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Anti-Stiff Bianchi Type VI_h String Cosmological Model in General Relativity

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ABSTRACT

Anti-stiffBianchi-type VI_h string cosmological model is studied for a perfect fluid. To derive the solution, we have considered $A = \beta B^n$ where A and B are metric potentials. The value of the deceleration parameter depends on the time which shows the accelerating phase of the universe. Further, we have studied various parameters for the model.

I. INTRODUCTION

The cosmic string is an important aspect in the study of the beginning of the universe. Cosmic strings appear as topological defects. Studying string cosmological models has been of great interest in general relativity. From the observational data, the universe is expanding and accelerating [1,2,3,4]. One of the greatest cosmological mysteries in the universe is structure formation. Cosmic strings may help to explain large structure formation in the universe [5,6]. Bianchi explains the accelerating phase and several observations in our present universe by developing a cosmological model. An earlier study looked at the exact solution to string cosmology in different space-time by Bali R. and Pradhan A [7], Bhattacharjee et al. [8], Chatterjee [9], Krori et al. [10]. Bianchi VI0 string cosmological model studied by Tikekar and Patel [11] and Bijan Saha [12] gave the solution for the Bianchi type-VI model. Sharma A et al. [13], R.Venkateswarlu et al[14], Ladke L.S. et al [15], and Gore et al [16] have discussed solutions for string cosmological models. Bali and Sharma [17] have investigated a stiff fluid magnetized Bianchi type-I cosmological model. Bali R and Pareek U [18] discussed the cosmological model for perfect fluid with string. Anti-stiff fluid cosmological models are studied by Bhoyar and Chirde [19]. For barotropic perfect fluid, Bali R et al [20] investigated Bianchi I cosmological model. Bali R et al [21]discussed Bianchi I cosmological model in the presence of string for barotropic perfect fluid in general relativity. Moreover, Das et al [22]prescribed the mostgeneral solution for the perfect fluid cosmological universe. Roy and Prasad [23], Tripathi et.al [24], Mishra et.al [25], Santhi et al [26] explored Bianchi type VIh model in different aspects. Motivated by the previous investigations, we have focused on the anti-stiff Bianchi type VIhstring cosmological model in general relativity.

II. THE METRIC AND FIELD EQUATIONS

Metric for the Bianchi type VI_h is given by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2hx} dy^2 + C^2 e^{2hx} dz^2 \tag{1}$$

where A, B and C are the metric potentials.

For string, the energy-momentum tensor can be written as

$$T_i^j = (\rho + p) v_i v^j + p g_i^j - \lambda x_i x^j \tag{2}$$

where

p is the pressure, ρ is the density

λ is string tension density

x_i is the unit space-like vector specifying string direction with the conditions

$$v_i v^i = -x_i x^i = -1, \text{ and } v^i x_i = 0 \tag{3}$$

The flow vector v^i satisfying

$$g_{ij} v^i v^j = -1 \tag{4}$$

The co-ordinates are considered as $v^1 = 0 = v^2 = v^3 \;\&\; v^4 = 1$ for the line element (1)

Further, x^i parallel to the x-axis so that

$$x^i = \left(\frac{1}{A}, 0, 0, 0 \right)$$

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The Einstein's field equation (in gravitational units $c=1, 8\pi G=1$)

$$R_i^j - \frac{R}{2} g_i^j = -T_i^j \tag{5}$$

The equation (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{h^2}{A^2} = \lambda - p \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{h^2}{A^2} = -p \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{h^2}{A^2} = -p \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{h^2}{A^2} = \rho \tag{9}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{10}$$

III. SOLUTION FOR THE FIELD EQUATIONS

Consider the following condition

$$A = \beta B^n \tag{11}$$

Where β is constant

Equation (10) leads to

$$C = \alpha B \tag{12}$$

where α is constant of integration

we assume $\alpha = 1$

$$B = C$$

Further, equation (6), (7) and (8) becomes

$$\frac{2B_{44}}{B} + \frac{B_4^{\ 2}}{B^2} + \frac{h^2}{A^2} = \lambda - p \tag{13}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{h^2}{A^2} = -p \tag{14}$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^2}{B^2} - \frac{h^2}{A^2} = \rho \tag{15}$$

For Anti-stiff Fluid

$$p + \rho = 0$$

From equations (14) and (15)

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{A_4 B_4}{AB} - \frac{B_4^{\ 2}}{B^2} = 0 \tag{16}$$

Using the condition

$$\frac{A_4}{A} = n \frac{B_4}{B}$$

Equation (16) leads to

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$$\frac{B_{44}}{B_4} + P\frac{B_4}{B} = 0 \tag{17}$$

$$where\ \mathrm{P} = \frac{n^2 - 2n - 1}{n+1}$$

Integrating equation (17)

$$B = (n+1)^{1/P+1}(lt+m)^{1/P+1}$$

Where *'l'* and *'m'* are constant of integration.

$$B = M\ T^{(1/P+1)} \tag{18}$$

where

$$M = (P+1)^{(1/P+1)}\ and\ T = lt + m$$

The line element becomes

$$ds^2 = -\frac{dT^2}{l^2} + \left(\beta M^P T^{1/P+1}\right)^2 dx^2 + M^2 T^{2/P+1}\left(e^{-2hx}dy^2 + e^{2hx}dz^2\right) \tag{19}$$

IV. PHYSICAL AND GEOMETRICAL FEATURES

The proposed model starts with a big bang and rotation (ω) is identically zero.
For the model, the deceleration parameter (q) can be written as

$$q = \frac{3P}{(n+2)M\ T} - 1\ ,\quad n \neq -2 \tag{20}$$

$q < 0$ (accelerating phase) for $T > \dfrac{3P}{M(n+2)}\ \dfrac{2-\sqrt{5}}{2} < n < \dfrac{2+\sqrt{5}}{2}$,

$q > 0$ (decelerating phase) $T < \dfrac{3P}{M(n+2)}\ n > \dfrac{2+\sqrt{5}}{2}$.

