psi} In\left(\frac{\mu_0}{\mu}\right) T \} - \frac{\delta}{T}\{ [\, e_{S_P} + e_{S_R} + e_P P t_1 + e_H[I_1 + I_2 + I_3 + I_4 + I_5] + e_D[Pt_1 - (a-bS)T - R\mu_r(t_2 - t_1)] + \\
& e_R R(t_2 - t_1) + e_S \mu_r R(t_2 - t_1)]\, [1 - \lambda(1 - e^{-mG})]\,] \} - G - C_P - (1-\psi) S_P \} - \lambda_2 \{ \mu_R - P - (1-\psi) H_P \} - \\
& \lambda_3 \{ S\omega - T - (1-\psi) e_P \}
\end{aligned}$$

From the Kuhn-Tucker necessary condition,

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

$$\frac{\partial L}{\partial \psi}=0;\frac{\partial L}{\partial S}=0;\frac{\partial L}{\partial P}=0;\frac{\partial L}{\partial S_P}=0;\frac{\partial L}{\partial S_R}=0;\frac{\partial L}{\partial C_D}=0;$$

$$\frac{\partial L}{\partial C_R}=0;\frac{\partial L}{\partial C_S}=0;\frac{\partial L}{\partial H_P}=0;\frac{\partial L}{\partial H_R}=0;\frac{\partial L}{\partial \lambda_1}=0;\frac{\partial L}{\partial \lambda_2}=0;\frac{\partial L}{\partial \lambda_3}=0$$

$$\frac{\partial L}{\partial \psi}=1-\lambda_1 S_P-\lambda_2 H_P-\lambda_3 e_P \geq 0$$

$$\begin{aligned}\frac{\partial L}{\partial S}=\lambda_1\Big\{&(a-b)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-b)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-b)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}+\omega\lambda_3\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial P}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[t_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}+\lambda_2\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial S_P}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)\Big\}\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial S_R}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial C_D}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial C_R}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial C_S}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial H_P}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}+(1-\psi)\lambda_2\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial H_R}=\lambda_1\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}\leq 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial \lambda_1}=\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}\geq 0\end{aligned}$$

$$\frac{\partial L}{\partial \lambda_2}=\mu R-P-(1-\psi)H_P\geq 0$$

$$\frac{\partial L}{\partial \lambda_3}=S\omega-T-(1-\psi)e_P\geq 0$$

and $\psi(1-\lambda_1 S_P-\lambda_2 H_P-\lambda_3 e_P)=0$

$$\begin{aligned}\lambda_1 S\Big\{&(a-b)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-b)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-b)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}+\omega\lambda_3=0\end{aligned}$$

$$\begin{aligned}\lambda_1 P\Big\{&S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[t_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}+\lambda_2=0\end{aligned}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

$$\begin{gathered}\lambda_1 S_P\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)\Big\}=0\end{gathered}$$

$$\begin{gathered}\lambda_1 S_R\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}=0\end{gathered}$$

$$\begin{gathered}\lambda_1 C_D\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}=0\end{gathered}$$

$$\begin{gathered}\lambda_1 C_R\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}=0\end{gathered}$$

$$\begin{gathered}\lambda_1 C_S\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}=0\end{gathered}$$

$$\begin{gathered}\lambda_1 H_P\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}+(1-\psi)\lambda_2=0\end{gathered}$$

$$\begin{gathered}\lambda_1 H_R\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+[I_4+I_5]+C_D[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}=0\end{gathered}$$

$$\begin{gathered}\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T- \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-C_P-(1-\psi)S_P\Big\}=0\end{gathered}$$

$$\lambda_2\{\mu R-P-(1-\psi)H_P\}=0$$

$$\lambda_3\{S\omega-T-(1-\psi)e_P\}=0$$

$$\begin{gathered}TC_1^*=S^*(a-bS^*)(1-\mu)-\frac{1}{T}\Big\{S_P^*+S_R^*+C_P P^*t_1+H_P^*[I_1+I_2+I_3]+H_R^*[I_4+I_5]+C_D^*[P^*t_1-(a-bS^*)T- \\ R\mu_r(t_2-t_1)]+C_R^*R(t_2-t_1)+C_S^*\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P P^*t_1+ \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[P^*t_1-(a-bS^*)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)] \\ [1-\lambda(1-e^{-mG})]\Big]\Big\}-G\end{gathered}$$

