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# Scrutinizing Disparate Ranking in Fuzzy Tri-Cum Biserial Bulk Queueing Model Tethered with a Common Server

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**ABSTRACT**

Fuzzy queuing frameworks shed data on openness and fluctuation in each and every domain. In this detailed study, we present a model for assessing system characteristics in a tri-cum biserial bulk queuing systems with a server that exists commonly in an ambiguous environment. The estimates of two distinct ranking prototypes of a given queuing model are compared in this research. For the model's validity, we acquired numerical illustrations in the tabular form, showing that a fuzzy queue can be more realistic than a crisp queue.

**Keywords:** Fuzzy Queuing System · Triangular Fuzzy Number · Trapezoidal Fuzzy Number · Fuzzy Ranking · Geometric Mean · Weighted distance · Bulk Arrival · Significant Measures.

## I Introduction

Queuing theory is an area of mathematics that examines and models people who wait in lines. Queuing system is utilized in an assortment of various day-to-day circumstances, including media transmission frameworks, computer networks, manufacturing firms, traffic control, clinical benefits, organizations, and different fields and claims to fame. Fuzzy set theory is a well-known idea for describing imprecision or uncertainty.

There are enormous applications in which the established model can be successfully utilised. For example, in the gaming club, there are three sections: $Sr_a$, $Sr_b$ and $Sr_c$. These categories contain a variety of games that can only be played in a team. A team must have at least two members and can have any number of players. The team can enter any of the parts and can travel from one to the other at any time. It's also feasible that they enter only one segment before exiting the $Sr_d$ portion. The team's motions can be combined in a variety of ways. As a result, situations may emerge where teams or customers must wait for a long period to receive the information. The study looks into how interlaced methodologies can be used to solve the vexing problem of queuing.

## II Literature Review

A few explores have been directed in the past that investigated the qualities of queuing models. To investigate the queue model, Maggu [11] broke down the different parts of stage type administration queues with two servers in bi-series. The transient way of behaving of a bi-series queuing model with servers in parallel was examined by Singh et al. [12]. Richa [8] presented an overview of phase service queueing models, emphasising how the findings could be applied to real-world scenarios. Kumar et al. [13] clarified different queuing boundaries in a muddled line network with two biserially associated subsystems with a typical server. Chen [10] strengthened his ability to participate the consistent state behaviour of lining frameworks with various batch sizes. In a steady-state scenario, Mittal and Gupta [2] devised a biseries bulk queuing model coupled to a common server. Agrawal & Singh [9] looked into various queue characteristics, utilizing a tri-cum biserial queue architecture connected to a common server. In a tri-cum biserial queuing model, Agrawal et al. [7] explored and made a bulk waiting line architecture.

The current time centres on the advancement of queuing theory applications and the ramifications of fuzzy logic in queuing frameworks. There are assortments of ways that can be used to take care of the issue of fuzzy numbers. Based on Zadeh's expansion rule, fuzzy queues are at first proposed. Various strategies such as maximizing and minimizing sets, sign distance method, centroid method, alpha cuts and defuzzification are used on grading fuzzy numbers. In the unit interval, yager [6] presented a strategy for ordering fuzzy subsets. Wang et al. [15] presented a new method of ranking fuzzy numbers according to their centres. To overcome the drawbacks of the magnitude technique, Asady [1] devised the modified distance minimization method. Qing-song Mao [4] used weighted distance to rank fuzzy numbers. Chutia. R and Chutia. B [5] proposed a new method for ranking parametric forms of fuzzy numbers that include value and ambiguity. Wang [14] have done broad examination on grading triangular and trapezoidal fuzzy numbers using relative

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preference relation. Nalla Veerraju et al. [3] proposed a novel ranking system based on defuzzification, which was inspired by the geometric mean and height notions.

This paper intends to build the tri-cum biserial bulk queuing model tethered with a common server with fixed clump size in fuzzy environment. In the field of queuing, a variety of ranking mechanisms have been detailed in order to improve decision-making in an uncertain circumstance. The two separate ranking prototypes such as weighted distance and geometric mean are utilized in this review for ranking triangular and trapezoidal fuzzy numbers.

## III Preliminaries

Definition: 2.1

Let $L$ be a classical set or a Universe. A fuzzy subset $\tilde{p}$ in $L$ is defined by a membership function $\varphi_{\tilde{p}} : L \to [0,1]$, where $\varphi_{\tilde{p}}(x)$ is the degree of $x$ in $\tilde{p}$ and it is represented as $\tilde{p} = \{ x, \varphi_{\tilde{p}}(x) / x \in L \}$.

