

S- Ideals in Almost Distributive Fuzzy Lattices

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Abstract: In an Almost Distributive Fuzzy Lattice (ADFL), the concept of S-ideals is introduced, and some significant features of these ideals are derived. S-ideals are used to categorise ADFLs. Prime ideals were also confirmed in ADFL. For the class of all S-ideals to become a DFL to the lattice of all ideals, a set of analogous requirements is stated, leading to the characterization of an ADFL.

Keywords: Almost Distributive Fuzzy Lattice (ADFL), ideal, prime ideal, S-ideal, maximal ideal.

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1. Introduction

U. M. Swamy and G. C. Rao, [5] proposed the concept of Almost Distributive Lattices (ADL) as a generalisation of an Almost Distributive Lattices (ADLs) [6], which was a common abstraction of almost all existing ring theoretic generalisations of a Boolean algebra on the one hand and distributive lattices on the other hand. However, L. A. Zadeh [7] introduced the concept of a fuzzy set in 1965. A fuzzy ordering, according to L.A. Zadeh [8], is a transitive fuzzy relation that is an elaboration of the concept of ordering. A fuzzy partial ordering is a reflexive and anti-symmetric fuzzy ordering in particular. In 1994, N. Ajmal and K. V. Thomas [1] created fuzzy sub lattices and built a fuzzy lattice as a fuzzy algebra. Based on Zadeh's fuzzy order concept, I. Chon [4] presented a novel notion of fuzzy lattices and investigated the level sets of fuzzy lattices in 2009. He also provided the notions of distributive and modular fuzzy lattices, as well as the essential characteristics of fuzzy lattices.

Berhanu et al. [2] proposed Almost Distributive Fuzzy Lattices (ADFLs) as a generalisation of Distributive Fuzzy Lattices and defined various characteristics of an ADL using I. Chon's fuzzy partial order relations and fuzzy lattices. B. Assaye and B. Tarekegn [10] extended the crisp notion to analogues of an ADFL, as well as the smallest ideal and smallest filter encompassing a non- empty subset of R of an ADFL. In [9], G.C. Rao and S. Ravi Kumar found that some results on an ADL's minimal prime ideal. [10] C.G. Rao, N. Rafi, and Ravi Kumar Bandaru [3] presented S-ideals and derived certain fundamental characteristics of Almost Distributive Lattices.

The concept of S-ideals is described using filters in an Almost Distributive Fuzzy Lattices in this paper (ADFL). According to numerous properties of these S-ideals, the set of all S-ideals of an ADFL produces a complete practically distributive fuzzy lattice. We establish a set of comparable requirements for the class of all S-ideals to create a fuzzy lattice of all ideals to characterise ADFL.

2. Preliminaries

We'll review some basic concepts and outcomes in this section.

Definition 2.1. [2] Let $(R, \vee, \wedge, 0)$ be an algebra of type $(2, 2, 0)$ and we call (R, A) is an Almost Distributive Fuzzy Lattice (ADFL) if the following condition satisfied:

- (1) $A(a, a \vee 0) = A(a \vee 0, a) = 1$;
- (2) $A(0, 0 \wedge a) = A(0 \wedge a, 0) = 1$;
- (3) $A((a \vee b) \wedge c, (a \wedge c) \vee (b \wedge c)) = A((a \wedge c) \vee (b \wedge c), (a \vee b) \wedge c) = 1$;
- (4) $A(a \wedge (b \vee c), (a \wedge b) \vee (a \wedge c)) = A((a \wedge b) \vee (a \wedge c), a \wedge (b \vee c)) = 1$;
- (5) $A(a \vee (b \wedge c), (a \vee b) \wedge (a \vee c)) = A((a \vee b) \wedge (a \vee c), a \vee (b \wedge c)) = 1$;
- (6) $A((a \vee b) \wedge b, b) = A(b, (a \vee b) \wedge b) = 1$, for all $a, b, c \in R$.

