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Independent Domination Number for 6-Alternative Snake graphs

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Abstract

Let $G\ (V, E)$ be a graph, V has a subset C, contains vertices with at least one vertex in V that is not in C, then C is the dominating set of G. If the vertices of C are not adjacent to each other, then C is an independent dominating set of G and so the minimum cardinality of C represents the IDN. As we already know about the concepts of framing n-Alternative Triangular Snake graph$nA(T_n)$, n-Alternative Double Triangular Snake graph $nA(D(T_n))$, n-Alternative Quadrilateral Snake graph$nA(Q_n)$ and n-Alternative Double Quadrilateral Snake graph$nA(D(Q_n))$. In this paper, we find the IDN for 6-Alternative Triangular Snake graph $6A(T_n)$ and 6-Alternative Quadrilateral Snake graph $6A(Q_n)$

Keywords: Domination set, ID, IDN, T_n, $A(T_n)$,$D(T_n)$**,** Q_n, $A(Q_n)$, $D(Q_n)$.

Introduction

In the past the ideas of domination, is started with the game of chess. Later the work was extended by various peoples as, Ahrens in 1901 [16] Berge in 1958 [2] and ore in 1962 [9], by 1972 Cockayne and Hedetniemi [3,4] gone through domination and commenced to review it, tereby a survey was published in 1975 and there came into existence for the topic independent domination number. Thereon, many researchers started to work in that. Thus, the cubic and regular graphs are overviewed by Goddard et al. [13,15], Kostichka [1] and Lam et al. [10]. While the sharp upper bounds of general graphs were done by Favaron [8] and the extension is given by Haviland [7]. Cockayne et al. [5] found its boundary and its complement, while Shiu et al. [14] gave for triangle-free graphs and thereby characterizing its upper bounds.

Definition 1. [11,19] The graph $G(V, E)$, has a subset C of V, contains vertices with at least one vertex in V that is not in C, then C is the dominating set of G.

Definition 2. [1,19] If the vertices of C are not adjacent to each other, then C is an independent dominating set of G and so the minimum cardinality of C represents the IDN.

Definition 3. [12,17] **Triangular snake (**T_n**):**

In the path P_n we add a vertex corresponding to each edge to form a triangle C_3.

Definition 4. [12,17] **Alternate triangular snake**$A(T_n)$**:**

In the path $v_1, v_2, \dots v_n$ we add a vertex a_i $to\ v_i and\ v_{i+1}$(alternately). So that each alternate edge forms a triangle C_3.

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Definition 5. [18] **n-Alternate triangular snake**$nA(T_n)$**:**

In the path $v_1, v_2, \dots v_n$ we add a new vertex a_i to $v_i and\ v_{i+1}, v_{i+1} and\ v_{i+2}, \dots v_{i+(n-1)} and\ v_{i+n}$ (n-alternately). So that each n-alternate edge forms a triangle C_3.

Definition 6. [7,17] **Quadrilateral snake**Q_n**:**

In the path $v_1, v_2, \dots v_n$ we add a new vertices $a_i and\ b_i$ corresponding to the edges of the path $v_i and\ v_{i+1}$ and by joining $a_i and\ b_i$ for i=1,2,...n-1, we get a cycle C_4.

Definition 7. [7,17] **Alternate quadrilateral snake**$A(Q_n)$**:**

In the path $v_1, v_2, \dots v_n$ we add a new vertices $a_i and\ b_i$corresponding to the edges of the path $v_i and\ v_{i+1}$ and by joining $a_i and\ b_i$ for i≡1(mod2) and I≤n-1then joining $b_i and\ c_i$ alternatively we get a cycle C_4.

Definition 8. [18] **n-Alternate quadrilateral snake**$nA(Q_n)$**:**

In the path $v_1, v_2, \dots v_n$ we add new vertices $a_i and\ b_i$ corresponding to the edges of the path $v_i and\ v_{i+1}, v_{i+1} and\ v_{i+2}, \dots v_{i+(n-1)} and\ v_{i+n}$ and by joining $a_i and\ b_i$ for i≡1(modn)alternatively we get a cycle C_4.

Theorem 1 [18]:

Let us take the graph $6A(T_n)$ with the path P_n , then i($6A(T_n)$)= $\left\lceil \frac{n}{4} \right\rceil$

Proof:

Procedure for $6A(T_n)$:

The $6A(T_n)$ graph is defined by adding a new vertex for every six edges of $(v_{i-1}, v_i), (v_i, v_{i+1}), (v_{i+1}, v_{i+2}), (v_{i+2}, v_{i+3}), (v_{i+3}, v_{i+4}), (v_{i+4}, v_{i+5})$ alternatively.

We label the root path of the vertices as $v_1, v_2, \dots v_n$ thereby we add a vertex $a_1, a_2, a_3, a_4, a_5, a_6$ to the corresponding edges $v_1\ to\ v_7$.

Leave the next six edges $v_7\ to\ v_{13}$.

This Alternative process from the vertex v_1 to the vertex v_{13} is named as A_1 [1st alternative].

