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An Optimal Algorithm for a Pythagorean Fuzzy Transportation Problem Using Decagonal Numbers

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Abstract: Pythagorean Transportation Problem (PTP) is applied on Capacity and Requirement of commodities taken from one source to the different destinations. In this paper Decagonal Pythagorean Fuzzy Numbers using Transportation problem by ranking method and Centroid Ranking Technique and Proposed Ranking Method.. The transportation cost can be minimized by using of Proposed Ranking Method under Score function Method. The procedure is illustrated with a numerical example.

Keywords: **P**ythagorean Transportation problems, Decagonal fuzzy numbers, ranking method, PRM, Initial Basic Feasible Solution, Optimal Solution.

1. Introduction:

Transportation problem (TP) plays with one or more factors (origins) to number of various warehouses (destinations).Transportation algorithm is one of the influential system to deliver the merchandise to the consumer in proficient platform. Transportation problem is less expensive commodities to the customers in more proficient approaches. Transportation problem provides the require advancement and cautious accessibility of raw equipments and finished items. The motto of transportation problem is to maximize the profit or to minimize the transportation cost.Prof. Zadeh introduced the concept of fuzzy set to develop with uncertainty and vagueness of things in real life situations. Fuzzy set apply to be incomplete due to the omission of non-membership function and omit of hesitation degree. Atanassov contributed the concept of Intuitionistic Fuzzy Set [IFS] in 1986, such that and Atanassov constructed the generalization of IFS which deals with the situation known as Intuitionistic Fuzzy Set. Anthor level of fuzzy subset was defined by Yager in 2013, known as Pythagorean Fuzzy set [PFS]. PFS insists uncertain and vague messages more efficiently than IFS, it is applied in solving decision making, engineering, science and real life problems. Pythagorean Fuzzy Set (PFS) is focused in a new source to deal with uncertainty considering the membership sign "μ" and non-membership sign "ν" satisfying the conditions ($\mu + \nu \leq 1$) or ($\mu + \nu \geq 1$), and also, it continues that ($\mu2 + \nu2 + \pi2$) = 1, where "π" is the Pythagorean fuzzy set index. Score function for the ranking level of Interval Valued Pythagorean Fuzzy Sets pointed out by Garg which idea next level of IFS. Lotfi Zadeh introduced fuzzy set which he expressed the goal of the author to point out the typical sign of a set and a proposition to put up Fuzziness in the point that it is in human language,(i.e) in human traditions, evaluation, and decisions making. Ỉntuitionistic Fuzzy Sets (ỈFSs) initiated by Atanassov who has highlighted these ideas and delivered a tool. The system incorporates both membership function "μ" and non-membership function "ν" with hesitation range "π" such that $\mu + \nu \leq 1$ and $\mu + \nu + \pi = 1$. He applied on Ỉntuitionistic Fuzzy Sets Next Type with the condition that the sum of the square of the membership and non-membership grade is less than or equal to one. A new theory encompassing philosophical belief which addresses nature and possibility of neutralities was given by Florentin Smarandache in 1995. A Neutrosophic set carrys truth membership, inderminancy membership and falsity membership. Wang et. al in 2005, defined an illustration of neutrosophic set known as single valued neutrosophic set. A single valued neutrosophic number is a special case of neutrosophic set which deals with uncertain and complex data Neutrosophic Set (NS) aimed by Smarandache which has the concept of which plays the inherent struggles that the output in Fuzzy Sets (FS) and İntuitionistic Fuzzy Sets (İFS) and Ƥythagorean Fuzzy Sets (ƤFS).

2.1: Definition: [11] The classical transportation problem concerns minimizing the cost of transporting a single product from sources to destinations. It is a path problem that gives in industrial logistics and is considered as a special method of linear programming problem. The total number of units produces at each source, the total number of units required at each destination and the cost to transport one unit from each source to each destination is the basic values. The objective will be minimizing the total cost of transporting the units produced at sources to meet the demands at destinations.

2.2:Ƒuzzy Set: [12] Ƒuzzy set Ά in Real is given to be a set {(x, (μΆ (x)) |x∈R }, μΆ(x): X tends to [0,1] is mapping called the degree of membership function of the fuzzy set Ά and μΆ(x) is called the membership value of x ∈X in the fuzzy set Ά . These membership values are represented by real numbers lying in [0, 1].

2.3:Definition: [9] Fuzzy number is formulated as a fuzzy set defining a fuzzy interval in the real number. Generally a fuzzy interval is represented by two end points p and r and q peak point as [p, q, r]. It is a fuzzy set the following conditions:

- Convex fuzzy set , normalized fuzzy set , It is defined in the real number
- It ís membership function is piecewise continuous.

2.4: Definition: [6] Let X is a Classical set, a Pythagorean fuzzy set is an object having the form

P = {(x,(τP (x), φP (x))/x ∈ X },where the function τP (x): X→ [0, 1] and φP (x): X→[0, 1] are the degree of membership and non-membership of the element x ∈ X to P, respectively.

Also for every x∈ X, it holds that $(\tau P(x))^2 + (\varphi P(x))^2 \leq 1$.

2.5: Definition: [6] Let $\alpha_1^p = (\tau_i^p, \varphi_u^p)$ and $\beta_1^p = (\tau_j^p, \varphi_v^p)$ be two Pythagorean Fuzzy Numbers (PFNs).

