JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

Some Special Types of Edges in Apicture Fuzzy Tree

N.RAJATHI[1] , V.ANUSUYA[2]andA.NAGOORGANI[3]

[1]PG and Research Department of Mathematics, SeethalakshmiRamaswami College Trichy-02, India.
[2]PG and Research Department of Mathematics, SeethalakshmiRamaswamiCollegeTrichy-02, India.
[3]PG&Research Department of Mathematics, JamalMohamedCollegeTrichy-20, India.
(Affiliated toBharathidasan University, Tiruchirappalli)
E-mail:n.rajianand@gmail.com

Received 2022 April 02**; Revised** 2022 May 20**; Accepted** 2022 June 18**.**

Abstract

In both theory and applications of fuzzy graphs, the concept of connectednessis significant. This study analyzes edges in a picture fuzzy tree into three typesbased on their strength such as robust edge, fragile edge and futile edge. Theadvantage of this classification is used to understand the fundamental structureofapicturefuzzytree.Sometheoremsandpropertiesrelatedtotheseedgeshavebeen provedwith examples.

KeyWords

picturefuzzy graph, PF-bridge,PictureFuzzy Tree, fragileedge, futileedge,robustedge.

Introduction

Fuzzy graphs were introduced by Rosenfeld [8], who has described the fuzzyanalogueofgraphtheoreticconceptslikepaths,connectedness,cyclesandtreesandestablishedsomeoftheir properties.Bhutani andRosenfeld[2]haveintroduced the concept of strong edges. Sunitha [10] classified strong edges in totwo types namely α-Strong, β-Strong and introduced two other types ofedgesinfuzzygraphswhicharenotstrongnamely δand δ^*edges.Intuitionistic fuzzy graph theory was introduced by Krassimir T.Atanassov [1]. Karunambigai and Parvathi [5] introduced intuitionistic fuzzygraph as a special case of Atanassov IFG. Parvathi. R [6] classified edges inIFGintothreetypes namelyα-strong,β-strongandδ– weakedgesbasedonitsstrength.

Cuong and Kreinovich[4] proposed the picture fuzzy set which is amodified version of fuzzy set and intuitionistic fuzzy set. It is an efficientmodel with uncertain real-life problems, in which intuitionistic fuzzy set mayfail to reveal satisfactory results. Picture fuzzy set allows the degree of positivemembership,degreeofneutralmembershipanddegreeofnegativemembership of an element. The concept of neutrality degree can be seen insituations when we face human opinions involving more answers of type: yes,abstain, no, refusal. Cen Zuo[3] introduced several types of Picture FuzzyGraphs such as regular PFG, strong PFG, complete PFG, connected PFG andcomplement PFG. In graph theory, edge analysis is not very important as alledges are strong. But in PFG, it is very important to find the nature of edges. Inthispaper,threetypesofedgessuchasrobust,fragileandfutileedge,dependingonthestrengthofconnectedness between two nodes are discussedwithsuitableillustrations.

This paperisorganizedasfollows:Section2containspreliminaries.Insection3,theconceptof robust,fragileandfutileedgesisintroducedwithexample.Itis alsoshownthatanedge(v_i, v_j)of$G$is anPF-bridgeiff itis aα-μstrong,α-ηstrongandα-γstrong. Insection4,sometheoremsandproportionsin picturefuzzy treearediscussed.

2. Preliminaries

Inthissection,somebasicdefinitionswhichareusedtoconstructtheoremsand properties relatedto thepicture fuzzy graph aregiven.

Definition2.1

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

A fuzzy graph $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ for all $v_i, v_j \in V$ and μ is a symmetric fuzzy relation on σ.

Definition2.2

An Intuitionistic Fuzzy Graph is of the form $G=(V,E)$ where

(i) $V = \{v_1, v_2 \ldots . . v_n\}$ such that the mapping $\mu_1: V \to [0, 1]$ is the degree of membership and the mapping $\gamma_1: V \to [0, 1]$ is the degree of non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V$ (i= 1,2..….n)

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ are such that

$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$

$\gamma_2(v_i, v_j) \leq \max[\gamma_1(v_i), \gamma_1(v_j)]$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1, \forall (v_i, v_j) \in E$.(i,j=1,2…..n).

Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

In an Intuitionistic Fuzzy Graph G, when $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j.

Definition2.3

A pair $G=(V,E)$ is known as Picture Fuzzy Graph (PFG) if

(i) $V = \{v_1, v_2 \ldots v_n\}$ such that $\mu_1: V \to [0, 1]$ $\eta_1: V \to [0, 1]$ and $\gamma_1: V \to [0,1]$ degree of Positive, neutral and negative membership function of the vertex $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \eta_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V$, $i = 1, 2 \ldots n$..

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$, $\eta_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \leq \min(\mu_1(v_i), \mu_1(v_j))$, $\eta_2(v_i, v_j) \leq \min(\eta_1(v_i), \eta_1(v_j))$

and $\gamma_2(v_i, v_j) \leq \max(\gamma_1(v_i), \gamma_2(v_j))$ where $0 \leq \mu_2(v_i, v_j) + \eta_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E, i, j = 1, 2 \ldots n$.

Here the 4-tuple $(v_i, \mu_{1i}, \eta_{1i}, \gamma_{1i})$ denotes the degree of positive membership, neutral membership and negative membership of the vertex v_i and the 4-tuple $(e_{ij}, \mu_{2ij}, \eta_{2ij}, \gamma_{2ij})$ denotes the degree of positive membership, neutral membership and negative membership of the edge relation $e_{ij} = (v_i, v_j)$.

Example 2.4

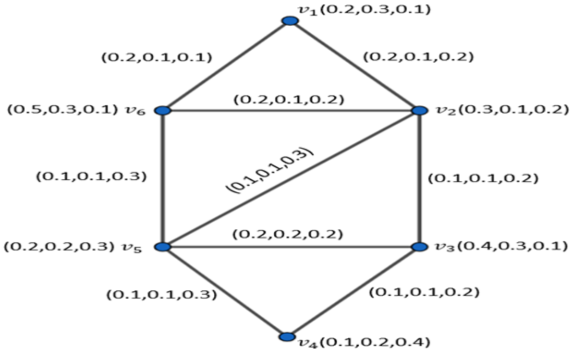

Figure2.1

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

Definition2.5

APicturefuzzygraphG=(V,E)issaidtobecomplete,if$\mu_2(v_i, v_j)$ =min($\mu_1(v_i)$, $\mu_1(v_j)$),$\eta_2(v_i, v_j)$ = min($\eta_1(v_i)$, $\eta_1(v_j)$)and$\gamma_2(v_i, v_j)$ =max($\gamma_1(v_i)$,$\gamma_1(v_j)$)forevery$v_i,v_j \epsilon V$.

Definition 2.6

Let G = (V, E) be the PFG. Then the vertex cardinality of V is defined by
$$|V| = \sum_{v_i \epsilon V} \frac{1 + \mu_1(v_i) + \eta_1(v_i) - \gamma_1(v_i)}{2}$$

for all $v_i \epsilon V$.It is also called the order of a PFG and it is denoted by p.

Definition 2.7

Let G = (V, E) be the PFG. Then the edge cardinality of E defined by

$$|E| = \sum_{(v_i, v_j) \epsilon E} \frac{1 + \mu_2(v_i, v_j) + \eta_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}$$

for all $(v_i, v_j) \epsilon E$.It is also called the size of a PFG.

Definition 2.8

Let G = (V, E) be the PFG. Then the cardinality of G is defined to be

$$|G| = |\sum_{v_i \epsilon V} \frac{1 + \mu_1(v_i) + \eta_1(v_i) - \gamma_1(v_i)}{2} + \sum_{(v_i v_j) \epsilon E} \frac{1 + \mu_2(v_i, v_j) + \eta_2(v_i, v_j) - \gamma_2(v_i, v_j)}{2}|$$

and it is denoted by q.