Definition 2.2. [10] Let L be an ADFL and I be any non empty subset of R . Then I is said to be an ideal of an ADFL L , if it satisfies the following axioms:

- (1) $a, b \in I$ implies that $a \vee b \in I$;
- (2) $a \in I, b \in R$ implies that $a \wedge b \in I$.

Definition 2.3. [12] A prime ideal of L is called a minimal prime ideal if it is a minimal element in the set of all prime ideals L ordered by set inclusion.

Theorem 2.4. [13] Let L be an ADL. Then a prime ideal P is minimal if and only if for any $x \in P$, there exist an element $y \notin P$ such that $x \wedge y = 0$.

Definition 2.5. [3] Let S be a sub-ADL of R . An ideal I of R is called a S -ideal of R if

$I = (I \cap S]$. An S -ideal I is called a S -prime ideal of R if, $I \cap S$ is a prime ideal of S , and S -maximal ideal if $I \cap S$ is a maximal ideal of S . It can be observed that every

S -maximal ideal is a S -prime ideal.

3. S -Ideals in Almost Distributive Fuzzy Lattices

The concept of S -ideals is expanded in Almost Distributive Fuzzy Lattice in this section. (R, A) stands for an ADFL $(R, \wedge, \vee, 0, 1)$ throughout this work.

Definition 3.1. Let (R, A) be an ADFL and S be a sub-ADFL. An ideal I_S of (R, A) is called a S -ideal if $I_S = (I \cap S] = \{x \in R \mid A(x \wedge s, 0) > 0 \forall s \in S\}$.

Example 3.2. Let $(R, A) = \{0, a, b, c, 1\}$ be an ADFL whose Hasse diagram is given in the following figure

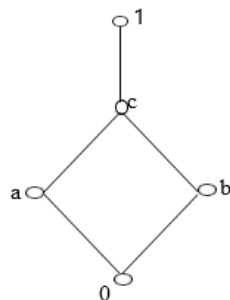


Figure 1: Hasse diagram of the ADFL $(R, A) = \{0, a, b, c, 1\}$.

Consider $I = \{0, a\}$ and $S = \{0, b, c, 1\}$

Clearly I is an ideal of (R, A) and S is an ideal of sub-ADFL (R, A)

$$\therefore I_S = (I \cap S) = \{0, a, b, c, 1\} = A(0, 0) > 0$$

Clearly (R, A) is a fuzzy poset, and if it satisfies the ADFL conditions. Therefore, I_S is a S -ideal in ADFL.

Theorem 3.3. Let (R, A) be a ADFL and S be a sub-ADFL (R, A) . Then the set

$$I_S = \{x \in R \mid A(x \wedge s, 0) > 0 \forall s \in S\} \text{ is an } S\text{-ideal of } (R, A).$$

Proof: Let (R, A) be an ADFL and S be a sub-ADFL (R, A) . By the definition S -ideal ADL, subsequent conditions.

$$x, y \in I \Rightarrow x \wedge y \in I \text{ and } x \in I, y \in R \Rightarrow x \wedge s \in I \cap S$$

$$\text{The Set } I_S \text{ is defined by } I_S = \{x \in R \mid A(x \wedge s, 0) > 0 \forall s \in S\}$$

Therefore, we get $A(x \wedge s, 0) > 0$ ($\because x$ is a dense element)

$$\Rightarrow A(0 \wedge s, 0) > 0$$

$$\Rightarrow A(0, 0) > 0.$$

Clearly $0 \in I_S$

$\therefore I_S$ is non-empty.

$$\text{Let } x, y \in I_S \text{ and } (y \wedge 0, 0) > 0, \forall s \in S.$$

$$\text{Therefore every } x_i \in I \text{ such that } \{A((\bigwedge_{i=1}^n x_i) \wedge s, 0) > 0, \forall s \in S\}$$

$$\Rightarrow I_S = (I \cap S) = \{A((\bigwedge_{i=1}^n x_i) \wedge s, 0) > 0 \forall s \in S\}$$

Hence $I_S = (I \cap S)$ is an I_S -ideal of (R, A) be an ADFL.

