JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2641 - 2646
https://publishoa.com
ISSN: 1309-3452

An Impact of Reneging, Server Breakdowns and Server Vacations of Fuzzy Queuing System Under Trisectional Fuzzification Approach

P. Yasodai[1] · W. Ritha[2]

ABSTRACT
Fuzzy queues are used in a variety of real-world applications, such as production systems, machines, service stations and computer networks. This paper examines the impacts of reneging, server breakdowns and server vacation on the various states of a batch arrivals fuzzy queuing system with a single server providing service to customers in three different fluctuating modes. To convert fuzzy rates into crisp numbers, we use the trisectional fuzzy trapezoidal approach. The system performance measures are calculated based on different values of α. In order to demonstrate the validity of the proposed method, a numerical example is solved successfully.

Keywords Fuzzy Queuing System · Trapezoidal Fuzzy Number · Batch Arrival · Reneging · Server Vacation · Server Breakdowns · Fluctuating Modes of Service · Trisectional Fuzzy · Trapezoidal Approach.

I Introduction

Fuzzy queuing structures cast light upon accessibility and heterogeneity in each and every domain. Queues and congestion are a natural occurrence that we come across in all the corners**.** Waiting line techniques form an integral part in the fields of operations research, transmission and interconnection frameworks, which was embedded in the twentieth century spread fleetly in all areas which latterly on flourished with constraints to apply randomness with a query in all possibilities to produce invention with optimum mileage.

Queuing models marked famed connection in real-time systems. The queuing techniques were largely delved by multitudinous investigators and have extended its mileage on uncertain grounds. If the server is busy, on vacation or undergoing maintenance, any batch arriving joins the queue. If the server is available, one customer from the approaching batch is able to use the service right away, while others must wait in line. In the event of a server outage, the customer whose service is disrupted is returned to the front of the queue. As soon as the server is repaired, it returns to mode 1 and attends to the customer. Customers that are impatient due to server downtime or repair may opt out of this queuing mechanism. The system may provide service with complete or reduced efficiency due to varying modes of service delivery.

II Literature Review

A batch arrival queuing system with two varying modes of services has been proposed by M. Baruah, K.C. Madan, and T. Eldabi [7]. Madan [5] investigated a batch arrival single server queuing system for general service in three varying service modes. Single server queue with working vacations was discussed by L.D. Servi and S.G. Finn [6]. Latterly, D. Wu and H. Takagi [4] extended general queue with multiple working vacations. Aside from fluctuating modes of service delivery, a queuing system may suffer a sudden breakdown, resulting in a stoppage until the machine is repaired. In this case, the customer whose service has been interrupted returns to the front of the line and waits for the repair process to be completed. Queue with server breakdowns has been proposed by Y. Tang [12] and W. J. Gray et. al [10]. Researchers such as R. F. Khalaf, K.C. Madan and C.A. Lukas [8] have studied batch arrival queues with server breakdowns. Customers who have waited in line for longer than expected frequently become irritated and depart without receiving service. Reneging is a term used to describe such behaviour. A. Choudhury and P. Medhi [1] presented a research on Multiserver Markovian Queuing Systems with Balking and Reneging. The effects of reneging, server breakdowns, and vacation on a batch arrival single Server queuing system with three fluctuating modes of service are devised by Samuel Ugochukwu Enogwe et al. [9].

Numerous investigators have probed into the queuing methods and have extended their distance on shaky basis. The queuing system in a fuzzy environment was proposed by well-known scholars such as Lofti A. Zadeh, R.J. Li and E.S. Lee, J.J. Buckley, R.S. Negi and E.S. Lee, and S.P. Chen. Y. J. Wang and H. S. Lee [11] suggested a new Method of Ranking Fuzzy Numbers with an Area Between the Centroid and Original Points. C.H. Cheng [2]

[1] Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli-620 002, Tamilnadu, India.
[2] Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli-620 002, Tamilnadu, India.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2641 - 2646
https://publishoa.com
ISSN: 1309-3452

have done substantial study on grading fuzzy numbers by distance method. C. S. Dinesh Bisht and Pankaj Kumar Srivastava [3] investigated how to optimise interval data using a trisectional fuzzy trapezoidal approach.