Definition 2.9

An edge(v_i, v_j) is called a strong edge, if $\mu_2(v_i, v_j) \geq \mu_2'^{\infty}(v_i, v_j), \eta_2(v_i, v_j) \geq \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2(v_i, v_j) \leq \gamma_2'^{\infty}(v_i, v_j)$ For every $v_i, v_j \epsilon V$.Where $\mu_2'^{\infty}(v_i, v_j), \eta_2'^{\infty}(v_i, v_j)$ and $\gamma_2'^{\infty}(v_i, v_j)$ is the strength of the connectedness between v_i and v_j in the picture fuzzy graph obtained from G by deleting the edge(v_i, v_j).

Definition2.10

Twovertices$v_i$and$v_j$aresaidtobeneighborsinPFGifeitheroneofthefollowingconditions hold.

i) $\mu_2(v_i,v_j)$>0,$\eta_2(v_i,v_j)$>0,$\gamma_2(v_i,v_j)$>0

ii) $\mu_2(v_i,v_j)$=0,$\eta_2(v_i,v_j)$≥0, $\gamma_2(v_i,v_j)$>0

iii) $\mu_2(v_i,v_j)$>0,$\eta_2(v_i,v_j)$=0, $\gamma_2(v_i,v_j)$≥0

iv) $\mu_2(v_i,v_j)$≥0,$\eta_2(v_i,v_j)$>0,$\gamma_2(v_i,v_j)$=0,$\forall v_i,v_j \epsilon V$.

Definition2.11

Let $v_i$beavertex in a Picturefuzzy graphG =(V,E)then

$N_s(v_i)$={$v_j \epsilon$V:(v_i,v_j)isastrongedge}iscalledstrongneighborhoodofv_i.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

$N_s[v_i]=N_s(v_i)\cup\{v_i\}$iscalledtheclosedstrongneighborhoodofv_i.

Definition2.12

Avertex$v_i\epsilon V$ofthePFG G=(V,E)is saidtobe anisolatedvertexif$\mu_2(v_i,v_j)=0$,$\eta_2(v_i,v_j)=0$and$\gamma_2(v_i,v_j)=0$forall$v_i\epsilon V i.e) N(u)=\varphi$.Thus, anisolated vertexdoes notdominate anyothervertexin G.

3. SomeClassification ofEdgesina PictureFuzzyTree

Dependingonthestrengthofedges(v_i,v_j)inaPFG,thefollowingthreedifferenttypes of edgesareintroduced.

Definition 2.2.1

An edge (v_i, v_j) in G with positive membership value $\mu_2(v_i, v_j)$, neutral membership value $\eta_2(v_i, v_j)$ and negative membership value $\gamma_2(v_i, v_j)$ is called

(i) α-μ strong edge if $\mu_2(v_i, v_j) > \mu_2'^{\infty}(v_i, v_j)$
(ii) α-η strong edge if $\eta_2(v_i, v_j) > \eta_2'^{\infty}(v_i, v_j)$
(iii) α-γ strong edge if $\gamma_2(v_i, v_j) < \gamma_2'^{\infty}(v_i, v_j)$
(iv) β-μ strong edge if $\mu_2(v_i, v_j) = \mu_2'^{\infty}(v_i, v_j)$
(v) β-η strong edge if $\eta_2(v_i, v_j) = \eta_2'^{\infty}(v_i, v_j)$
(vi) β-γ strong edge if $\gamma_2(v_i, v_j) = \gamma_2'^{\infty}(v_i, v_j)$
(vii) δ-μ weak edge if $\mu_2(v_i, v_j) < \mu_2'^{\infty}(v_i, v_j)$
(viii) δ-η weak edge if $\eta_2(v_i, v_j) < \eta_2'^{\infty}(v_i, v_j)$
(ix) δ-γ weak edge if $\gamma_2(v_i, v_j) > \gamma_2'^{\infty}(v_i, v_j)$, for all $v_i, v_j \in V$.

With the above definitions, edges are classified into three types:

(i) Robust edge (ii) Fragile edge (iii) Futile edge.