Definition: 2.2

A triangular fuzzy number is denoted by an ordered triple as $\tilde{p} = (p,q,r)$ whose membership function $\varphi_{\tilde{p}}(x)$ is given by:

$$\varphi_{\tilde{p}}(x) = \begin{cases} \frac{(x-p)}{(q-p)}, & p \le x \le q \\ \frac{(r-x)}{(r-q)}, & q \le x \le r \\ 0, & otherwise \end{cases}$$

Definition: 2.3

A trapezoidal fuzzy number is denoted by an ordered quadruple as $\tilde{p} = (p,q,r,s)$ whose membership function $\varphi_{\tilde{p}}(x)$ is given by:

$$\varphi_{\tilde{p}}(x) = \begin{cases} \frac{(x-p)}{(q-p)}, & p \le x \le q \\ 1, & q \le x \le r \\ \frac{(s-x)}{(s-r)}, & r \le x \le s \\ 0, & otherwise \end{cases}$$

Definition: 2.4

Height of a fuzzy set $\tilde{p}$ is defined as $H(\tilde{p}) = \max(\varphi_{\tilde{p}}(x))$.

## IV Fuzzy Ranking Prototypes

Ranking based on Weighted distance

The concept of fuzzy distance underpins weighted distance ranking. This ranking index corrects the flaws in prior approaches.

(i) Let $u = (L, M, R)$ represents a triangular fuzzy number. The ranking index of $u$ is defined as $R(u) = (1-q)\left(\frac{2}{3}(M-L)+L\right) + q\left(R+\frac{2}{3}(M-R)\right)$, $q \in (0,1)$.

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(ii) Let $v=(L, M_1, M_2, R)$ denotes the trapezoidal fuzzy number. The ranking index of $v$ is defined as $R(v)=(1-q)\left(\frac{2}{3}(M_1-L)+L\right)+q\left(R+\frac{2}{3}(M_2-R)\right)$, $q\in(0,1)$.

Ranking based on Geometric Mean

To order fuzzy numbers, a new defuzzification technique based on the concepts of geometric mean and height of a fuzzy number is proposed. This method is thought to be concrete and easier to use when ranking fuzzy numbers.

(i) If $\tilde{P}=(p, q, r)$ be a triangular fuzzy number then its defuzzified value is

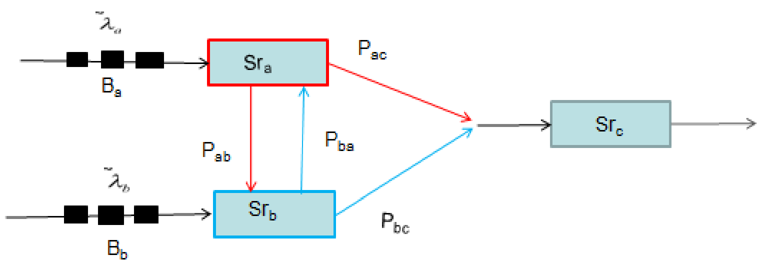
$$D_{\tilde{P}}=\exp\left[\frac{\int_p^q\left(\frac{x-p}{q-p}\right)\ln x\,dx+\int_q^r\left(\frac{r-x}{r-q}\right)\ln x\,dx}{\int_p^q\left(\frac{x-p}{q-p}\right)dx+\int_q^r\left(\frac{r-x}{r-q}\right)dx}\right]\bullet H(\tilde{p})\,\forall\, x>0.$$

(ii) If $\tilde{P}=(p, q, r, s)$ be a trapezoidal fuzzy number then its defuzzified value is

$$D_{\tilde{P}}=\exp\left[\frac{\int_p^q\left(\frac{x-p}{q-p}\right)\ln x\,dx+\int_q^r\ln x\,dx+\int_r^s\left(\frac{s-x}{s-r}\right)\ln x\,dx}{\int_p^q\left(\frac{x-p}{q-p}\right)dx+\int_q^r dx+\int_r^s\left(\frac{s-x}{s-r}\right)dx}\right]\bullet H(\tilde{p})\,\forall\, x>0.$$

