

Bipolar Fuzzy Graph on Certain Topological Indices

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ABSTRACT

Fuzzy information plays a significant part in everyday life, and information is typically denoted by fuzzy graphs, where structure of graph is utilized to portray the related design of the fuzzy information. Bipolar fuzzy graph (BFG) indices were discussed in this article. Zagreb first and second index, Randic and Harmonic and Estrada index were discussed. Union and intersection for those indices are determined. Lower and Upper bound for first Zagreb and Randic indices for BFG is proved.

Keywords— Bipolar fuzzy graph, Incidence bipolar fuzzy graph, Topology index, Fuzzy graph, Graph

I. INTRODUCTION

The motivation behind why the numerical hypothesis and application innovation in fuzzy system are centered by investigators and has a quickly established in the previous 60 years is fundamentally due to the broad and significant uses and foundation in human culture and certain run-through [1-6]. The advancement of numerical hypothesis of fuzzy to a great extent relies upon the advancement of fuzzy designing innovation and application. Designing innovation in fuzzy is not just the main thrust for the advancement of fuzzy arithmetic hypothesis, yet additionally the center of fuzzy math helping the public creation practice. Application of fuzzy is more and more increasing, it includes fuzzy controller [7-9], representation of knowledge in fuzzy [10-12], Coding and fuzzy information [13-17], fuzzy evaluation, prediction and decision technology [18-20] and so on.

The researchers gradually attract the topological indices on fuzzy graph. These lists of indices play an important part in decision making system in fuzzy and in fuzzy reasoning. Molecule bond connectivity index is presented by E. Estrada and et al, (1990) [21]. First zagreb index was presented by Trinajstić, N and Gutman, I in (1972) [22]. The Geometric-Arithmetic index was presented by B. Furtula and D. Vukicevic in (2009) [23].

If $J(G)$ be the jump graph in G somewhere two vertices are adjoining and just relating edges are not neighboring in G . If $G = (\mathcal{E}, \tau, \varsigma)$ where $\mathcal{E}$ and τ are BFS of vertex and edges respectively and ς is the BFI of G with both positive and negative membership values.

$\mu_{\varsigma}^P(q, e) \leq \min(\mu_{\mathcal{E}}^P(q), \mu_{\tau}^P(e))$ and $\mu_{\varsigma}^N(q, e) \geq \max(\mu_{\mathcal{E}}^N(q), \mu_{\tau}^N(e))$ for every $q \in V$ and $e \in E$.

The rest of the paper is organized as follows. Basic definition of index's and theorems are explained in section II. union and intersection for topological indices are presented in section III. Discussion are given in section IV. Concluding remarks are given in section V.

II. PRELIMINARIES

2.1 Definition -

The Zagreb index of first is denoted by $Z(G)$, where G is the BFG with node set then it's defined as

$$Z(G) = (Z^P(G), Z^N(G)) = \sum_{i=1}^n [\rho^P(e_i) d^{P^2}(e_i), \rho^N(e_i) d^{N^2}(e_i)] \text{ for all } e_i \in V.$$

2.2 Definition-

The Zagreb index of second is denoted by $Z^*(G)$, where G is the BFG with node set then it's defined as

$$Z^*(G) = (Z^{*P}(G), Z^{*N}(G)) = \frac{1}{2} \sum [\rho^P(e_i) d^P(e_i) \rho^P(c_j) d^P(c_j), \rho^N(e_i) d^2(e_i) \rho^N(c_j) d^N(c_j)] \text{ for all } i \neq j \text{ and } \nu(e_i, c_j) \in E.$$

2.3 Definition -

Let G be the BFG then the Randic index is defined as

$$RI(G) = (RI^P(G), RI^N(G)) = \frac{1}{2} \sum \left[\left(\rho^P(e_i) d^P(e_i) \rho^P(c_j) d^P(c_j) \right)^{-\frac{1}{2}}, \left(\rho^N(e_i) d^2(e_i) \rho^N(c_j) d^N(c_j) \right)^{-\frac{1}{2}} \right]$$

for all $i \neq j$ and $v(e_i, c_j) \in E$.