This work aims to construct a batch arrival queuing model in a fuzzy environment with three varying modes of service, highlighting the effects of reneging, server outages, and vacation. The trapezoidal fuzzy number is converted into a crisp number using a trisectional fuzzy trapezoidal technique.

Trapezoidal fuzzy number

A trapezoidal fuzzy number given by $\tilde{C} = (c_1, c_2, c_3, c_4)$ has a membership function

$$\varphi_c(x) = \begin{cases} \frac{x-c_1}{c_2-c_1}, & c_1 \le x \le c_2 \\ 1 \quad, & c_2 \le x \le c_3 \\ \frac{x-c_4}{c_3-c_4}, & c_3 \le x \le c_4 \\ 0 \quad, & \text{otherwise} \end{cases}$$

III Proposed Trisectional fuzzy trapezoidal approach

To fuzzify the given interval data to a trapezoidal fuzzy number, the proposed trisectional technique is applied. Consider an interval data (L, R). The trisection of this interval is taken as $d = \frac{(R-L)}{3}$. The required trapezoidal fuzzy number will be $(L, L+d, L+2d, R)$.

Consider the fuzzy number $\tilde{P} = (a_1, a_2, a_3, a_4)$, which is a normal trapezoidal fuzzy number as represented in Fig. 1. Extend the line that connects $Q(a_1, 0)$ and $S(a_2, 1)$ as QSP and the line connects $R(a_4, 0)$ and $T(a_3, 1)$ as RTP. P is the point where the extended lines QS and RT meet. The coordinates for intersection point *P* is (*x*, *y*), where

$$x = \frac{a_1 a_3 - a_2 a_4}{a_3 - a_4 - a_2 + a_1}, \quad y = \frac{x - a_2}{a_2 - a_1} + 1$$

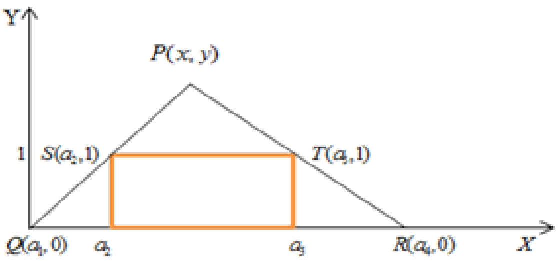

Fig. 1

Normal trapezoidal fuzzy number

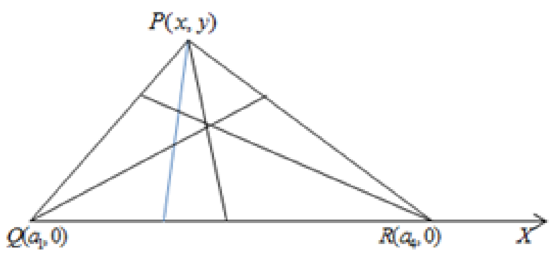

Fig. 2. In-Centre

The proposed ranking technique is based on the concept of the centre of the in circle of a triangle (in-centre) PQR (Fig. 2) and is defined as

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2641 - 2646
https://publishoa.com
ISSN: 1309-3452

$$R(\tilde{P}) = \frac{ax + ba_1 + ca_4}{a + b + c},$$

$$a = a_4 - a_1$$

$$b = \sqrt{y^2 + (a_4 - x)^2}$$

$$c = \sqrt{y^2 + (x - a_1)^2}$$

IV Model Description

We consider a batch arrival single server fuzzy queuing system with three service modes that fluctuate. In the queuing literature, there is no model that incorporates batch arrivals, three variable modes of service, server breakdown, vacation and reneging. This type of model could be useful in a production line when raw materials are arrived in batches of varying sizes rather than as single units. The machine that makes an item may require three different service modes, such as quick, normal, or slow. Also, due to mechanical or job-related issues, the machine producing items may unexpectedly break down, and the operation is halted either for preparatory checks of raw materials or for maintenance or repair. Customers who are dissatisfied with the wastage may quit the queue during periods of server breakdown or vacation.
Based on the assumptions stated below, reneging, server breakdowns and vacations in a single server fuzzy queue significant measures are being derived.