## V Description of the Model

In this queuing model, three servers are related in parallel in tri cum biseries way, which might be similarly related with a not unusual place server in series. The queues related to the servers $Sr_a$, $Sr_b$, $Sr_c$ and $Sr_d$ are $Q_a$, $Q_b$, $Q_c$ and $Q_d$ respectively. The customers entered the system with imply fuzzy arrival rates $\tilde{\lambda}_a$, $\tilde{\lambda}_b$ and $\tilde{\lambda}_c$ arrive in batches of constant sizes $B_a$, $B_b$ and $B_c$ comply with the Poisson process and be part of the queues $Q_a$, $Q_b$, $Q_c$ respectively. The customers $n_a$ coming at fuzzy arrival rate $\tilde{\lambda}_a$ after entirety of provider at server $Sr_a$ can use the power to be had at server $Sr_b$ or $Sr_c$ (both or either of two) with the probabilities $p_{ab}$ and $p_{ac}$ or at once can use the power to be had at server $Sr_d$ with the probability $p_{ad}$ to such an quantity that $p_{ab}+p_{ac}+p_{ad}=1$. A comparable criterion will follow to the ones clients who entered in servers $Sr_b$ and $Sr_c$. After availing the provider at server $Sr_d$ the customer is allowed to go out the system. Customers' servicing patterns are assumed to follow the Poisson law with fuzzy rates $\tilde{\mu}_a$, $\tilde{\mu}_b$, $\tilde{\mu}_c$ and $\tilde{\mu}_d$ with $p_{ab}+p_{ac}+p_{ad}=1$, $p_{ba}+p_{bc}+p_{bd}=1$ and $p_{ca}+p_{cb}+p_{cd}=1$.




**Tri-Cum Biserial Queuing Network with Three servers**

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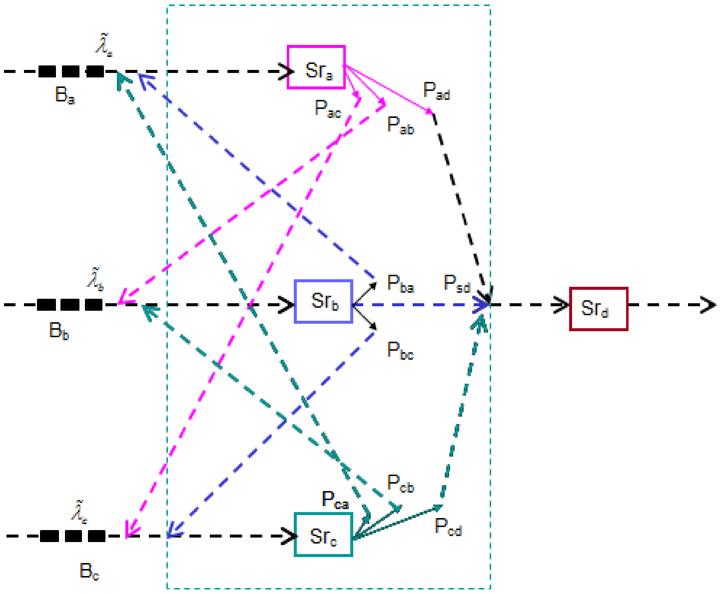


**Tri-Cum Biserial Queuing Network**

Nomenclature

Fuzzy arrival rates: $\tilde{\lambda}_a$, $\tilde{\lambda}_b$, $\tilde{\lambda}_c$

Fuzzy service rates: $\tilde{\mu}_a$, $\tilde{\mu}_b$, $\tilde{\mu}_c$, $\tilde{\mu}_d$

Probabilities: $p_{ab}$, $p_{ac}$, $p_{ad}$, $p_{ba}$, $p_{bc}$, $p_{bd}$, $p_{ca}$, $p_{cb}$, $p_{cd}$

Batch sizes: $B_a$, $B_b$, $B_c$

Queues: $Q_a$, $Q_b$, $Q_c$, $Q_d$

Servers: $Sr_a$, $Sr_b$, $Sr_c$, $Sr_d$

Number of customers: $n_a$, $n_b$, $n_c$, $n_d$

Traffic intensities of servers: $\tilde{\rho}_a$, $\tilde{\rho}_b$, $\tilde{\rho}_c$, $\tilde{\rho}_d$

Mean Queue Length: $L$

Fluctuation in Queue length: $V_{ar}$

Customer's average wait time: $E_{wt}$

Mathematical Analysis

The governing differential equation in steady-state is as follows:

$$\begin{aligned}\left(\tilde{\lambda}_a+\tilde{\lambda}_b+\tilde{\lambda}_c+\tilde{\mu}_a+\tilde{\mu}_b+\tilde{\mu}_c+\tilde{\mu}_d\right)P_{n_a,n_b,n_c,n_d} &= \tilde{\lambda}_a P_{n_a-B_a,n_b,n_c,n_d}+\tilde{\lambda}_b P_{n_a,n_b-B_b,n_c,n_d}+\tilde{\lambda}_c P_{n_a,n_b,n_c-B_c,n_d}\\ &+\tilde{\mu}_a p_{ab}P_{n_a+1,n_b-1,n_c,n_d}+\tilde{\mu}_a p_{ac}P_{n_a+1,n_b,n_c-1,n_d}\\ &+\tilde{\mu}_a p_{ad}P_{n_a+1,n_b,n_c,n_d-1}+\tilde{\mu}_b p_{ba}P_{n_a-1,n_b+1,n_c,n_d}\\ &+\tilde{\mu}_b p_{bc}P_{n_a,n_b+1,n_c-1,n_d}+\tilde{\mu}_b p_{bd}P_{n_a,n_b+1,n_c,n_d-1}\\ &+\tilde{\mu}_c p_{cb}P_{n_a,n_b-1,n_c+1,n_d}+\tilde{\mu}_c p_{ca}P_{n_a-1,n_b,n_c+1,n_d}\\ &+\tilde{\mu}_c p_{cd}P_{n_a,n_b,n_c+1,n_d-1}+\tilde{\mu}_d P_{n_a,n_b,n_c,n_d+1}\end{aligned} \quad \text{... (1)}$$