6.2 Considering preservation investment $(\xi > 0)$ and without considering carbon reduction investment $(G = 0)$

The proposed inventory model is,

$$\begin{gathered}\tilde{M}in\,TC_2=S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T- \quad s.t \quad \mu\tilde{R}\le\tilde{P} \\ R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+ \quad \tilde{S}\lambda\le T \\ e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\xi \quad \forall\, S,P>0\end{gathered}$$

which reduces to the following equation

$Max\ \alpha$

Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

$$\begin{aligned}
s.t\quad & S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi\le C_P+(1-\psi)S_P\\
& \mu R\le P+(1-\psi)H_P\\
& S\omega\le T+(1-\psi)e_P\\
& \forall S,P>0\ \&\ \psi\in[0,1]
\end{aligned}$$

The corresponding Lagrangian function is,

$$\begin{aligned}
& L(\psi,S,P,S_P,S_R,C_D,C_R,C_S,H_P,H_R,\lambda_1,\lambda_2,\lambda_3)=\psi-\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+\\
& H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\\
& \frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+\\
& e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\xi-C_P-(1-\psi)S_P\Big\}-\lambda_2\big\{\mu_R-P-(1-\psi)H_P\big\}-\\
& \lambda_3\big\{S\omega-T-(1-\psi)e_P\big\}
\end{aligned}$$

From the Kuhn-Tucker necessary condition

$$\frac{\partial L}{\partial\psi}=0;\frac{\partial L}{\partial S}=0;\frac{\partial L}{\partial P}=0;\frac{\partial L}{\partial S_P}=0;\frac{\partial L}{\partial S_R}=0;\frac{\partial L}{\partial C_D}=0;\frac{\partial L}{\partial C_R}=0;$$

$$\frac{\partial L}{\partial C_S}=0;\frac{\partial L}{\partial H_P}=0;\frac{\partial L}{\partial H_R}=0;\frac{\partial L}{\partial\lambda_1}=0;\frac{\partial L}{\partial\lambda_2}=0;\frac{\partial L}{\partial\lambda_3}=0$$

$$\frac{\partial L}{\partial\psi}=1-\lambda_1S_P-\lambda_2H_P-\lambda_3e_P\ge 0$$

$$\begin{aligned}
& \frac{\partial L}{\partial S}=\lambda_1\Big\{(a-b)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-b)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-b)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}+\omega\lambda_3\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial P}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[t_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}+\lambda_2\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial S_P}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial S_R}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial C_D}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial C_R}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+R(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial C_S}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial H_P}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}+(1-\psi)\lambda_2\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial H_R}=\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial\lambda_1}=\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\Big\}-\\
& \xi-C_P-(1-\psi)S_P\Big\}\ge 0
\end{aligned}$$

$$\frac{\partial L}{\partial\lambda_2}=\mu R-P-(1-\psi)H_P\ge 0$$

$$\frac{\partial L}{\partial\lambda_3}=S\omega-T-(1-\psi)e_P\ge 0$$

and $\psi(1-\lambda_1S_P-\lambda_2H_P-\lambda_3e_P)=0$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

$$\begin{aligned}&\lambda_1 S\Big\{(a-b)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-b)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-b)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}+\omega\lambda_3=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 P\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[t_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}+\lambda_2=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 S_P\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)\Big\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 S_R\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 C_D\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 C_R\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 C_S\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 H_P\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}+(1-\psi)\lambda_2=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 H_R\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\\&\xi-C_P-(1-\psi)S_P\Big\}=0\end{aligned}$$