Assumptions

a. Customers arrive in batches of varying sizes in accordance with a compound Poisson process with the fuzzy arrival rate $\tilde{\lambda}$.
b. There is a single server that provides service in three fluctuating modes. The time it takes to provide service follows an exponential with fuzzy rates $\tilde{\mu}_1, \tilde{\mu}_2$ and $\tilde{\mu}_3$.
c. Customer receives service one by one based on First-Come, First-Served queue discipline.
d. The probability of the server providing service in mode 1, mode 2, mode 3 are p_1, p_2 and p_3 respectively where $p_1 + p_2 + p_3 = 1$.
e. Service is provided through a single channel with finite capacity.
f. After each service completion, the server may take a vacation of a random length with Probability $\tilde{\varphi}$ or may continue to serve the next customer with probability $(1 - \tilde{\varphi})$.
g. The reneging time is distributed exponentially with the fuzzy parameter $\tilde{\gamma}$ and System vacation completion rate is $\tilde{\eta}$.
h. System Breakdown rate is α and System fuzzy repair rate is $\tilde{\beta}$.

Significant Measures

1. Probability that the server is providing service in mode 1, mode 2 and mode 3 at a random point of time

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2641 - 2646
https://publishoa.com
ISSN: 1309-3452

$$P^{(1)}(1)=\frac{p_1 Q\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\frac{1}{\tilde{\mu}_1+a}}{\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\tilde{\mu}_3+a}\right]\left[1-\frac{\tilde{\varphi}(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\eta}}\right]-\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\left[\frac{p_1}{\tilde{\mu}_1+a}+\frac{p_2}{\tilde{\mu}_2+a}+\frac{p_3}{\tilde{\mu}_3+a}\right]}$$

$$P^{(2)}(1)=\frac{p_2 Q\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\frac{1}{\tilde{\mu}_2+a}}{\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\tilde{\mu}_3+a}\right]\left[1-\frac{\tilde{\varphi}(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\eta}}\right]-\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\left[\frac{p_1}{\tilde{\mu}_1+a}+\frac{p_2}{\tilde{\mu}_2+a}+\frac{p_3}{\tilde{\mu}_3+a}\right]}$$

$$P^{(3)}(1)=\frac{p_3 Q\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\frac{1}{\tilde{\mu}_3+a}}{\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\tilde{\mu}_3+a}\right]\left[1-\frac{\tilde{\varphi}(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\eta}}\right]-\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\left[\frac{p_1}{\tilde{\mu}_1+a}+\frac{p_2}{\tilde{\mu}_2+a}+\frac{p_3}{\tilde{\mu}_3+a}\right]}$$

2. Probability that the server is under repairs at random point of time

$$R(1)=\frac{a\,Q\left[\frac{p_1\tilde{\mu}_1}{\mu_1+m}+\frac{p_2\tilde{\mu}_2}{\mu_2+m}+\frac{p_3\tilde{\mu}_3}{\mu_3+m}\right]\left[1-\frac{\tilde{\varphi}(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\eta}}\right]\frac{1}{\tilde{\beta}}}{\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\tilde{\mu}_3+a}\right]\left[1-\frac{\tilde{\varphi}(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\eta}}\right]-\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\left[\frac{p_1}{\tilde{\mu}_1+a}+\frac{p_2}{\tilde{\mu}_2+a}+\frac{p_3}{\tilde{\mu}_3+a}\right]}$$

3. Probability that the server is on vacation at random point of time

$$V(1)=\frac{\tilde{\varphi}Q\left[\frac{p_1\tilde{\mu}_1}{\mu_1+a}+\frac{p_2\tilde{\mu}_2}{\mu_2+a}+\frac{p_3\tilde{\mu}_3}{\mu_3+a}\right]\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\frac{1}{\tilde{\eta}}}{\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\tilde{\mu}_3+a}\right]\left[1-\frac{\tilde{\varphi}(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\eta}}\right]-\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\left[\frac{p_1}{\tilde{\mu}_1+a}+\frac{p_2}{\tilde{\mu}_2+a}+\frac{p_3}{\tilde{\mu}_3+a}\right]}$$