The expansion (θ), shear (σ), pressure (p), density (ρ), string tension density (λ), volume (V), Hubble parameter (H) for the model (21) are given by

$$\theta = \left(\frac{n+2}{P+2}\right)\frac{M}{T^{(P/P+1)}} \tag{21}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{n-1}{P+2}\right).\frac{M}{T^{(P/P+1)}} \tag{22}$$

$$p = \frac{h^2}{\beta^2 M^{2n} T^{(2n/P+1)}} - \frac{(2n+1)M^2}{(P+2)^2 T^{(2P/P+1)}} \tag{23}$$

$$\rho = \frac{(2n+1)M^2}{(P+2)^2 T^{(2P/P+1)}} - \frac{h^2}{\beta^2 M^{2n} T^{(2n/P+1)}} \tag{24}$$

$$\lambda = \frac{1}{T^2}\left[\frac{2h^2}{\beta^2 M^{2n} T^{n/(P+1)}} - \frac{2(P+n)l^2}{(P+1)^2}\right] \tag{25}$$

$$V = \beta\ M^{(n+2)} T^{\left(\frac{n+2}{P+1}\right)} \tag{26}$$

$$H = \frac{1}{3}\left(\frac{n+2}{P+2}\right)\frac{M}{T^{(P/P+1)}} \tag{27}$$

From figure 1, initially, the volume V of the universe is zero. However, as time increases, the volume of the universe increases.

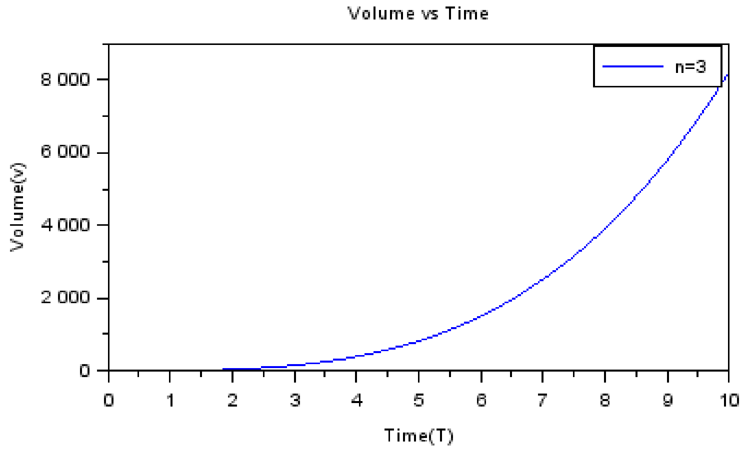

Figure1. The plot of volume versus Time

Figure 2 shows that the Hubble parameter value decreases as time increases, i.e. $H \to 0$ as $T \to \infty$

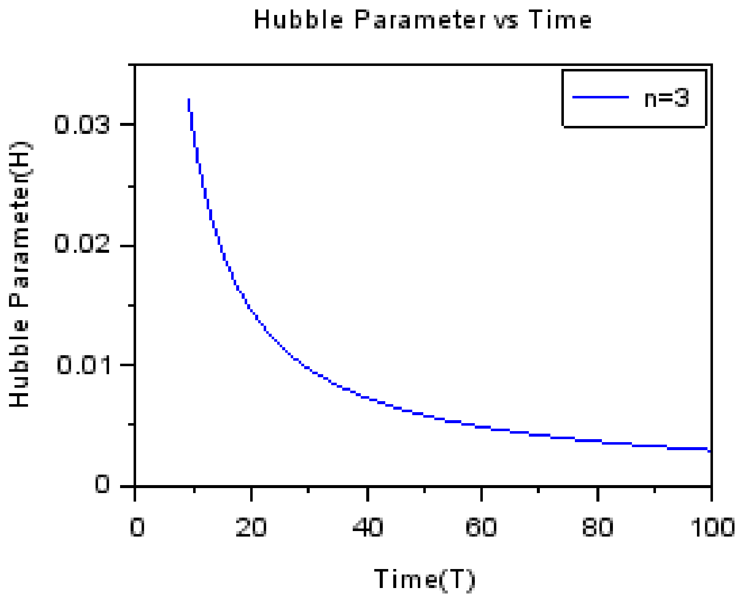

Figure2. The plot of Hubble Parameter versus Time

V. CONCLUSION

We have obtained the solution for the anti-stiff Bianchi type VIh cosmological model with the string. The model describes the evolution of the universe. In recent observations of Type Ia supernova, the present universe is accelerated and expanding. For our model, the value of the deceleration parameter depends on the time which shows the accelerating phase of the universe. As the time increases, the expansion (θ) for the model decreases and expansion stops $T \to \infty$. As $\frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}}\left(\frac{n-1}{n+2}\right), \ n \neq -2$ is finite, anisotropy for the model is maintained. Moreover, the model isotropizesfor $n = 1$. Hubble's parameter is decreasing and vanishes as $T \to \infty$. Further, as the universe expands, the energy density of the fluid decreases. Our model approaches a flat universe during the late time. At the initial epoch, the universe is dominated

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by the String as string tension density $\lambda \to \infty,\ T \to 0$. Further, the string will disappear from the universe as time increases i.e. $\lambda \to 0,\ T \to \infty$

The model described herein agrees well with the recent observations and can be considered a promising approach to explaining the observed universe as it represents a non-rotating, expanding and shearing universe.

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