JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1884-1886
https://publishoa.com
ISSN: 1309-3452

Characterization of Double Domination of Star Graph Families

V.G. RAJU[1] M. BABU [2,*] AND M. HAJ MEERAL[3]

1. Research Scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai – 600117, Tamil Nadu, India. Email- masilamaniviji13@gmail.com
2. Assistant Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai – 600117, Tamil Nadu, India. Email- mbabu5689@gmail.com
* corresponding Author: mbabu5689@gmail.com
3. Assistant Professor, Department of Mathematics, The Quaide Milleth College for Men, Chennai – 600100, Tamil Nadu, India.

Received 2022 April 02**; Revised** 2022 May 20**; Accepted** 2022 June 18**.**

Abstract

A subset Sof V (G) is a "double dominating set" for G if S dominates every vertex of G at least twice. The minimum cardinality of a double dominating set is called double domination number and is denoted by γ_{dd}. In this paper, the double domination number of central, middle and total graphs of star graph is found.

1. INTRODUCTION

Let G = (V, E) be a simple undirected graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by d(u). The minimum and maximum degree of G is denoted by δ(G) and Δ(G) respectively. A path on n vertices is denoted by P_n. The graph with $V(B_{n,n})=\{u_1,u_2,u_3,\ldots.u_n,v_1,v_2,v_3,\ldots v_n\}$and $E(B_{n,n}) =\{uu_i,vv_i,uv: 1 \le i \le n\}$ is called the n-bistar and is denoted by $B_{n,n}$.

The central graph of G, denoted by C(G) is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in C(G).

The middle graph M(G) of a graph G is defined as follows: The vertex set of M(G) is V(G) ∪ E(G). Two vertices x, y in the vertex set of M(G) are adjacent in M(G) if either (i) x, y are in E(G) and x, y are adjacent in G or (ii) x is in V(G), y is in E(G) and x, y are incident in G. In other words, M(G) is obtained by subdividing each edge of G exactly once and joining all these newly added middle vertices of adjacent edges of G.

The total graph T(G) of a graph G is defined as a graph with vertex set V(G) ∪E(G) and two vertices x, y of T(G) are adjacent in T(G) if either (i) x, y are in V(G) and x is adjacent to y in G or (ii) x, y are in E(G) and x, y are adjacent in G or (iii) x is in V(G), y is in E(G) and x, y are incident in G.

A subset Sof V (G) is a "double dominating set" for G if S dominates every vertex of G at least twice. The minimum cardinality of a double dominating set is called double domination number and is denoted by γ_{dd}.

Here it is assumed that the vertex set $V(k_{1,n})$ of the star graph $k_{1,n}$is $\{v, v_1, v_2 \ldots, v_n\}$, where $v_1, v_2 \ldots, v_n$ are the pendent vertices of $k_{1,n}$ and the center vertex v is adjacent to vi, $1 \le i \le n$.

Theorem 1.1

For star graph $k_{1,n}$, $n \ge 2$, γ_{dd} $[C(k_{1,n})] = n + 1$.

Proof

By the definition of central graph, C($k_{1,n}$) is obtained by subdividing each edge vvi of $k_{1,n}$ exactly once by the vertex c_i, $1 \leq i \leq n$ in C($k_{1,n}$) and joining all the non-adjacent vertices $v_i v_j$ of $k_{1,n}$,$1 \leq i,j \leq n$. Let $V_1 = \{v_1, v_2 \ldots, v_n\}$and V′ = $\{c_1, c_2 \ldots, c_n\}$. Then $V(C(K_{1,n}) = V_1 \cup V' \cup \{v\}$, the set V′ is linearly independent and v_i is adjacent to v and v_i, $i = 1, 2, \ldots, n$. Also observe that induced subgraph < V_1 > is complete in $C(k_{1,n})$.

To find double domination number of $C(k_{1,n})$.

As $C(k_{1,n})$ contains a clique $< v_1 >$ of order n and since the double domination number of a complete graph is two. Without loss of generality, let $v_i and\ v_j$ be arbitrary elements of double dominating set in $< V_1 >$.

Since v is adjacent to all $d_i, 1 \leq i \leq n$, and deg of v is greater than or equal to all vertices of the vertex v along with vertices $v_i and\ v_j$adjacent to $d_i and\ d_j$ and remaining vertices of $V', d_t \neq\ d_i and\ d_j$,for $1 \leq t \leq n, t \neq i\ and\ t \neq j$ forms double dominating set.