4. Probability that the server is idle but available in the system

$$Q=\frac{\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\tilde{\mu}_3+a}\right]\left[1-\frac{\tilde{\varphi}(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\eta}}\right]-\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\beta}\right]\left[\frac{p_1}{\tilde{\mu}_1+a}+\frac{p_2}{\tilde{\mu}_2+a}+\frac{p_3}{\tilde{\mu}_3+a}\right]}{\left[1+\frac{\tilde{\varphi}\gamma}{\tilde{\beta}}+\frac{\tilde{a}}{\tilde{\beta}}\right]\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\mu_3+a}\right]}$$

5. The Utilization factor

$$\rho=\frac{\left[\tilde{\lambda}+\frac{a(\tilde{\lambda}-\tilde{\gamma})}{\tilde{\beta}}\right]\left[\frac{p_1}{\tilde{\mu}_1+a}+\frac{p_2}{\tilde{\mu}_2+a}+\frac{p_3}{\tilde{\mu}_3+a}\right]+\left[\frac{\tilde{\varphi}\tilde{\lambda}}{\tilde{\eta}}-\frac{a}{\tilde{\beta}}\right]\left[\frac{p_1\mu_1}{\tilde{\mu}_1+a}+\frac{p_2\mu_2}{\tilde{\mu}_2+a}+\frac{p_3\mu_3}{\tilde{\mu}_3+a}\right]}{\left[1+\frac{\tilde{\varphi}\gamma}{\tilde{\beta}}+\frac{a}{\tilde{\beta}}\right]\left[\frac{p_1\tilde{\mu}_1}{\tilde{\mu}_1+a}+\frac{p_2\tilde{\mu}_2}{\tilde{\mu}_2+a}+\frac{p_3\tilde{\mu}_3}{\tilde{\mu}_3+a}\right]}$$

V Numerical Illustration

Consider the arrival rate, service rate, vacation rate, system repair rate, reneging rate and Vacation completion rate, which are all Trapezoidal fuzzy numbers with the probability providing services of three modes are $p_1 = 0.34, p_2 = 0.33, p_3 = 0.33$ respectively. Let a be a breakdown rate and it varies from 2 to 8.

Let $\tilde{\lambda} = [2,3,4,5]$; $\tilde{\mu}_1 = [1,2,3,4]$; $\tilde{\mu}_2 = [5,6,7,8]$; $\tilde{\mu}_3 = [9,10,11,12]$; $\varphi = [0.4,0.5,0.6,0.7]$; $\tilde{\gamma} = [2,4,6,8]$; $\tilde{\eta} = [1,3,5,7]$; $\tilde{\beta} = [6,8,12,14]$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2641 - 2646
https://publishoa.com
ISSN: 1309-3452

We apply trisectional fuzzy trapezoidal ranking approach of a given trapezoidal fuzzy number $\tilde{\lambda} = [2,3,4,5]$ R($\tilde{\lambda}$) = *R*(2, 3, 4, 5).

$$x = \frac{a_1 a_3 - a_2 a_4}{a_3 - a_4 - a_2 + a_1} = 3.5$$

$$y = \frac{x - a_2}{a_2 - a_1} + 1 = 1.5$$

$$R(\tilde{\lambda}) = \frac{ax + ba_1 + ca_4}{a + b + c} = 3.5$$

$$a = a_4 - a_1 = 3$$

$$b = \sqrt{y^2 + (a_4 - x)^2} = 2.1213$$

$$c = \sqrt{y^2 + (x - a_1)^2} = 2.1213$$

Recourse of this fuzzy ranking, we get

$$R(\mu_1) = R[1,2,3,4] = 2.5$$

$$R(\tilde{\mu}_2) = R[5,6,7,8] = 6.5$$

$$R(\mu_3) = [9,10,11,12] = 10.5$$

$$R(\tilde{\varphi}) = [0.4, 0.5, 0.6, 0.7] = 0.55$$

$$R(\gamma) = [2,4,6,8] = 5$$

$$R(\eta) = [1,3,5,7] = 4$$

$$R(\tilde{\beta}) = [6,8,12,14] = 10$$

We compute the significant measures of described fuzzy queuing model and tabulate the values with respect to ‘ α ’.