$$\lambda_2\{\mu R-P-(1-\psi)H_P\}=0$$

$$\lambda_3\{S\omega-T-(1-\psi)e_P\}=0$$

$$\begin{aligned}&TC_2^*=S^*(a-bS^*)(1-\mu)-\frac{1}{T}\Big\{S_P^*+S_R^*+C_P P^*t_1+H_P^*[I_1+I_2+I_3]+H_R^*[I_4+I_5]+C_D^*[P^*t_1-(a-bS^*)T-\\&\quad R\mu_r(t_2-t_1)]+C_R^*R(t_2-t_1)+C_S^*\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P P^*t_1+\\&\quad e_H[I_1+I_2+I_3+I_4+I_5]+e_D[P^*t_1-(a-bS^*)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\Big\}-\xi\end{aligned}$$

6.3 Taking into consideration both the preservation investment $(\xi > 0)$ and carbon reduction investment $(G > 0)$

The proposed inventory model is,

$$\begin{aligned}&\tilde{M}in\,TC_3=S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&\quad R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big]-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_P Pt_1+\\&\quad e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&\quad[1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi\end{aligned}\qquad \begin{aligned}s.t\quad &\mu\tilde{R}\le\tilde{P}\\&\tilde{S}\lambda\le T\\&\forall\, S,P>0\end{aligned}$$

which reduces to the following equation

$Max\ \alpha$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

$$
\begin{aligned}
s.t\quad & S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi\le C_P+(1-\psi)S_P\\
& \mu R\le P+(1-\psi)H_P\\
& S\omega\le T+(1-\psi)e_P\\
& \forall\, S,P>0\ \&\ \psi\in[0,1]
\end{aligned}
$$

The corresponding Lagrangian function is,

$$
\begin{aligned}
& L(\psi,S,P,S_P,S_R,C_D,C_R,C_S,H_P,H_R,\lambda_1,\lambda_2,\lambda_3)=\psi-\lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+\\
& H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\\
& \frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+\\
& e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)][1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}-\lambda_2\{\mu_R-P-(1-\psi)H_P\}-\\
& \lambda_3\{S\omega-T-(1-\psi)e_P\}
\end{aligned}
$$

From the Kuhn – Tucker necessary condition,

$$
\frac{\partial L}{\partial \psi}=0;\frac{\partial L}{\partial S}=0;\frac{\partial L}{\partial P}=0;\frac{\partial L}{\partial S_P}=0;\frac{\partial L}{\partial S_R}=0;\frac{\partial L}{\partial C_D}=0;\frac{\partial L}{\partial C_R}=0;\frac{\partial L}{\partial C_S}=0;
$$

$$
\frac{\partial L}{\partial H_P}=0;\frac{\partial L}{\partial H_R}=0;\frac{\partial L}{\partial \lambda_1}=0;\frac{\partial L}{\partial \lambda_2}=0;\frac{\partial L}{\partial \lambda_3}=0
$$

$$
\frac{\partial L}{\partial \psi}=1-\lambda_1S_P-\lambda_2H_P-\lambda_3e_P\ge 0
$$

$$
\begin{aligned}
\frac{\partial L}{\partial S}=\ & \lambda_1\Big\{(a-b)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-b)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-b)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}+\omega\lambda_3\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial P}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[t_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}+\lambda_2\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial S_P}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial S_R}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial C_D}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial C_R}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+R(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial C_S}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial H_P}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}+(1-\psi)\lambda_2\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial H_R}=\ & \lambda_1\Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}\le 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial \lambda_1}=\ & \Big\{S(a-bS)(1-\mu)-\frac{1}{T}\Big\{S_P+S_R+C_PPt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\
& R\mu_r(t_2-t_1)]+C_RR(t_2-t_1)+C_S\mu_rR(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\Big\}-\frac{\delta}{T}\Big\{\Big[e_{S_P}+e_{S_R}+e_PPt_1+\\
& e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_RR(t_2-t_1)+e_S\mu_rR(t_2-t_1)]\\
& [1-\lambda(1-e^{-mG})]\Big]\Big\}-G-\xi-C_P-(1-\psi)S_P\Big\}\ge 0
\end{aligned}
$$