To solve the governing equation, partial generating functions are considered as

$$g(z_1,z_2,z_3,z_4)=\sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty}\sum_{n_c=0}^{\infty}\sum_{n_d=0}^{\infty}P_{n_a,n_b,n_c,n_d}\ z_1^{n_a}z_2^{n_b}z_3^{n_c}z_4^{n_d} \quad \text{... (2)}$$

Such that $|z_1|\le 1, |z_2|\le 1, |z_3|\le 1, |z_4|\le 1$

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$$g_{n_b,n_c,n_d}(z_1)=\sum_{n_a=0}^{\infty}P_{n_a,n_b,n_c,n_d}\ z_1^{n_a}$$

$$g_{n_c,n_d}(z_1,z_2)=\sum_{n_b=0}^{\infty}g_{n_b,n_c,n_d}(z_1)\cdot z_2^{n_b}$$

$$g_{n_d}(z_1,z_2,z_3)=\sum_{n_c=0}^{\infty}g_{n_c,n_d}(z_1,z_2)\cdot z_3^{n_c}$$

$$g(z_1,z_2,z_3,z_4)=\sum_{n_d=0}^{\infty}g_{n_d}(z_1,z_2,z_3)\cdot z_4^{n_d}$$

Consider $g(z_1,z_2,z_3,z_4)=\dfrac{\theta}{\omega}$ ... (3)

$$\theta=\tilde{\mu}_a\left\{1-\frac{p_{ab}z_2}{z_1}-\frac{p_{ac}z_3}{z_1}-\frac{p_{ad}z_4}{z_1}\right\}g_a+\tilde{\mu}_b\left\{1-\frac{p_{ba}z_1}{z_2}-\frac{p_{bc}z_3}{z_2}-\frac{p_{bd}z_4}{z_2}\right\}g_b$$
$$+\tilde{\mu}_c\left\{1-\frac{p_{ca}z_1}{z_3}-\frac{p_{cb}z_2}{z_3}-\frac{p_{cd}z_4}{z_3}\right\}g_c+\tilde{\mu}_d\left\{1-\frac{1}{z_4}\right\}g_d$$

And

$$\omega=\tilde{\lambda}_a\left(1-z_1^{B_a}\right)+\tilde{\lambda}_b\left(1-z_2^{B_b}\right)+\tilde{\lambda}_c\left(1-z_3^{B_c}\right)+\tilde{\mu}_a\left\{1-\frac{p_{ab}z_2}{z_1}-\frac{p_{ac}z_3}{z_1}-\frac{p_{ad}z_4}{z_1}\right\}$$
$$+\tilde{\mu}_b\left\{1-\frac{p_{ba}z_1}{z_2}-\frac{p_{bc}z_3}{z_2}-\frac{p_{bd}z_4}{z_2}\right\}+\tilde{\mu}_c\left\{1-\frac{p_{ca}z_1}{z_3}-\frac{p_{cb}z_2}{z_3}-\frac{p_{cd}z_4}{z_3}\right\}+\tilde{\mu}_d\left\{1-\frac{1}{z_4}\right\}$$

Assuming $g(z_2,z_3,z_4)=g_a$, $g(z_1,z_3,z_4)=g_b$, $g(z_1,z_2,z_4)=g_c$, $g(z_1,z_2,z_3)=g_d$

Since $g(1,1,1,1)=1$. Let us contemplate $z_1=1$ as $z_2\to1, z_3\to1, z_4\to1$

$$\tilde{\mu}_a g_a-\tilde{\mu}_b p_{ba}g_b-\tilde{\mu}_c p_{ca}g_c=-\tilde{\lambda}_a B_a+\tilde{\mu}_a-\tilde{\mu}_b p_{ba}-\tilde{\mu}_c p_{ca} \quad \text{... (4)}$$

Again differentiating numerator and denominator of equation (3) separately with respect to $z_2$ by considering $z_2=1$ as $z_1\to1,\ z_3\to1,\ z_4\to1$ we get,

$$-\tilde{\mu}_a p_{ab}g_a+\tilde{\mu}_b g_b-\tilde{\mu}_c p_{cb}g_c=-\tilde{\lambda}_b B_b-\tilde{\mu}_a p_{ab}+\tilde{\mu}_b-\tilde{\mu}_c p_{cb} \quad \text{... (5)}$$