Definition3.1

An edge(v_i,v_j)iscalled asaμ-strongedge if itiseitherα-μstrong orβ-

μstrong.

Definition3.2

An edge(v_i,v_j)iscalled asaη-strongedgeifitiseitherα-ηstrong orβ-

ηstrong.

Definition3.3

An edge(v_i,v_j)iscalled as aγ-strongedge if itiseitherα-γstrong orβ-

γstrong.

Definition3.4

Anedge iscalleda robustedge ifitisμ-strong,η-strongandγ-strong.

Definition3.5

Anedgeiscalledafragileedgeifitiseitherδ-μweakorδ-ηweakorδ-

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

γweak.

Definition 3.6

Anedge(v_i,v_j)iscalledasafutileedgeifitisδ-μweak,δ-ηweakand

δ-γweak.

Example3.7

Infigure3.1,theedges(v_1,v_3),(v_2,v_3),(v_1,v_4)and(v_3,v_5)arerobustedges.(v_4,v_5)isafragileedge and(v_1,v_2)isafutile edge.

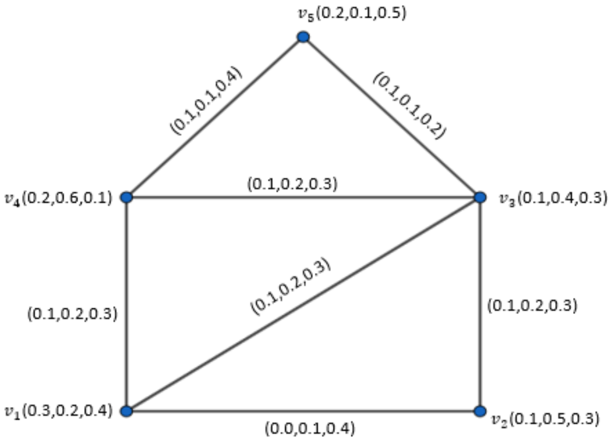

Figure3.1

Theorem 3.9

Let$G$=(V,E)beaPFG.Thenanedge(v_i,v_j)beanPF-bridgeiffitis

α-μstrong,α-ηstrong,α-γstrong.

Proof

Let$G$beaPFG. Then by thedefinition ofthe PF-bridge, wehave

$\mu_2'^{\infty}(v_i,v_j)<\mu_2^{\infty}(v_i,v_j)$,$\eta_2'^{\infty}(v_i,v_j)<\eta_2^{\infty}(v_i,v_j)and\gamma_2'^{\infty}(v_i,v_j)>$

$\gamma_2^{\infty}(v_i,v_j)$,forall$v_i$,$v_j\in V$ -- (1)

Since$G$is theconnected PFG, thestrength of theconnectedness betweenv_i

and$v_j$is$\mu_2(v_i,v_j)\geq\mu_2^{\infty}(v_i,v_j)$, $\eta_2(v_i,v_j)\geq\eta_2^{\infty}(v_i,v_j)$ and$\eta_2(v_i,v_j)\geq$

$\eta_2^{\infty}(v_i,v_j)$,forallv_i,$v_j\in V$ --(2)

Comparing(1)and(2),wehave

$\mu_2(v_i,v_j)>\mu_2'^{\infty}(v_i,v_j)$,$\eta_2(v_i,v_j)>\eta_2'^{\infty}(v_i,v_j)and\gamma_2(v_i,v_j)<\gamma_2'^{\infty}(v_i,v_j)$.

Conversely, suppose (v_i, v_j) is a α-μstrong, α-ηstrong and α-γstrong. Thenby definition, it follows that $v_iv_j$is the unique strongest path from $v_i$to $v_j$andthe removal of (v_i, v_j) will reduce the strength of the connectedness between $v_i$and

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

v_j.Thus (v_i,v_j)isaPF-bridge.

Corollary3.10

Let $G = (V, E)$ be a PFG. Then an edge (v_i, v_j) of $G$is a PF-bridge if it isrobustedge.

Remark 3.11

Ifanedge(v_i,v_j) inthePFGG=(V,E)isapicturefuzzybridge.