$$
\frac{\partial L}{\partial \lambda_2}=\mu R-P-(1-\psi)H_P\ge 0
$$

$$
\frac{\partial L}{\partial \lambda_3}=S\omega-T-(1-\psi)e_P\ge 0
$$

and $\psi(1-\lambda_1S_P-\lambda_2H_P-\lambda_3e_P)=0$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

$$\begin{aligned}&\lambda_1 S\{(a-b)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-b)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-b)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}+\omega\lambda_3=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 P\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[t_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}+\lambda_2=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 S_P\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 S_R\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 C_D\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 C_R\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 C_S\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 H_P\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}+(1-\psi)\lambda_2=0\end{aligned}$$

$$\begin{aligned}&\lambda_1 H_R\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}=0\end{aligned}$$

$$\begin{aligned}&\lambda_1\{S(a-bS)(1-\mu)-\frac{1}{T}\{S_P+S_R+C_P Pt_1+H_P[I_1+I_2+I_3]+H_R[I_4+I_5]+C_D[Pt_1-(a-bS)T-\\&R\mu_r(t_2-t_1)]+C_R R(t_2-t_1)+C_S\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P Pt_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[Pt_1-(a-bS)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi-C_P-(1-\psi)S_P\}=0\end{aligned}$$

$$\lambda_2\{\mu R-P-(1-\psi)H_P\}=0$$

$$\lambda_3\{S\omega-T-(1-\psi)e_P\}=0$$

$$\begin{aligned}TC_3^*=\;&S^*(a-bS^*)(1-\mu)-\frac{1}{T}\{S_P^*+S_R^*+C_P P^* t_1+H_P^*[I_1+I_2+I_3]+H_R^*[I_4+I_5]+C_D^*[P^* t_1-(a-bS^*)T-\\&R\mu_r(t_2-t_1)]+C_R^* R(t_2-t_1)+C_S^*\mu_r R(t_2-t_1)+\frac{\omega}{\psi}In\left(\frac{\mu_0}{\mu}\right)T\}-\frac{\delta}{T}\{[e_{S_P}+e_{S_R}+e_P P^* t_1+\\&e_H[I_1+I_2+I_3+I_4+I_5]+e_D[P^* t_1-(a-bS^*)T-R\mu_r(t_2-t_1)]+e_R R(t_2-t_1)+e_S\mu_r R(t_2-t_1)]\\&[1-\lambda(1-e^{-mG})]]\}-G-\xi\end{aligned}$$

7 Numerical Example

7.1 Crisp Model

7.1.1 In the absence of preservation technology investment $(\xi = 0)$ and in the presence of carbon reduction investment $(G > 0)$

T = 8 unit/time; t_2 = 7 unit/time; t_1 = 2.647 unit/time; P = 100 unit; R = 30 unit; S_P = \$300; S_R = \$100; C_P = \$30; C_D = \$0.5; C_R = \$5; C_S = \$4; H_P = \$1; H_R = \$0.5; μ_0 = 0.3; μ = 0.02; μ_r = 0.2; ψ = 2; ω = 200; S = \$5/kg; e_{A_P} = 100 kg/setup; e_{A_R} = 50 kg/setup; e_P = 5 kg/unit; e_R = 20 kg/unit; e_H = 1 kg/unit; e_S = 10 kg/unit; e_D = 3 kg/unit; λ = 0.1; δ = 5; m = 0.2; a = 100; b = 0.2; G = 20.