Again differentiating numerator and denominator of equation (3) independently with respect to $z_3$ by taking $z_3=1$ as $z_1\to1,\ z_2\to1,\ z_4\to1$ we get,

$$-\tilde{\mu}_a p_{ac}g_a-\tilde{\mu}_b p_{bc}g_b+\tilde{\mu}_c g_c=-\tilde{\lambda}_c B_c-\tilde{\mu}_a p_{ac}-\tilde{\mu}_b p_{bc}+\tilde{\mu}_c \quad \text{... (6)}$$

Again differentiating numerator and denominator of equation (3) separately with respect to $z_4$ by taking $z_4=1$ as $z_1\to1,\ z_2\to1,\ z_3\to1$ we get,

$$-\tilde{\mu}_a p_{ad}g_a-\tilde{\mu}_b p_{bd}g_b-\tilde{\mu}_c p_{cd}g_c+\tilde{\mu}_d g_d=-\tilde{\mu}_a p_{ad}-\tilde{\mu}_b p_{bd}-\tilde{\mu}_c p_{cd}+\tilde{\mu}_d \quad \text{... (7)}$$

Solving equations (4)-(7), we get the traffic intensities of servers $\tilde{\rho}_a$, $\tilde{\rho}_b$, $\tilde{\rho}_c$ and $\tilde{\rho}_d$.

The solution of the queuing model in steady-state is represented as

$$P_{n_a,n_b,n_c,n_d}=\tilde{\rho}_a^{n_a}\tilde{\rho}_b^{n_b}\tilde{\rho}_c^{n_c}\tilde{\rho}_d^{n_d}\left(1-\tilde{\rho}_a\right)\left(1-\tilde{\rho}_b\right)\left(1-\tilde{\rho}_c\right)\left(1-\tilde{\rho}_d\right)$$

**Queue Characteristics**

1. Traffic Intensities or Utilization of server

$$\tilde{\rho}_a=\frac{\tilde{\lambda}_a B_a\left(1-p_{bc}p_{cb}\right)+\tilde{\lambda}_b B_b\left(p_{ba}+p_{bc}p_{ca}\right)+\tilde{\lambda}_c B_c\left\{p_{ca}\left(1-p_{bc}p_{cb}\right)+p_{cb}\left(p_{ba}+p_{bc}p_{ca}\right)\right\}}{\tilde{\mu}_a\left\{\left(1-p_{ac}p_{ca}\right)\left(1-p_{bc}p_{cb}\right)-\left(p_{ab}+p_{ac}p_{cb}\right)\left(p_{ba}+p_{bc}p_{ca}\right)\right\}}$$

$$\tilde{\rho}_b=\frac{\tilde{\lambda}_a B_a\left\{p_{ab}\left(1-p_{ca}p_{ac}\right)+p_{ac}\left(p_{cb}+p_{ca}p_{ab}\right)\right\}+\tilde{\lambda}_b B_b\left(1-p_{ca}p_{ac}\right)+\tilde{\lambda}_c B_c\left(p_{cb}+p_{ca}p_{ab}\right)}{\tilde{\mu}_b\left\{\left(1-p_{ba}p_{ab}\right)\left(1-p_{ca}p_{ac}\right)-\left(p_{bc}+p_{ba}p_{ac}\right)\left(p_{cb}+p_{ca}p_{ab}\right)\right\}}$$

$$\tilde{\rho}_c=\frac{\tilde{\lambda}_a B_a\left(p_{ac}+p_{ab}p_{bc}\right)+\tilde{\lambda}_b B_b\left\{p_{bc}\left(1-p_{ab}p_{ba}\right)+p_{ba}\left(p_{ac}+p_{ab}p_{bc}\right)\right\}+\tilde{\lambda}_c B_c\left(1-p_{ab}p_{ba}\right)}{\tilde{\mu}_c\left\{\left(1-p_{cb}p_{bc}\right)\left(1-p_{ab}p_{ba}\right)-\left(p_{ca}+p_{cb}p_{ba}\right)\left(p_{ac}+p_{ab}p_{bc}\right)\right\}}$$

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$$\tilde{\rho}_d = \frac{p_{ad}}{\tilde{\mu}_d}\tilde{\rho}_a\tilde{\mu}_a + \frac{p_{bd}}{\tilde{\mu}_d}\tilde{\rho}_b\tilde{\mu}_b + \frac{p_{cd}}{\tilde{\mu}_d}\tilde{\rho}_c\tilde{\mu}_c$$

where $\tilde{\rho}_a = 1 - g_a$, $\tilde{\rho}_b = 1 - g_b$, $\tilde{\rho}_c = 1 - g_c$, $\tilde{\rho}_d = 1 - g_d$