2.4 Definition –

Let G be the BFG then the Harmonic index is defined as

$$HI(G) = (HI^P(G), HI^N(G)) = \frac{1}{2} \sum \left[\frac{1}{\rho^P(e_i) d^P(e_i) + \rho^P(c_j) d^P(c_j)}, \frac{1}{\rho^N(e_i) d^2(e_i) + \rho^N(c_j) d^N(c_j)} \right]$$

for all $i \neq j$ and $v(e_i, c_j) \in E$.

2.5 Definition -

The Estrada index of BFG is defined by the order and the eigenvalue of the adjacency matrix

$$ESI(G) = (ESI^P(G), ESI^N(G)) = \left[m^P \times \sum e^{\lambda_i^P}, m^N \times \sum e^{\lambda_i^N} \right], \text{ for all } i = 1, 2, \dots, n.$$

2.1 Theorem –

If two BFG are isomorphic, then the topological indices of those graphs are equal.

Proof:

Let G and F be the two isomorphic BFG. In G all the membership value of edges and vertex coincide with F. consequently, the graph shape may additionally vary however collection of edges and vertices are identical offers the identical topological indices value.

2.1 Corollary –

The topological value of two BFG are similar, then those graph will no longer be isomorphic.

2.2 Theorem –

If the particular edge of the membership value is removed from the connected BFG G, then its split into more than one graph and it's disconnected. Then (i) $Z^P(G_1) < Z^P(G_2), Z^N(G_1) > Z^N(G_2)$

$$(ii) Z^{*P}(G_1) < Z^{*P}(G_2), Z^{*N}(G_1) > Z^{*N}(G_2)$$

$$(iii) RI^P(G_1) < RI^P(G_2), RI^N(G_1) > RI^N(G_2)$$

$$(iv) HI^P(G_1) < HI^P(G_2), HI^N(G_1) > HI^N(G_2) \text{ and}$$

$$(v) ESI^P(G_1) < ESI^P(G_2), ESI^N(G_1) > ESI^N(G_2)$$

Proof.

Consider G as connected BFG splitted into more than one graph such as G_1 and G_2 . Then G_1 and G_2 be the partial BFG $Z(G-u) < Z(G)$ such that $Z^P(G_1), Z^P(G_2) < Z^P(G)$ by eliminating the membership value which is chosen from the edge in G.

Similarly for negative membership $Z^N(G_1), Z^N(G_2) > Z^N(G)$. Since $V(G_1) \leq V(G_2)$ which suggests that

$Z^P(G_1) < Z^P(G_2), Z^N(G_1) > Z^N(G_2)$. The excess cases are inconsequentially evident by the accompanying the above strategy.

2.2 Corollary –

Consider P6 is the BFG to such an extent that by eliminating the specific edge which split in to two BFG. The nodes and edges membership value for both the BFG are equivalent and isomorphic.

$$\text{Then (i) } Z^P(G_1) = Z^P(G_2), Z^N(G_1) = Z^N(G_2) \text{ (ii) } Z^{*P}(G_1) = Z^{*P}(G_2), Z^{*N}(G_1) = Z^{*N}(G_2) \text{ (iii)}$$

$$RI^P(G_1) = RI^P(G_2), RI^N(G_1) = RI^N(G_2) \text{ (iv) } HI^P(G_1) = HI^P(G_2), HI^N(G_1) = HI^N(G_2) \text{ and}$$

$$(v) ESI^P(G_1) = ESI^P(G_2), ESI^N(G_1) = ESI^N(G_2)$$

III. TOPOLOGICAL INDICES FOR UNION AND INTERSECTION

Here, we examine about the outcome of topological indices by union and intersection of the BFG. Consider the two BFG then union of the graph will be (i) $Z(G \cup H) \geq Z(G)$, (ii) $Z^*(G \cup H) \geq Z^*(G)$, (iii) $RI(G \cup H) \leq RI(G)$,