Theconverseneed not betrue.

Proposition 3.12

Inanedge(v_i,v_j)in$G$isanPF-bridge,then

$\mu_2(v_i,v_j)=\mu_2^\infty(v_i,v_j),\eta_2(v_i,v_j)=\eta_2^\infty(v_i,v_j) and \gamma_2(v_i,v_j)=\gamma_2^\infty(v_i,v_j)$

Forall$v_i,v_j \in V$.

Proof

By theorem 3.9, picture fuzzy bridge is α-μ strong, α-η strong and α-γ strong.

$\mu_2(v_i,v_j) > \mu_2'^\infty(v_i,v_j)$,$\eta_2(v_i,v_j) > \eta_2'^\infty(v_i,v_j)and\gamma_2(v_i,v_j) < \gamma_2'^\infty(v_i,v_j)$.

Therefore,max$(\mu_2(v_i,v_j),\mu_2\ (v_i,v_j))=\mu_2(v_i,v_j)$

max$(\eta_2(v_i,v_j),\eta_2(v_i,v_j))=\eta_2(v_i,v_j)$

min $(\gamma_2(v_i,v_j),\gamma_2(v_i,v_j))=\gamma_2(v_i,v_j)$

Bythedefinition2.11andremark 2.12,

$\mu_2^\infty(v_i,v_j)=\mu_2(v_i,v_j)$, $\eta_2^\infty(v_i,v_j)=\eta_2(v_i,v_j)and\gamma_2^\infty(v_i,v_j)=\gamma_2(v_i,v_j)$

Theorem 3.13

Let G= (V, E) be the Picture fuzzy graph G. An edge (v_i, v_j) in a Picture fuzzytree G is a robust edge iff (v_i, v_j) is an edge of the picture fuzzy spanning tree FofG.

Proof

Let(v_i,v_j)bearobustedgein G.Then by definition,

$\mu_2(v_i,v_j) > \mu_2'^\infty(v_i,v_j)$

$\eta_2(v_i,v_j)>\eta_2'^\infty(v_i,v_j)$-------------------(1)

$\gamma_2(v_i,v_j)<\gamma_2'^\infty(v_i,v_j)$

Assume that (v_i, v_j) is not in the picture fuzzy spanning tree F, then by thedefinition ofthepicturefuzzytree,

$\mu_{F2}(v_i,v_j)>\mu_2(v_i,v_j)$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

$\eta_{F2}(v_i,v_j) > \eta_2(v_i,v_j)$------------------------(2)

$\gamma_{F2}(v_i,v_j) < \gamma_2(v_i,v_j)$

Also $\mu_{F2}(v_i,v_j) \leq \mu_2{}'^{\infty}(v_i,v_j)$

$\eta_{F2}(v_i,v_j) \leq \eta_2{}'^{\infty}(v_i,v_j)$ -----------------------(3)

$\gamma_{F2}(v_i,v_j) \geq \gamma_2{}'^{\infty}(v_i,v_j)$

From(2)and(3)

Weget $\mu_2(v_i,v_j) < \mu_2{}'^{\infty}(v_i,v_j)$

$\eta_2(v_i,v_j) < \eta_2{}'^{\infty}(v_i, v_j)$

$\gamma_2(v_i,v_j) > \gamma_2{}'^{\infty}(v_i,v_j)$

which contradicts (1). Hence (v_i, v_j) is in the picture fuzzy spanning tree.Conversely,let(v_i,v_j) bethe vertexinthepicture fuzzyspanningtreeF.Then$(v_i,v_j)$is apicturefuzzybridgeandtheedge$(v_i,v_j)$is theuniquestrongestv_i-$v_j$path.

Hence$\mu_2(v_i,v_j) > \mu_2{}'^{\infty}(v_i,v_j)$

$\eta_2(v_i,v_j) > \eta_2{}'^{\infty}(v_i, v_j)$

$\gamma_2(v_i,v_j) < \gamma_2{}'^{\infty}(v_i, v_j)$

whichimplies that(v_i,v_j)isarobustedge.