The solution exists if $\tilde{\rho}_a, \tilde{\rho}_b, \tilde{\rho}_c, \tilde{\rho}_d < 1$

2. Mean Queue Length

$$L = \frac{\tilde{\rho}_a}{(1-\tilde{\rho}_a)} + \frac{\tilde{\rho}_b}{(1-\tilde{\rho}_b)} + \frac{\tilde{\rho}_c}{(1-\tilde{\rho}_c)} + \frac{\tilde{\rho}_d}{(1-\tilde{\rho}_d)}$$

3. Fluctuation in queue length

$$V_{ar} = \frac{\tilde{\rho}_a}{(1-\tilde{\rho}_a)^2} + \frac{\tilde{\rho}_b}{(1-\tilde{\rho}_b)^2} + \frac{\tilde{\rho}_c}{(1-\tilde{\rho}_c)^2} + \frac{\tilde{\rho}_d}{(1-\tilde{\rho}_d)^2}$$

4. Customers' average wait time

$$E_{wt} = \frac{L}{\lambda_{sum}}, \text{ where } \lambda_{sum} = \tilde{\lambda}_a + \tilde{\lambda}_b + \tilde{\lambda}_c$$

## VI Numerical Illustration

Let us contemplate the arrival rate and service rate are fuzzy numbers. The details of the various input parameters that were used to calculate the various queuing performance measures are shown in the table below.

Table 1 Input Parameters

| $p_{ab}$ | $p_{ac}$ | $p_{ad}$ | $p_{ba}$ | $p_{bc}$ | $p_{bd}$ | $p_{ca}$ | $p_{cb}$ | $p_{cd}$ |
|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.3 | 0.4 | 0.3 | 0.3 | 0.4 | 0.3 | 0.3 | 0.4 |

For Triangular fuzzy number

Let $\tilde{\lambda}_a = [1,2,3]$; $\tilde{\lambda}_b = [1,3,5]$; $\tilde{\lambda}_c = [2,4,6]$; $\tilde{\mu}_a = [26,28,30]$; $\tilde{\mu}_b = [27,29,31]$; $\tilde{\mu}_c = [30,31,32]$ and $\tilde{\mu}_d = [30,32,34]$.

Table 2 Ranking Triangular Fuzzy Numbers

| **Fuzzy numbers** | **Ranking Index** | |
|---|---|---|
| | **Weighted Distance** | **Geometric Mean** |
| $\tilde{\lambda}_a = [1,2,3]$ | 2 | 1.9562 |
| $\tilde{\lambda}_b = [1,3,5]$ | 3 | 2.8797 |
| $\tilde{\lambda}_c = [2,4,6]$ | 4 | 3.9131 |
| $\tilde{\mu}_a = [26,28,30]$ | 28 | 27.9915 |
| $\tilde{\mu}_b = [27,29,31]$ | 29 | 28.9798 |
| $\tilde{\mu}_c = [30,31,32]$ | 31 | 30.9385 |
| $\tilde{\mu}_d = [30,32,34]$ | 32 | 31.9893 |

We compute the performance measures of given described fuzzy queuing model and tabulate the values with respect to different combinations of batch size using different ranking prototypes of triangular fuzzy number.

Table 3 Performance Measures Vs Various Batch Sizes

| **Batch Size** | | | **Performance Measures** | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | **Weighted Distance** | | | **Geometric Mean** | | |
| $B_a$ | $B_b$ | $B_c$ | $L$ | $V_{ar}$ | $E_{wt}$ | $L$ | $V_{ar}$ | $E_{wt}$ |
| 3 | 2 | 2 | 5.6169 | 13.6019 | 0.6241 | 5.2707 | 12.2958 | 0.6024 |
| 2 | 3 | 2 | 6.4419 | 17.2069 | 0.7158 | 5.9616 | 15.1386 | 0.6814 |
| 2 | 2 | 3 | 7.4570 | 22.2256 | 0.8286 | 6.8534 | 19.3274 | 0.7833 |

The pictorial representation of the queue characteristics is demonstrated in the following figures.