Lemma 3.4. For any Ideal I of an ADFL (R, A) , then I_S is a S -ideal in (R, A) .

Proof: Clearly $0 \in I_S$. Let $x, y \in I_S$. Then, $A((a \wedge s), 0) = A((b \wedge u), 0) > 0$

for some $s, u \in S$.

$$\text{Now } A((a \vee b) \wedge (s \wedge u), 0) > 0$$

$$\Rightarrow A(((a \wedge s \wedge u) \vee (b \wedge s \wedge u)), 0) > 0$$

$$\Rightarrow A(((0 \wedge u) \vee (b \wedge u \wedge s)), 0) > 0 \quad (\because a \wedge s = b \wedge u = 0)$$

$$\Rightarrow A((0 \vee (s \wedge 0)), 0) > 0$$

$$\Rightarrow A((0 \vee 0), 0) > 0$$

$$\Rightarrow A(0, 0) > 0$$

Hence $A((a \vee b), 0) \in I_S$. Again, let $a \in I_S$ and $x \in R$.

Then $A((a \wedge s), 0) > 0$ for some $s \in S$.

$$\text{Now } A((a \wedge x) \wedge s, 0) > 0$$

$$\Rightarrow A((x \wedge 0), 0) > 0$$

$$\Rightarrow A(0, 0) > 0$$

So $A((a \wedge x), 0) \in I_S$.

Thus I_S is an S -ideal in (R, A) .

Definition 3.5. For any ideal I is an ADFL (R, A) , the set

$I_S = (I \cap S) = \{x \in R \mid A(x \wedge s, 0) > 0 \forall s \in S\}$ is an S -ideal of (R, A) . It is called an annihilator S -ideal of (R, A) .

Remark 3.6. For any ideal I of ADFL (R, A) .

The set $I_S = \{x \in R \mid A(x \wedge s, 0) > 0 \forall s \in S\}$. This I_S is an S -ideal of ADFL (R, A) .

The set $S(L)$ is set of all S -ideals of (R, A) is a bounded ADFL with least element $\{0\}$ and greater element (R, A) , for any $I_S, J_S \in S(L)$, the supremum element of I_S and J_S is defined by

$I_S \wedge J_S = \{A((a \wedge b), 0) > 0 \mid a \in I_S, b \in J_S\}$ and the infimum element is defined by

$I_S \vee J_S = \{A((a \vee b), 0) > 0 \mid a \in I_S, b \in J_S\}$.

Theorem 3:8. Let I_S and J_S be any two S -ideals of an ADFL (R, A) . Then $I_S \wedge J_S$ and $I_S \vee J_S$ are also S -ideals of (R, A) , where $I_S \wedge J_S = I_S \cap J_S$ and $I_S \vee J_S = \{x \vee y \mid x \in I_S \text{ and } y \in J_S\}$.

Proof: Let (R, A) be a ADFL and I_S and J_S be any two S -ideals of ADFL (R, A) . Since I_S and J_S be non-empty, take x and y from I_S and J_S respectively.

Now, since $(x \wedge y) \in I_S, J_S$ and $x = x \vee (x \wedge y) \in I_S \vee J_S$ and $y = (x \wedge y) \wedge y \in I_S \vee J_S$, $x \in I_S \vee J_S$ and $y \in I_S \vee J_S$. Hence $I_S \vee J_S \neq \varnothing$

i) Let $a, b \in I_S \vee J_S$. Then there exist $x, y \in I_S$ and $u, v \in J_S$ such that $a = x \vee u$ and $b = y \vee v$.