Corollary3.14

LetG=(V,E)bethepicture fuzzytree.Anedge(v_i,v_j)isarobust edgeiff$(v_i,v_j)$is apicturefuzzybridgeofG.

Theorem 3.15

A picture fuzzy graph G = (V, E) is a picture fuzzy tree iff fragile edge doesnotexist in G.

Proof

Let G= (V, E) be a picture fuzzy tree and let F be the maximum picturefuzzyspanning tree.Then bytheorem3.13,everyedgein Fis arobustedge.

Suppose(v_i,v_j)isafragileedgeinG.Thentheedge(v_i,v_j)doesnotbelongto Fand by the definition ofpicturefuzzy tree,

$\mu_2(v_i,v_j) < \mu_2{}'^{\infty}(v_i,v_j)$ 2

$\eta_2(v_i,v_j) < \eta_2{}'^{\infty}(v_i,v_j)and\gamma_2(v_i,v_j) > \gamma_2{}'^{\infty}(v_i,v_j)$-------------------(1)
2

Also$\mu_{F2}(v_i,v_j) \leq \mu'^{\infty}(v_i,v_j)$ 2 2

$\eta_{F2}(v_i,v_j) \leq \eta'^{\infty}(v_i,v_j)$ --------------------(2)

$\gamma_{F2}(v_i,v_j) \geq \gamma'^{\infty}(v_i,v_j)$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 2026-2034
https://publishoa.com
ISSN: 1309-3452

From(1) and(2)

$\mu_2(v_i,v_j)<\mu'^{\infty}(v_i,v_j)$

$\eta 2(v_i,v_j)<\eta'^{\infty}(v_i,v_j)$ 2

$\gamma_2(v_i,v_j)>\gamma'^{\infty}(v_i,v_j)$

Which implies that (v_i, v_j) is a futile edge which is a contradiction. Hencethefragileedgedoes not existsin G.

Conversely, suppose let us assume that there is no fragile edge in G. IftherearenopicturefuzzycyclesinG,thenGisapicturefuzzytree.Nowassumethat $C_P$be the picture fuzzy cycle in G. Then $C_P$will contain only robust edgesandfutileedges.Also,everyedgeof$C_P$cannotbearobustedge.Therefore,thereexistsat least one futile edgeinC_P. Itallowsthat G is apicturefuzzytree.

Remark 3.16

Apicturefuzzy treeGcan havefutileedge.

Theorem 3.17

Let G= (V, E) be a picture fuzzy tree iff there exists a unique strong pathbetweenany two verticesin G.

Proof

LetG=(V,E)beapicturefuzzytree.LetFbethemaximumspanning

tree.Thenbytheorem3.13,anedge(v_i,v_j)inthepicturefuzzygraphG=(V,E)isa robustedgeiffit isan edgeof themaximum spanningtreeF.sinceFisapicture fuzzy tree, it contains a unique path between any two vertices. But itcontainsalltheverticesofG.Therefore∃auniquestrongpathbetweenanytwoverticesofFand henceitis a uniquestrong path in G.