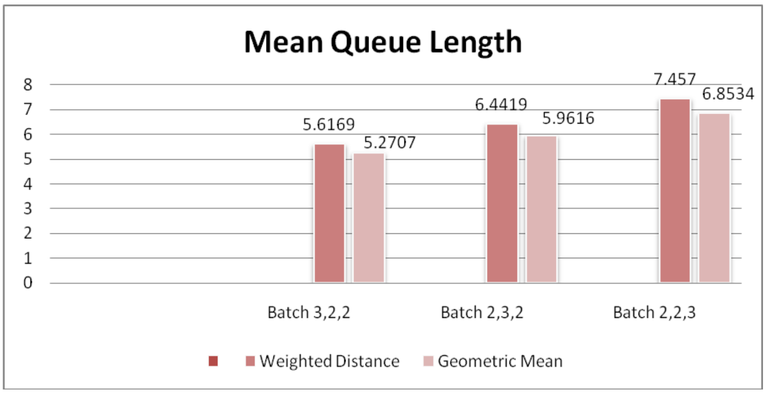


Fig. 1 Graphical representation of queue length

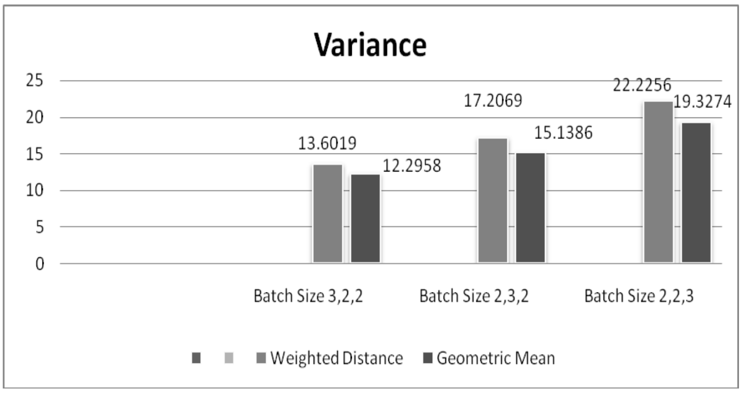


Fig. 2 Graphical representation of variance

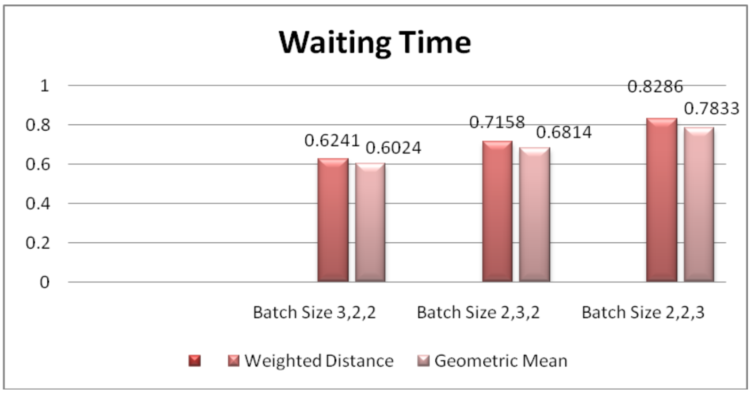


Fig. 3 Graphical representation of waiting time

For Trapezoidal fuzzy number

Let $\tilde{\lambda}_a = [1,2,3,4]$; $\tilde{\lambda}_b = [1,3,5,7]$; $\tilde{\lambda}_c = [2,4,6,8]$; $\tilde{\mu}_a = [25,27,29,31]$; $\tilde{\mu}_b = [28.5,29.5, 30.5,31.5]$; $\tilde{\mu}_c = [30.5,31.5,32.5,33.5]$ and $\tilde{\mu}_d = [31,33,35,37]$.

Table 4 Ranking Trapezoidal Fuzzy Numbers

| **Fuzzy numbers** | **Ranking Index** | |
|---|---|---|
| | **Weighted Distance** | **Geometric Mean** |
| $\tilde{\lambda}_a = [1,2,3,4]$ | 2.5 | 2.4114 |
| $\tilde{\lambda}_b = [1,3,5,7]$ | 4 | 3.7685 |
| $\tilde{\lambda}_c = [2,4,6,8]$ | 5 | 4.8225 |
| $\tilde{\mu}_a = [25,27,29,31]$ | 28 | 27.9691 |

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| | | |
|---|---|---|
| $\tilde{\mu}_b = [28.5, 29.5, 30.5, 31.5]$ | 30 | 29.9926 |
| $\tilde{\mu}_c = [30.5, 31.5, 32.5, 33.5]$ | 32 | 31.9989 |
| $\tilde{\mu}_d = [31, 33, 35, 37]$ | 34 | 33.9843 |

Table 5 Performance Measures Vs Various Batch Sizes

| **Batch Size** | | | **Performance Measures** | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | **Weighted Distance** | | | **Geometric Mean** | | |
| $B_a$ | $B_b$ | $B_c$ | $L$ | $V_{ar}$ | $E_{wt}$ | $L$ | $V_{ar}$ | $E_{wt}$ |
| 3 | 2 | 2 | 10.3445 | 37.4431 | 0.8995 | 8.9320 | 29.0998 | 0.8118 |
| 2 | 3 | 2 | 13.5642 | 63.4937 | 1.1795 | 11.0793 | 43.5575 | 1.0070 |
| 2 | 2 | 3 | 16.4955 | 94.2638 | 1.4344 | 13.3077 | 62.4357 | 1.2095 |

Utilizing multiple ranking methods of trapezoidal fuzzy numbers, we find out the significant measures of the stated fuzzy queuing model with respect to various permutations of batch size. With regard to the data in the preceding tables, it is clear that the queue length, variances as well as customer's average wait time increases for three different combinations of batch sizes of $B_a$, $B_b$ and $B_c$. The pictorial illustration of the queue traits is confirmed within side the following figures.