Again, add six new vertices $a_7, a_8, a_9, a_{10}, a_{11}, a_{12}$ to the corresponding edges $v_{13}\ to\ v_{19}$.

This Alternative process from the vertex v_1 to the vertex v_{19} is named as A_2 [2nd alternative].

Leave the next six edges $v_{19}\ to\ v_{25}$.

Continue this process till A_n.

This graph is named as G and now we find the set C (ID set) for a graph $6A(T_n)$, such that V-C has vertices which is adjacent to at least one vertex in C and thus the vertices in C are independent.

The process goes in such a way, so that we get the IDN for $6A(T_n)$ as (i.e.,)

i($6A(T_n)$)= $\left\lceil \frac{n}{4} \right\rceil$

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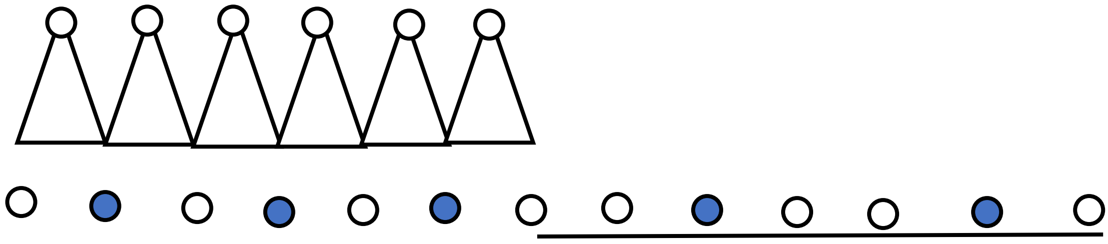

Figure-1 6-Alternative Triangular Snake graph

The highlighted vertices represent the Independent Domination.

Theorem 2 [18]:

Let us take the graph $6A(Q_n)$ with the path P_n, then i($6A(Q_n)$)= $\left\lceil\frac{n}{3}\right\rceil$

Proof:

Procedure for $6A(Q_n)$:

The $6A(T_n)$ graph is defined by adding six new pairs of vertices for every six edges of $(v_{i-1}, v_i), (v_i, v_{i+1}), (v_{i+1}, v_{i+2}), (v_{i+2}, v_{i+3}), (v_{i+3}, v_{i+4}), (v_{i+4}, v_{i+5})$ alternatively.

We label the root path of the vertices as $v_1, v_2, \dots v_n$ thereby we add six pairs of vertices $(a_1, b_1), (a_2, b_2), (a_3, b_3), (a_4, b_4), (a_5, b_5)$ *and* (a_6, b_6) to the corresponding edges v_1 *to* v_7.

Leave the next six edges v_7 *to* v_{13}.

This Alternative process from the vertex v_1 to the vertex v_{13} is named as A_1 [1st alternative].

Again, add six new pairs of vertices $(a_7, b_7), (a_8, b_8), (a_9, b_9), (a_{10}, b_{10}), (a_{11}, b_{11})$ *and* (a_{12}, b_{12}) to the corresponding edges v_{13} *to* v_{19}.

This Alternative process from the vertex v_1 to the vertex v_{19} is named as A_2 [2nd alternative].

Leave the next six edges v_{19} *to* v_{25}.

Continue this process till A_n.

This graph is named as G and now we find the set C (ID set) from a graph $6A(Q_n)$, such that V-C has vertices which is adjacent to atleast one vertex in C and thus the vertices in C are independent.

The process goes in such a way, so that we get the IDN for $6A(Q_n)$ as (i.e.,)

i($6A(Q_n)$)= $\left\lceil\frac{n}{3}\right\rceil$

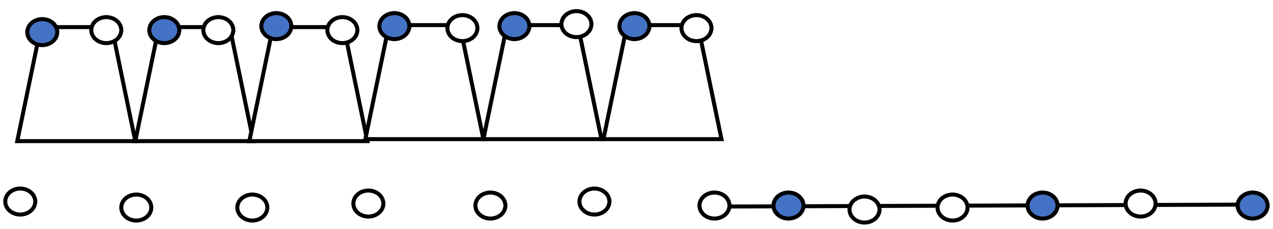

Figure-2 6-Alternative Quadrilateral Snake graph

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The highlighted vertices represent the Independent Domination.

Conclusion

We have taken some special types of snake graphs as our reference and found the extension for n-Alternate Triangular and n-Alternate Quadrilateral snake graph. For the n-Alternative Snake graph, the work from 2-Alternative to 5-Alternative Snake graphs have been done. Here in this paper we have found the particular graph namely, 6-Alternate Triangular Snake graph and 6-Alternate Quadrilateral Snake graph.

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