(iv) $HI(G \cup H) \leq HI(G)$

3.1 Theorem–

The union of Zagreb index of first, Zagreb index of second, Radic and harmonic for BFG is

- (i) $Z(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \geq Z(H_j^P, H_j^N)$, then $Z(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \leq \left(\sum_{j=1}^m Z(H_j^P), \sum_{j=1}^m Z(H_j^N) \right)$
- (ii) $Z^*(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \geq Z^*(H_j^P, H_j^N)$, then $Z^*(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \leq \left(\sum_{j=1}^m Z^*(H_j^P), \sum_{j=1}^m Z^*(H_j^N) \right)$
- (iii) $RI(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \leq RI(H_j^P, H_j^N)$, then $RI(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \geq \left(\sum_{j=1}^m RI(H_j^P), \sum_{j=1}^m RI(H_j^N) \right)$
- (iv) $HI(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \geq HI(H_j^P, H_j^N)$, then $HI(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \geq \left(\sum_{j=1}^m HI(H_j^P), \sum_{j=1}^m HI(H_j^N) \right)$

Proof.

Let H_j be the BFG for all $j=1, 2, \dots, m$. Our primary point is to show that $Z(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \leq (\cup_{j=1}^m Z(H_j^P), \cup_{j=1}^m Z(H_j^N))$. By demonstrating the

- (i) comparative methodology can be made to demonstrate
- (ii) Assume think about the two BFG H_j and H_k then at that point the union of those

graphs are done by the following stages.

Stage 1. Let $H_j = (V_{H_j}, E_{H_j}, \mu_{H_j}, \rho_{H_j})$ and $H_k = (V_{H_k}, E_{H_k}, \mu_{H_k}, \rho_{H_k})$ be the two BFGs. Considering $V_{H_j} \neq V_{H_k}$ then at that point $V_{H_j} \cap V_{H_k} = \phi$ which suggests that $\mu_{H_j} \cap \mu_{H_k} = \phi$ furthermore, $\rho_{H_j} \cap \rho_{H_k} = \phi$.Subsequently $H_j \cap H_k = \phi$. Then, at that point the index of Zagreb firstkind for the union of BFGs H_j and H_k is $Z((H_j^P \cup H_k^P), (H_j^N \cup H_k^N)) = ((Z(H_j^P) + Z(H_k^P)), (Z(H_j^N) + Z(H_k^N)))$. Subsequently

$$Z(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) = \left(\sum_{j=1}^m Z(H_j^P), \sum_{j=1}^m Z(H_j^N) \right).$$

Stage 2. Let $H_j = (V_{H_j}, E_{H_j}, \mu_{H_j}, \rho_{H_j})$ and $H_k = (V_{H_k}, E_{H_k}, \mu_{H_k}, \rho_{H_k})$ be the two BFGs. Considering $V_{H_j} \neq V_{H_k}$ then at that point $V_{H_j} \cap V_{H_k} = \phi$ which suggests that $\mu_{H_j} \cap \mu_{H_k} = \phi$ furthermore, $\rho_{H_j} \cap \rho_{H_k} = \phi$.

Subsequently

Thus $Z(\cup_{j=1}^m H_j^P, \cup_{j=1}^m H_j^N) \leq \left(\sum_{j=1}^m Z(H_j^P), \sum_{j=1}^m Z(H_j^N) \right)$. Thus the theorem is demonstrated for (i) and (ii). By

following same system union of two BFGs the indices of Randić and Harmonic can be proved.