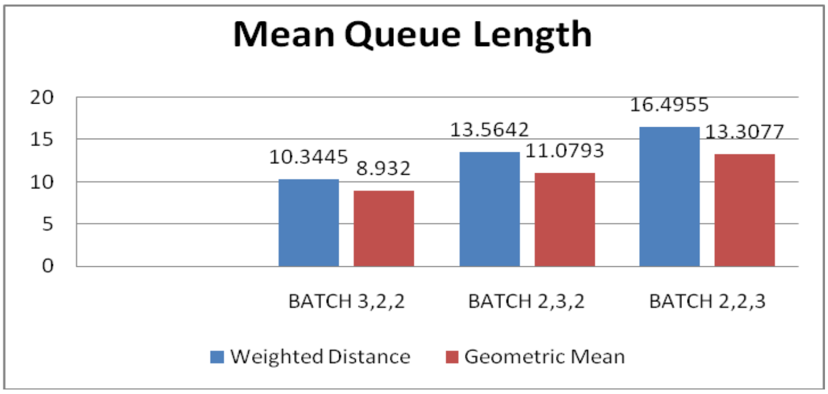


Fig. 4 Graphical representation of queue length

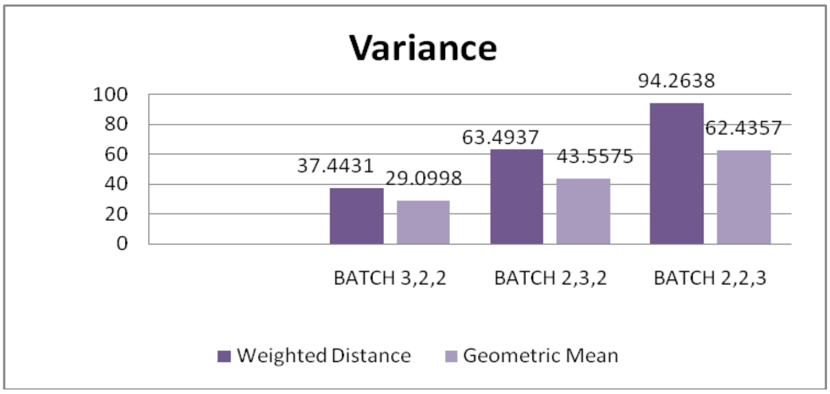


Fig. 5 Graphical representation of variance

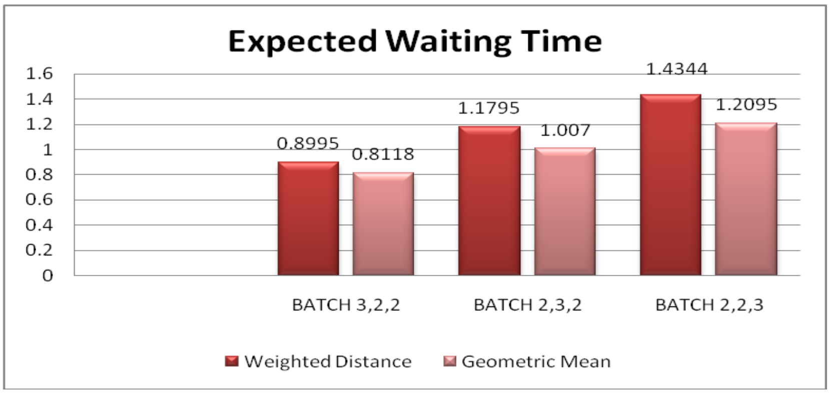


Fig. 6 Graphical representation of waiting time

## VII Conclusion

Fuzzy queuing model has attracted a huge load of thought due to its utilitarian applications, in reality. Two alternative fuzzy rankings, such as weighted distance and geometric mean, can be used to provide significant estimations of a tri-cum biserial fuzzy bulk queuing model linked with a common server. The two separate ranking prototypes are examined in this review to observe crucial measures that indicate whether one leads to better outcomes. It likewise gives context for comparing fuzzy numbers and assists the decision maker in selecting the most appropriate fuzzy ranking.

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