Table 1 Steady-state Characteristics with respect to ‘α’

α	ρ	Q	$P^1(1)$	$P^2(1)$	$P^3(1)$	$V(1)$	$R(1)$
2	0.6159	0.3927	0.2282	0.1173	0.0798	0.2984	0.1636
3	0.4826	0.3984	0.1879	0.1056	0.0743	0.2663	0.2298
4	0.3698	0.3990	0.1577	0.0949	0.0687	0.2381	0.2881
5	0.2716	0.3975	0.1342	0.0850	0.0630	0.2130	0.3398
6	0.1854	0.3947	0.1151	0.0760	0.0576	0.1907	0.3860
7	0.1091	0.3910	0.0995	0.0680	0.0524	0.1706	0.4276
8	0.0409	0.3869	0.0862	0.0606	0.0475	0.1524	0.4650

With regard to the data in the preceding table, it is clear that the system breakdown rate increases, then the utilization factor and the probability of server vacation decreases. Also three fluctuating mode of service experiences a decrease in it probabilities of rendering service. From our results, the probability that server is under repair increases provided the constant repair parameter whenever there is high rate of breakdowns.

VI Conclusion

The richness of fuzzy queuing systems in modifying complex data associated with ambiguity is reflected in their name. The effects of reneging, server vacation and breakdown on a single server batch arrival fuzzy queue with three varying modes of service are investigated in this work. To rank supplied trapezoidal fuzzy numbers, a trisectional fuzzy trapezoidal technique is used. The projected ranking algorithms may rank fuzzy numbers in practise. This tactic not only produces crisp results, but it also does it more precisely than most other methods.

References

1. A. Choudhury, and P. Medhi, “Balking and Reneging in Multiserver Markovian Queuing Systems”, International Journal of Mathematics in Operations Research, 2011, 3, pp. 377-394.

2. C.H. Cheng, "A New Approach for Ranking fuzzy numbers by distance method", Fuzzy Sets and Systems, 1998, **95**, 307-317.
3. C. S. Dinesh Bisht, Pankaj Kumar Srivastava, "Trisectional Fuzzy Trapezoidal Approach to Optimize Interval data based Transportation Problem", Journal of King Saud University-Science, **32**, 2020, 195-199.
4. D. Wu and H. Tagaki, "M/G/1 Queue with Multiple Working Vacation", Performance Evaluation, **63**, 2006, 654-681.
5. K.C. Madan, "On a Single Server Queue with Arrivals in Batches of Variable Size, General Service in Three Fluctuating Modes, Balking, Random Breakdowns and Stand-By Server during Breakdown Periods", Revista Investigacion Opereracional, 2014, 35, pp. 189-200.
6. L.D. Servi and S.G. Finn, "M/M/1 Queues with Working Vacations (M/M/1/WV)", Performance Evaluation, **50**, 2002, 41-52.
7. M. Baruah, K.C. Madan and T. Eldabi, "A Batch Arrival Single Server Queue with Server Providing General Service in Two Fluctuating Modes and Reneging during Vacation and Breakdowns", Journal of Probability and Statistics, 2014.
8. R. F. Khalaf, K.C. Madan and C.A. Lukas, "An $M^{[X]}$/G/1 Queue with Bernoulli Schedule Vacation Times, Random Breakdowns, General Delay Times and General Repair Times", Applied Mathematical Sciences, 2011, **5**, 35-51.
9. Samuel Ugochukwu Enogwe, Happiness Onyebuchi Obiora-Ilouno, "Effects of Reneging, Server Breakdowns and Vacation on a Batch Arrival Single Server Queuing System with Three Fluctuating Modes of Service", Open Journal of Optimization, 2020, **9**, 105-128.
10. W. J. Gray, P. P. Wang and M. Scott," A Queuing Model with Multiple Types of Server Breakdowns", Quality Technology and Quantitative Management, 2004, **1**, 245-255.
11. Y. J. Wang, and H. S. Lee, "The Revised Method of Ranking Fuzzy Numbers with an Area between the Centroid and Original Points". Computers and Mathematics with Applications, **55**(9) (2008), 2033-2042.
12. Y. Tang, "Single Server M/G/1 Queuing system Subject to Breakdowns-Some Reliability and Queuing Problems", Microelectronics Reliability, 2003, **149**, 588-613.