Then $(x \vee u) \wedge (a \vee b) \in I_S$ and $(y \vee v) \wedge (a \vee b) \in J_S$

Now, $I_S = A(a \vee b, (x \vee u) \vee (y \vee v))$

$$= A(a \vee b, (x \vee u) \vee (y \vee v)) \wedge (a \vee b))$$

$$= A\left(a \vee b, (((x \vee y) \vee (u \vee v)) \wedge (a \vee b))\right) > 0$$

and similarly, $I_S = A(((x \vee u) \vee (y \vee v)), a \vee b)$

$$= A(((x \vee y) \vee (u \vee v)) \wedge (a \vee b), a \vee b) > 0$$

Hence $a \vee b = (((x \vee y) \vee (u \vee v)) \wedge (a \vee b)) \in I_S \vee J_S$

ii) Let $a \in I_S \vee J_S$ and $x \in R$. Then there exists $x \in I_S$ and $u \in J_S$ such that $a = x \vee u$.

Since I_S and J_S are S -ideals of (R, A) , $a \wedge x \in I_S$ and $a \wedge u \in J_S$ and it implies that

$$(a \wedge x) \vee (a \wedge u) \in I_S \vee J_S$$

$$\text{Now, } A(a \wedge c, (x \wedge c) \vee (u \wedge c)) = A(a \wedge c, ((x \vee u) \wedge c))$$

$$= A(a \wedge c, a \wedge c) > 0$$

$$= A(0, 0) > 0$$

Hence $a \wedge c \in I_S \vee J_S$.

Thus $I_S \vee J_S$ is a S -ideal of (R, A) . To show $I_S \wedge J_S$ is a S -ideal of (R, A) . Then $a, b \in I_S$ and $a, b \in J_S$ and it follows that $a \vee b \in I_S$ and $a \vee b \in J_S$. Hence $a \vee b \in I_S \wedge J_S$.

iii) Let $a \in I_S \wedge J_S$ and $c \in R$, then $a \in I_S$ and $a \in J_S$ since $a \wedge c \in I_S, J_S$, $a \wedge c \in I_S \wedge J_S$ (By the definition of ideal). Hence $I_S \wedge J_S$ is a S -ideals of (R, A) .

Definitions 3.9. A S -ideal $M \subseteq L$ is said to be a maximal S -ideal there exist any S -ideal, I_S in (R, A) such that $M \not\subseteq I_S$. Every proper S -ideal of (R, A) is contained in the maximal S -ideal.

Theorem 3.10. If (R, A) is an ADFL contained the maximal S -ideal. Then every proper S -ideal of R is contained in a maximal S -ideal of R .

Proof: Let (R, A) is an ADFL contained the maximal S -ideal. Therefore, there exist any S -ideal I_S in (R, A) such that $M \not\subseteq I_S$ not smaller than any other element in R . Let M of R is a proper S -ideal it contained a maximal element. This implies M of R does not contained in any proper S -ideal of R . Therefore, M is a maximal of S -ideal of ADFL. Hence every proper S -ideal of R is contained in a maximal ideal M .

Theorem 3.14. Let (R, A) be a ADFL and I_S be an S -ideal of (R, A) . Then

$(I \cap S] = \{x \in R | A(x, (\bigwedge_{i=1}^n s_i) \wedge x) > 0 \forall s_i \in S, x \in R \text{ and } n \in N\}$ is the smallest

S -ideal of (R, A) containing S .

Proof: Let I_S be a S -ideal of (R, A) . Since $S \neq \varphi$, there exist $S \in I_S \subset (R, A)$ such that

$$\begin{aligned} A(s, (\bigwedge_{i=1}^n s_i) \wedge s) &= A(s, (s \vee s \vee \dots \vee s) \wedge s) \\ &= A(s, s \wedge s) \\ &= A(s, s) > 0 \text{ where } s_i = s \forall i = 1, 2, \dots, n \end{aligned}$$

Hence $S \in (I \cap S]$ and it implies that $(I \cap S] \neq \varphi$

Let $a, b \in (I \cap S]$.