The total cost TC_1 = Rs.1086317.82

7.1.2 Considering preservation investment $(\xi > 0)$ and without considering carbon reduction investment $(G = 0)$

T = 8 unit/time; t_2 = 7 unit/time; t_1 = 3.280 unit/time; P = 100 unit; R = 30 unit; S_P = \$300; S_R = \$100; C_P = \$30; C_D = \$0.5; C_R = \$5; C_S = \$4; H_P = \$1; H_R = \$0.5; μ_0 = 0.3; μ = 0.02; μ_r = 0.2; ψ = 2; ω = 200; S = \$5/kg;

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

e_P = \$5 kg/unit; e_{A_P} = 100 kg/setup; e_{A_R} = 50 kg/setup; e_R = 20 kg/unit; e_H = 1 kg/unit; e_S = 10 kg/unit; e_D = 3 kg/unit; λ = 0.1; δ = 5; m = 0.2; a = 100; b = 0.2; ξ = 1700

The total cost TC_2 = Rs.1997809.97

7.1.3 Taking into consideration both the preservation investment $(\xi > 0)$ and carbon reduction investment $(G > 0)$

T = 8 unit/time; t_2 = 7 unit/time; t_1 = 3.206 unit/time; P = 100 unit; R = 30 unit; S_P = \$300; S_R = \$100; C_P = \$30; C_D = \$0.5; C_R = \$5; C_S = \$4; H_P = \$1; H_R = \$0.5; μ_0 = 0.3; μ = 0.02; μ_r = 0.2; ψ = 2; ω = 200; S = \$5/kg; e_P = \$5 kg/unit; e_{A_P} = 100 kg/setup; e_{A_R} = 50 kg/setup; e_R = 20 kg/unit; e_H = 1 kg/unit; e_S = 10 kg/unit; e_D = 3 kg/unit; λ = 0.1; δ = 5; m = 0.2; a = 100; b = 0.2; G = 20; ξ = 1600

The total cost TC_3 = Rs.1755724.56

7.2 Fuzzy Model

7.2.1 In the absence of preservation technology investment $(\xi = 0)$ and in the presence of carbon reduction investment $(G > 0)$

T = 8 unit/time; t_2 = 7 unit/time; t_1 = 2.647 unit/time; P = 183.3333 unit; R = 30 unit; S_P = \$508.3333; S_R = \$308.3333; C_P = \$30; C_D = \$0.9167; C_R = \$14.1667; C_S = \$12.3333; H_P = \$1.9167; H_R = \$1.3333; μ_0 = 0.3; μ = 0.02; μ_r = 0.2; ψ = 2; ω = 200; S = \$13.3333/kg; e_{A_P} = 100 kg/setup; e_{A_R} = 50 kg/setup; e_P = 5 kg/unit; e_R = 20 kg/unit; e_H = 1 kg/unit; e_S = 10 kg/unit; e_D = 3 kg/unit; λ = 0.1; δ = 5; m = 0.2; a = 100; b = 0.2; G = 20

The total cost TC_1^* = Rs.876088.9611

7.2.2 Considering preservation investment $(\xi > 0)$ and without considering carbon reduction investment $(G = 0)$

T = 8 unit/time; t_2 = 7 unit/time; t_1 = 3.280 unit/time; P = 86.6667 unit; R = 30 unit; S_P = \$266.6667; S_R = \$66.6667; C_P = \$30; C_D = \$0.4333; C_R = \$3.5333; C_S = \$2.6667; H_P = \$0.8533; H_R = \$0.3667; μ_0 = 0.3; μ = 0.02; μ_r = 0.2; ψ = 2; ω = 200; S = \$3.6667/kg; e_{A_P} = 100 kg/setup; e_{A_R} = 50 kg/setup; e_P = 5 kg/unit; e_R = 20 kg/unit; e_H = 1 kg/unit; e_S = 10 kg/unit; e_D = 3 kg/unit; λ = 0.1; δ = 5; m = 0.2; a = 100; b = 0.2; ξ = 1700