3.2 Theorem –

The intersection of Zagreb index of first, Zagreb index of second, Radic and harmonic for BFG is

- (i) $Z(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \leq Z(H_j^P, H_j^N)$, then $Z(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \leq \left(\sum_{j=1}^m Z(H_j^P), \sum_{j=1}^m Z(H_j^N) \right)$
- (ii) $Z^*(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \leq Z^*(H_j^P, H_j^N)$, then $Z^*(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \leq \left(\sum_{j=1}^m Z^*(H_j^P), \sum_{j=1}^m Z^*(H_j^N) \right)$

$$\begin{aligned}
& Z * (\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \leq \left(\sum_{j=1}^m Z * (H_j^P), \sum_{j=1}^m Z * (H_j^N) \right) \\
\text{(iii)} \quad & RI(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \geq RI(H_j^P, H_j^N), \text{ then } RI(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \geq \left(\sum_{j=1}^m RI(H_j^P), \sum_{j=1}^m RI(H_j^N) \right) \\
\text{(iv)} \quad & HI(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \geq HI(H_j^P, H_j^N), \text{ then } HI(\cap_{j=1}^m H_j^P, \cap_{j=1}^m H_j^N) \geq \left(\sum_{j=1}^m HI(H_j^P), \sum_{j=1}^m HI(H_j^N) \right)
\end{aligned}$$

Proof.

The convergence of any two BFGs has the minimum membership value such that all cases are fulfilled by the definition of BFGs. Thus, (i), (ii), (iii) and (iv) are inconsequentially evident.

3.3 Theorem—

Let G_1 and G_2 be the two BFG, then first Zagreb index of G with bounds are

$$\begin{aligned}
Z(G) & \geq 2fw(\Delta_{G_2}^N + 1) + fw[\Delta_{G_2}^N + f - 2[\Delta_{G_1}^N - 1] + v] \\
& + 2 \left[\left[\frac{f(f-1)}{2} \right] - f(\Delta_{G_1}^N - 1) \right] [f - 1 - 2[\Delta_{G_1}^N - 1] + v]
\end{aligned}$$

and

$$\begin{aligned}
Z(G) & \leq 2fw(\Delta_{G_2}^P + 1) + fw[\Delta_{G_2}^P + f - 2[\Delta_{G_1}^P - 1] + v] \\
& + 2 \left[\left[\frac{f(f-1)}{2} \right] - f(\Delta_{G_1}^P - 1) \right] [f - 1 - 2[\Delta_{G_1}^P - 1] + v]
\end{aligned}$$

Proof. Consider

$$\begin{aligned}
Z(G) & = f \sum_{ab \in E(G_2)} [(\deg_{G_2}^N a + 1) + (\deg_{G_2}^N b + 1)] \\
& + \sum_{e \in V(J(G_1))} \sum_{a \in V(G_2)} [(\deg_{G_2}^N a + 1) + (\deg_{J(G_1)_2}^N e + v)] \\
& + \sum_{eh \in E(J(G_1))} [(\deg_{J(G_1)_2}^N e + v) + (\deg_{J(G_1)_2}^N h + v)] \\
& = fw[(\deg_{G_2}^N a + 1) + (\deg_{G_2}^N b + 1)] + fw[(\deg_{G_2}^N a + 1) + (\deg_{J(G_1)_2}^N e + v)] \\
& + \left[\left[\frac{f(f-1)}{2} \right] - f \left[\frac{\deg_{G_1}^N a + \deg_{G_1}^N b - 2}{2} \right] \right] [(\deg_{J(G_1)_2}^N e + v) + (\deg_{J(G_1)_2}^N h + v)] \\
& \geq fw[(\Delta_{G_2}^N + 1) + (\Delta_{G_2}^N + 1)] + fw[\Delta_{G_2}^N + 1 + f - 1 - [\Delta_{G_1}^N + \Delta_{G_1}^N - 2] + v] \\
& + \left[\left[\frac{f(f-1)}{2} \right] - f \left[\frac{\Delta_{G_1}^N + \Delta_{G_1}^N - 2}{2} \right] \right] [2[(f-1) - [\Delta_{G_1}^N + \Delta_{G_1}^N - 2] + v]] \\
& \geq 2fw(\Delta_{G_2}^N + 1) + fw[\Delta_{G_2}^N + f - 2[\Delta_{G_1}^N - 1] + v] \\
& + 2 \left[\left[\frac{f(f-1)}{2} \right] - f \left[\frac{2(\Delta_{G_1}^N - 1)}{2} \right] \right] [f - 1 - 2[\Delta_{G_1}^N - 1] + v] \\
& \geq 2fw(\Delta_{G_2}^N + 1) + fw[\Delta_{G_2}^N + f - 2[\Delta_{G_1}^N - 1] + v] \\
& + 2 \left[\left[\frac{f(f-1)}{2} \right] - f(\Delta_{G_1}^N - 1) \right] [f - 1 - 2[\Delta_{G_1}^N - 1] + v]
\end{aligned}$$