Then $A(a, (\bigwedge_{i=1}^n s_i) \wedge a) > 0$ and $A(b, (\bigwedge_{j=1}^m t_j) \wedge b) > 0$ for some $s_i, t_j \in S$.

Since (R, A) is a ADFL, $a = \bigwedge_{i=1}^n s_i \wedge a$ and $b = \bigwedge_{j=1}^m t_j \wedge b$

Now let $\{r_k | k = 1, 2, \dots, n + m\} = \{s_i | i = 1, 2, \dots, n\} \cup \{t_j | j = 1, \dots, m\}$

Hence, we have $\bigwedge_{k=1}^{n+m} r_k = \bigwedge_{i=1}^n s_i \vee \bigwedge_{j=1}^m t_j$ where $s_i, t_j \in S$ and it implies that

$r_k \in S, \forall k = 1, 2, \dots, n + m$

$$\begin{aligned} \text{Then } A(a \vee b, ((\bigwedge_{k=1}^{n+m} r_k) \wedge (a \vee b))) &= A(a \vee b, ((\bigwedge_{i=1}^n s_i \vee \bigwedge_{j=1}^m t_j) \wedge (a \vee b))) \\ &= A(a \vee b, ((\bigwedge_{i=1}^n s_i \vee \bigwedge_{j=1}^m t_j) \wedge a) \vee ((\bigwedge_{i=1}^n s_i \vee \bigwedge_{j=1}^m t_j) \wedge b)) \\ &= A(a \vee b, ((\bigwedge_{i=1}^n s_i) \wedge a) \vee ((\bigwedge_{j=1}^m t_j) \wedge a)) \vee (((\bigwedge_{i=1}^n s_i) \wedge b) \vee ((\bigwedge_{j=1}^m t_j) \wedge b))) \\ &= A(a \vee b, ((a \vee (\bigwedge_{j=1}^m t_j) \wedge a)) \vee (((\bigwedge_{i=1}^n s_i) \wedge b) \vee b)) \\ &= A(a \vee b, a \vee b) > 0 \\ &= A(0, 0) > 0 \end{aligned}$$

Hence $a, b \in (I \cap S]$

Finally, let $a \in (I \cap S]$ and $x \in R$ Then $A(a, (\bigwedge_{i=1}^n s_i) \wedge a) > 0$ for some $s_i \in S$

Now, $A(a \wedge x, (\bigwedge_{i=1}^n s_i) \wedge (a \wedge x)) = A(a \wedge x, (\bigwedge_{i=1}^n s_i) \wedge a) \wedge x$

$$= A(a \wedge x, a \wedge x) > 0$$

Hence $x \in (I \cap S]$

Thus $(I \cap S]$ is an S – ideal of $I_S = (R, A)$

Now, let $s \in S$, then $A(s_i (\bigwedge_{i=1}^n s_i) \wedge s) = 1 > 0$. where $s_i = S \forall i = 1, 2 \dots n$

Hence $s \in (I \cap S]$ and it follows that $S \subset (I \cap S]$

To show $(I \cap S]$ is the smallest S – ideal containing S .

Let us consider an S – ideal I_S if (R, A) such that $S \subseteq I_S$

Let $a \in (I \cap S]$. Then $A(a, (\bigwedge_{i=1}^n s_i) \wedge a) > 0$ where $s_i \in S$ (since $S \subseteq I_S$), $s_i \in I_S$ and I_S is an S – ideal $\bigwedge_{i=1}^n s_i \in I_S$.

On the other hand, $((\bigwedge_{i=1}^n s_i) \wedge a, a \wedge (\bigwedge_{i=1}^n s_i)) \in I_S$ and then $a = (\bigwedge_{i=1}^n s_i) \wedge a \in I_S$.

Hence $a \in I_S$ and it follows that $(I \cap S] \subseteq I_S$

Thus $(I \cap S]$ is the smallest S – ideal of (R, A) containing S . Hence proved.

In the following theorem we proved the characterization of S – ideals in ADFL.