The total cost TC_2^* = Rs.1878968.091

7.2.3 Taking into consideration both the preservation investment $(\xi > 0)$ and carbon reduction investment $(G > 0)$

T = 8 unit/time; t_2 = 7 unit/time; t_1 = 3.206 unit/time; P = 216.6667 unit; R = 30 unit; S_P = \$591.6667; S_R = \$391.6667; C_P = \$30; C_D = \$1.0833; C_R = \$17.8333; C_S = \$15.6667; H_P = \$2.2833; H_R = \$1.6667; μ_0 = 0.3; μ = 0.02; μ_r = 0.2; ψ = 2; ω = 200; S = \$16.6667/kg; e_P = \$5 kg/unit; e_{A_P} = 100 kg/setup; e_{A_R} = 50 kg/setup; e_R = 20 kg/unit; e_H = 1 kg/unit; e_S = 10 kg/unit; e_D = 3 kg/unit; λ = 0.1; δ = 5; m = 0.2; a = 100; b = 0.2; G = 20; ξ = 1600

The total cost TC_3^* = Rs.1578341.598

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2696 - 2708
https://publishoa.com
ISSN: 1309-3452

Conclusion

Environmental protection is the art of conserving natural resources and the existing natural environment and protection from individuals, organizations and governments. Worldwide carbon emissions are constantly increasing with increasing concerns to reduce carbon emission and protecting the environment. Many companies are looking to address these issues caused mainly by the manufacturing process in a production inventory system. Likewise the deterioration of items is yet another confrontation that causes emissions and poses high costs to companies. To formulate these issues, a carbon tax policy and an obligatory investment in quality improvement is achieved through three distinct models to analyse the cases with and without investment in preservation and green technologies. It is vivid that manufacturing, remanufacturing, disposal, warehousing, setup and scrapping operations promote carbon emission. An optimal solution for all the above cases can be attained by investing on the pertinent technology that can benefit the company. So, producers should resolve to accure maximum revenue or promote sustainability. Also, preservation technology leads to an optimal value of the total profit by investigating different technologies. An efficient analysis of deterioration in carbon emission to enhance total profit can be outlined by employing Non-Linear programming technique.

References

Bazan, E., Jaber, El Saadany, A.M., 2015. Carbon emissions and energy effects on manufacturing – remanufacturing inventory models. Comput. Ind. Eng. **88**, 307-316.

Cao, K., Xu, X., Wu, Q., Zhang, Q., 2017. Optimal production and carbon emission reduction level under cap and trade and low carbon subsidy policies. J. Clean. Prod. **167**, 505-513.

Halat, K., Hafezalkotob, A., 2019. Modeling carbon regulation policies in inventory decisions of a multi-stage green supply chain: a game theory approach. Comput. Ind. Eng. **128**, 807-830.

Kim, M.S., Kim, J.S., Sarkar, B., Sarkar, M., Iqbal, M.W., 2018. An improved way to calculate imperfect items during long-run production in an integrated inventory model with backorders. J. Manuf. Syst. **47**, 153-167.

Lu, C.J., Yang, C.T., Yen.H.F. 2020. Stakelberg game approach for sustainable production inventory model with collaborative investment in technology for reducing carbon emissions. J., Clean. Prod. **270**, 121963.

Mishra, U., Wu, J.Z., Sarkar, B., 2020a. A sustainable production inventory model for a controllable carbon emissions rate under shortages. J. Clean. Prod. **256**, 120268.

Mishra, U., Wu, J.Z., Sarkar, B., 2021. Optimum sustainable inventory management with backorder and deterioration under controllable carbon emissions. J. Clean. Prod. **279**, 123699.

Wangsa, I.D., Tiwari, S., Wee, H.M., Reong, S., 2020. A sustainable vendor-buyer inventory system considering transportation, loading and unloading activities. J. Clean. Prod. **271**, 122120.

Yang, Y., Chi, H., Zhou, W., Fan, T., Piramuthu, S., 2020. Deterioration control decision support for perishable inventory management decision support systems, p. 113308.

Sarkar, B., Sarkar, M., Ganguly, B., Cardenas-Barron, L.E., 2021. Combined effects of carbon emission and production quality improvement for fixed lifetime products in a sustainable supply chain management. Int. J. Prod. Econ. **231**, 107867.