4. AlgorithmtoComputeStrongEdgesinaPictureFuzzyTree

1. InputtheverticesV=$\{v_1,v_2,\cdots,v_n\}$withtheirmembershipvalues $\mu_1,\eta_1$and$\gamma_1$foranyv$_i\in$V.
2. Inputtheiredges $E = \{(v_i, v_j)/v_i, v_j\in V\}$ with theirmembershipvalues$\mu_2(v_i,v_j),\eta_2(v_i,v_j),\gamma_2(v_i,v_j)$forany$(v_i,v_j)\in E$
3. InitializeallpossiblepathsP$_i$, $\forall i=1,2,\cdots,n$
4. $S\mu_2(v_i,v_j)=\min\{\mu_2(v_i, v_j)/v_i, v_j\in V\}$
5. $S\eta_2(v_i,v_j)=\min\{\eta_2(v_i,v_j)/v_i,v_j\in V\}$
6. $S\gamma_2(vi,vj)=\max\{\gamma_2(vi,vj)/vi,vj\in V\}$
7. $\mu_2'^{\infty}(vi,vj)=\max\{S_{\mu 2}(vi,vj)/vi,vj \in V\}$
8. $\eta_2'^{\infty}(v\mathbf{i},v\mathbf{j})=\max\{S_{\eta 2}(v\mathbf{i},v\mathbf{j})/v\mathbf{i},v\mathbf{j}\in V\}$
9. $\gamma'_{2}{}^{\infty}(vi,vj)=\min\{S_{\gamma 2}(vi,vj)/vi,vj\in V\}$
10. Verifythecondition$\mu_2(v_\mathbf{i},v_\mathbf{j})\geq\mu'^{\infty}(v_\mathbf{i},v_\mathbf{j})\&\&\eta_2(v_\mathbf{i},v_\mathbf{j})\geq\eta_2'^{\infty}(v_\mathbf{i},v_\mathbf{j})\&\&\gamma_2(v_\mathbf{i},v_\mathbf{j})\leq\gamma_2'^{\infty}(v_\mathbf{i}, vj)$ holds.
11. Iftheconditionholds,determinetheedge(v_i,v_j)isastrongedge.

12. Ifnotdeterminetheedge(v_i,v_j)isnotastrongedge.

13. Repeatthesamestepswithremainingpairofverticesv$_i$,$v_j \in$V.

Conclusion

Inthispaper,theedgesina picture fuzzy treeareclassifiedintorobustedge,fragileedgeand futile edge based on their strength.Some properties and theorems on PF-bridge and picture fuzzy tree are also studied with examples. An algorithm hasbeenprovidedto computethe strong edgein picturefuzzy graph.

References

1. Atanassov.KonIntuitionisticfuzzygraphsand intuitionisticfuzzyrelations.Proceedings ofthe VI IFSA worldcongress. SaoPaulo,Brazil, July1995,Vol.1.551 -554.
2. K.R. Bhutani, A. Rosenfeld , strong arcs in fuzzy graphs, information science 152 (2003) 319 – 322.
3. Cen zuo, Anitapal and Arindamdey. New concepts of picture Fuzzy graphs with application. MDPI , Basel, Switzerland, Vol 7, May 2019, pp: 1-18.
4. Cuong, B.C , Kreinovich. V. Picture fuzzy sets – a new concept for computational intelligence problems in proceedings of the 2013Third world congress on information and communication technologies,Hanoi , Vietnam, Dec. 2013, pp: 15-18.
5. Karunambigai, M.G, Parvathi.R, Intuitionistic fuzzy graphs, Proceedings of 9th fuzzy days, International conference on computational Intelligence, theory and applications, Springer- Verlag, Vol.20, 2006, pp:139-150.
6. Karunambigai, M.G., Parvathi, R., &Buvaneswari, R, Arcs in intuitionistic fuzzy graphs,8th Int. Workshop on IFSs, BanskaBystrica,Notes on intuitionistic fuzzy set, Vol.18, Issue 4, Oct 2012, pp: 48-58.
7. Parvathi. R, Karunambigai,M.G, Atanassov. K.T, Operations on Intuitionistic fuzzy graphs, Proceedings of IEEE international conference on fuzzy systems, Aug 2009, pp:1396-1401.
8. Rosenfeld. A, Fuzzy graphs.In fuzzy sets and their applications to cognitive and decision processes,Academic press, 1975,pp: 77-95.
9. Sameena. K.Sunitha.M.S, strong arcs and maximum spanning trees in fuzzy graphs, International journal of mathematics science, vol 5, 2006, pp:17-20.
10. Sunitha.M.S,Mathew. S, Types of arcs in a fuzzy graph, Information sciences, Vol.179, 2009, pp:1760-1768.
11. Tom. M, Sunitha , M.S and Mathew. S, Notes on types of arcs in a fuzzy graph, Journal of uncertainty in mathematics science, Vol 2014, 2014, pp: 1-4.
12. Zadeh,L.A.Fuzzysets.Inf.Control, Vol 8, 1965, pp:338–353.