Theorem 3.15. Let (R, A) be an ADFL. Then the following are equivalent.

1. (R, A) is an DFL.
2. Every ideal is an S – ideal.
3. Every annihilator ideal is an S – ideal.
4. For $x \in (R, A)$, I is an S – ideal.

Proof: (1) \Rightarrow (2).

Assume that (R, A) is an DFL. Let I be an ideal of (R, A) .

Consider the set $I_S = \{x \in R | A(x \wedge s, 0) > 0 \forall s \in S\}$.

First, we prove that I_S is a S – ideal of (R, A) .

Let $x, y \in I_S$ then $A(x \wedge s, 0) > 0$ and $A(y \wedge s, 0) > 0 \forall s \in S$. Since (R, A) is an ADFL.

\therefore We get $A(0, x \wedge s) > 0$ and $A(0, y \wedge s) > 0 \forall s \in S$.

This implies $x \wedge s = 0$ and $y \wedge s = 0 \forall s \in S$

$$A((x \vee y) \wedge s, 0) = A((x \wedge s) \vee (y \wedge s), 0)$$

$$= A((0 \vee 0), 0)$$

$$= A(0, 0) > 0$$

\therefore We get $x \vee y \in I_S$

Again let $x \in I_S$ and $r \in R$. Then we get $A(x \wedge s, 0) > 0$ and $A(r \wedge s, 0) > 0$

$$\begin{aligned}\Rightarrow A((x \vee r) \wedge s, 0) &= A((x \wedge s) \vee (r \wedge s), 0) \\ &= A((0 \vee 0), 0) \\ &= A(0, 0) > 0\end{aligned}$$

\therefore We get $x \vee r \in I_S$. Therefore I_S is an ideal of (R, A) .

Now, we show that $I = I_S$.

Let $x \in I_S$. Then $x \wedge f = 0$ for some $f \in I_S$. Hence $x \in (I \cap S]$

Now $f \in I_S \Rightarrow A(x \wedge f, 0) > 0$ for some $x \in I \Rightarrow A(0, 0) > 0 \forall x \in I$

\therefore We get $x \wedge f \in I$. Therefore $I_S \subseteq I$ -----(1)

Conversely, let $x \in I$ since (R, A) be an ADFL there exist $y \in R$ such that

$\{y \in R | A(y \wedge x, 0) > 0 \forall x \in I\}$. Since $x \in I$, we get $y \in I_S$. Also $x \in I_S = y$.

Hence $x \in I_S$. Thus $I \subseteq I_S$ -----(2).

Therefore (1) and (2) we get $I_S = I$.

Thus I is an S -ideal.

Hence every ideal is an S -ideal.

(2) \Rightarrow (3), Since every annihilator S -ideal is an ideal, it is clear.

(3) \Rightarrow (4), Since I_S is an annihilator ideal, it is obvious.

(4) \Rightarrow (1), Let $x \in R, I$ is an S -ideal. This implies $I = I_S$ of (R, A) .

Let $x, y \in R$, since $A(x \wedge y, (x \wedge y) \wedge y) = A((x \wedge y) \wedge y, x \wedge y) = 1$

then $(x, x \wedge y) \in I$. Also $A(y \wedge y, y \wedge y) = 1$. Hence $(y, y) \in I$.

Since I is an S -ideal on (R, A) implies $(x \vee y, (x \wedge y) \vee y) \in I$.

$$\Rightarrow A((x \vee y) \wedge y, y \wedge y) = A(y \vee y, (x \vee y) \wedge y) = 1$$

$\therefore (R, A)$ is an Almost Distributive Fuzzy Lattice.

Hence (R, A) is an ADFL. Hence proved.

Conclusion: Several features of S -ideals in ADFL were proved in this paper. S -ideals theorems in ADFL were also studied. Finally, we conclude that the results of our S -ideals characterization reveal a link between ADL and ADFL. Future study will focus on the isomorphism of ADFL as well as the congruence of ideals and filters in ADFL.

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