Similarly,

$$Z(G) \leq 2fw(\Delta_{G_2}^P + 1) + fv[\Delta_{G_2}^P + f - 2[\Delta_{G_1}^P - 1] + v] \\ + 2\left[\left[\frac{f(f-1)}{2}\right] - f(\Delta_{G_1}^P - 1)\right][f - 1 - 2[\Delta_{G_1}^P - 1] + v]$$

3.4 Theorem–

Let G_1 be the BFG with minimum degree Δ and maximum degree δ with bounds are

$$n(\delta^N)^\alpha (\Delta^N)^{1+\alpha} / 2 \leq RI(G_1) \leq n(\delta^P)^\alpha (\Delta^P)^{1+\alpha} / 2$$

Proof.

Consider

$$RI(G_1) = \sum_{e_i c_j \in E} \left[(\rho^P(e_i) d^P(e_i) \rho^P(c_j) d^P(c_j))^{\frac{-1}{2}}, (\rho^N(e_i) d^2(e_i) \rho^N(c_j) d^N(c_j))^{\frac{-1}{2}} \right] \\ \geq \sum_{e_i c_j \in E} [\delta^N \rho^N(c_j) d^N(c_j)]^\alpha \\ = \delta^N \sum_{c_j \in E} [\rho^N(c_j) d^N(c_j)]^{1+\alpha} / 2 \\ \geq n(\delta^N)^\alpha (\Delta^N)^{1+\alpha} / 2$$

$$\text{Similarly, } RI(G_1) \leq n(\delta^P)^\alpha (\Delta^P)^{1+\alpha} / 2$$

IV. DISCUSSION

In this work, we study the Zagreb first and second index file of BFGs and a few ends are acquired. The hypotheses are positively affect the improvement of computerized reasoning, design acknowledgment, and fuzzy AI related fields. Due to the amazingly wide scope of utilizations of fuzzy graph information, this research will further develop the practical application abilities of fuzzy organized information in different disciplines. The outcomes give devices, techniques and application strategies in different fields like large information, block chain, cloud administrations, modern and mining computerization, chemistry, medication, materials science and sociology.

Researchers use graphs to represent sub-atomic constructions: vertices represent molecules, and edges between vertices represent compound connections between atoms. The graph is respected an atomic chart, and the properties of chemicals are concentrated by characterizing the topological index on the graph.

In particular, the accompanying points can be considered as the future research bearings: Different type of distance based topological indices ought to be presented in fuzzy graph, and definition of the distance ought to be considered in particular settings. Fuzzy topological indices can be applied in decision making.

V. CONCLUSION

BFG for Zagreb first and second index, Randic, Harmonic and Estrada index were defined. Union and intersection for those indices are determined. Consequently, it has wide application in the field of science and software engineering. The new meaning of topological indices in BFGs are examined. For the first Zagreb index the lower and upper bound are defined.

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