Therefore, $\gamma_{dd}\left(C(k_{1,n})\right) = n + 1$.

Theorem 1. 2

For star graph $k_{1,n}, n \geq 2, \gamma_{dd}(M(k_{1,n})) = n + 2$.

Proof

By definition of middle graph $M(k_{1,n})$ is obtained by subdividing each edge v_i of $k_{1,n}$ exactly once by the vertex $d_i, 1 \leq i \leq n$ in $M(k_{1,n})$ and joining all these middle vertices c_i and c_j of adj. Edges of $k_{1,n}, 1 \leq i, j \leq n$.

Let $V_1 = \{v_1, v_2, \ldots v_n\}$ and $V' = \{d_1, d_2, \ldots d_n\}$. $V(M(k_{1,n})) = V_1 \cup V' \cup \{v\}$ and c_i is adjacent to $v_i, i = 1,2 \ldots n$. It is evident that induced subgraph $< V' >$ is complete in $M(k_{1,n})$. And double domination number of complete graph is 2. Let $d_i and\ d_j$ be two vertices of $V'$for $i \neq j$. Now, all the pendent vertices $v_1, v_2, \ldots v_n$ are in double dominating set.$d_i and\ d_j$ dominates all the vertices of V' and the root vertex v. All the pendant vertices $\{v_1, v_2, \ldots v_n\} - V$ dominates itself. Therefore, $\{d_i, d_j, v_1, v_2, \ldots v_n\}$ forms double dominating set of $M(k_{1,n})$ which implies $\gamma_{dd}(M(k_{1,n})) = n + 2$.

Theorem 1. 3

For star graph $k_{1,n}, n \geq 2, \gamma_{dd}(T(k_{1,n})) = n + 2$.

Proof

By definition of total graph T$(k_{1,n})$ is obtained by subdividing each edge v_i of $k_{1,n}$ exactly once by the vertex $d_i, 1 \leq i \leq n$ in T$(k_{1,n})$ and joining all these middle vertices c_i and c_j of adjacent edges of $k_{1,n}, 1 \leq i, j \leq n$ and also joining the adjacent vertices v and v_i of $k_{1,n}$, $1 \leq i \leq n$ in T$(k_{1,n})$. Let $V_1 = \{v_1, v_2, \ldots v_n\}$ and $V' = \{d_1, d_2, \ldots d_n\}$. $V(T(k_{1,n})) = V_1 \cup V' \cup \{v\}$ and c_i is adjacent to $v_i, i = 1,2 \ldots n$. It is evident that induced subgraph $< V' >$ is complete in $T(k_{1,n})$. The result is obtained by the method followed in previous theorem.

Conclusion

In this paper, the characterization of Central, middle and total star graph double domination number is derived after identifying the double domination set of the above mentioned graphs. There is scope for extending to various families of graphs which has vast applications in network security.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1884-1886
https://publishoa.com
ISSN: 1309-3452

References

1. Namasivayam. P (2008): **Studies in strong double domination in graphs**, Ph.D., thesis, Manonmaniam Sundaranar University, Tirunelveli, India.
2. Tersa W.Haynes, Stephen T.Hedetniemi and Peter J.Slater(1998): **Fundamentals of domination in graphs**, Marcel Dekker, Reading mass.
3. Tersa W.Haynes, Stephen T.Hedetniemi and Peter J.Slater(1998): **Domination in graphs,Advanced Topics,** Marcel Dekker, Reading mass.
4. D. Michalak(1983): **On Middle and Total graphs with Coarseness Number Equal 1**, Lecture Notes in Mathematics, Volume 1018: Graph Theory, Springer-Verlag, Berlin, 139 – 150.
5. Kavitha K and David N.G(2012): **Dominator Coloring on Star and Double Star Graph Families**, International Journal of Computer Applications (0975 – 888) Volume 48– No.3.pp 22-25
6. G. Mahadevan, A. Selvam Avadayappan and M. Hajmeeral (2012), **Non Split Dom Strong Domination Number Of A Graph,** International Journal Of Computational Engineering, 2 (8): pp. 39-46.
7. Manjula. C. Gudgeri , Varsha ,Double Domination Number of Some Families of Graph, International Journal of Recent Technology and Engineering (IJRTE) ISSN: 2277-3878 (Online), Volume-9 Issue-2, July 2020:pp 161-164