Then the arithmetic operations are as follows:

(i) Additive property: $\alpha_1^p \oplus \beta_1^p = (\sqrt{}(\tau_i^p)^2 + (\tau_j^p)^2 - (\tau_i^p)^2 (\tau_j^p))^2, \varphi_u^p . \varphi_v^p)$

(ii) Multiplicative property: $\alpha_1^p \otimes \beta_1^p = (\tau_i^p) . \partial_s^p \sqrt{}(\varphi_u^p)^2 + (\varphi_v^p)^2 - (\varphi_u^p))^2 (\varphi_v^p)^2)$

(iii) Scalar product: $k\alpha_1^p = (\sqrt{}1 - (1 - \tau_i^p), (\varphi_u^p)^k)$

where k is nonnegative constant .i.e.k > 0

2.6: Definition: (Comparison of two PFNs)

Let $\alpha_1^p = (\tau_i^p, \varphi_u^p)$ and $\beta_1^p = (\tau_j^p, \varphi_v^p)$ be two Pythagorean Fuzzy Numbers such that the score and accuracy function are as follows:

(i) Score function: $S(\alpha_1^p) = \frac{1}{2}(1 - (\tau_i^p)^2 - (\varphi_u^p)^2)$

(ii) Accuracy function: $(\alpha_1^p) = (\tau_i^p)^2 + (\varphi_u^p)^2$

Then the following five cases arise:

Case 1: If $\alpha_1^p > \beta_1^p$ iff $S(\alpha_1^p) > S(\beta_1^p)$

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Case 2: If $\alpha_1^p < \beta_1^p$ iff S(α_1^p)< S(β_1^p)

Case 3: If S(α_1^p)= S(β_1^p) and H(α_1^p)< H(β_1^p) ,then $\alpha_1^p < \beta_1^p$

Case 4: If S(α_1^p)= S(β_1^p) and H(α_1^p)>H(β_1^p) ,then $\alpha_1^p > \beta_1^p$

Case 5: If S(α_1^p)= S(β_1^p) and H(α_1^p)= H(β_1^p) ,then $\alpha_1^p = \beta_1^p$

2.7: Definition:[7] Ranking Technique

Let P = {(x , (τ_P (x), φ_P (x))} a Pythagorean fuzzy number. The ranking R of˜ on the set of Pythagorean fuzzy number is defined as follows:

R(P) = $\{(\tau P(x))^2 + (\varphi P(x))^2\}/2$

2.8: Definition – Ranking Technique: [14] A positioning procedure that fulfills praise, linearity and addictive homes offers consequences that depend upon human instinct. If $\alpha_i^p$is a fuzzy number after that the place is distinct by

R(α_i^p) = $\frac{1}{\alpha_i^p} \sum \alpha_i^p$, i = 1 to 10.

3.1:Mathematical Formulation of Pythagorean fuzzy transportation Problem

The Pythagorean fuzzy transportation problem can be represented in the form of n x n cost table $[C_{ij}]$after defuzzification as given below.

The costs $[C_{ij}]$ = (τ_P (x), φ_P (x)) are Pythagorean fuzzy numbers. The goal is to minimize the

Pythagorean fuzzy cost incurred in transportation effectively.

The Pythagorean fuzzy transportation problem can be mathematically expressed as

Min Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} C^p{}_{ij} x^p{}_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a^p{}_i$, i = 1,2,……,m.

$\sum_{i=1}^{m} x_{ij} = b^p{}_j$, j = 1,2,……,n.

$x_{ij} \geq 0$, for all i and j.

A Pythagorean fuzzy transportation problem is said to be balanced if the total Pythagorean fuzzy supply from all sources equal to the total Pythagorean fuzzy demand in all destination

$\sum_{j=1}^{n} a^p{}_i = \sum_{j=1}^{n} b^p{}_j$, otherwise is called unbalanced.

3.2:Definition: [13] Pythagorean Decagonal Fuzzy Number (PDFN):

A Pythagorean Decagonal fuzzy number $(a_p^1, a_p^2, a_p^3, a_p^4, a_p^5, a_p^6, a_p^7, a_p^8, a_p^9, a_p^{10})$can be defined as and the membership function is defined as

$$\mu_{A_p}(x) = \begin{cases} \frac{1}{4}\frac{(x-a_p^1)}{(a_p^2-a_p^1)}, & a_p^1 \le x \le a_p^2 \\ \frac{1}{4}+\frac{1}{4}\frac{(x-a_p^2)}{(a_p^3-a_p^2)}, & a_p^2 \le x \le a_p^3 \\ \frac{1}{2}+\frac{1}{4}\frac{(x-a_p^3)}{(a_p^4-a_p^3)}, & a_p^3 \le x \le a_p^4 \\ \frac{3}{4}+\frac{1}{4}\frac{(x-a_p^4)}{(a_p^5-a_p^4)}, & a_p^4 \le x \le a_p^5 \\ 1, & a_p^5 \le x \le a_p^6 \\ 1-\frac{1}{4}\frac{(x-a_p^6)}{(a_p^7-a_p^6)}, & a_p^6 \le x \le a_p^7 \\ \frac{3}{4}-\frac{1}{4}\frac{(x-a_p^7)}{(a_p^8-a_p^7)}, & a_p^7 \le x \le a_p^8 \\ \frac{1}{2}-\frac{1}{4}\frac{(x-a_p^8)}{(a_p^9-a_p^8)}, & a_p^8 \le x \le a_p^9 \\ \frac{1}{4}\frac{(x-a_p^9)}{(a_p^{10}-a_p^9)}, & a_p^9 \le x \le a_p^{10} \\ 0, & otherwise \end{cases}$$

3.3: Proposed approach of Pythagorean decagonal fuzzy Transportation Problem

Step 1: Test whether the given Pythagorean decagonal fuzzy transportation problem is balanced

or not.

(i) If it is a balanced (i.e., the total supply is equal to the total demand) then go to step 3.
(ii) If it is an unbalanced (i.e., the total supply is not equal to the total demand) then go to step 2.

Step 2: Introduce dummy rows and /or dummy columns with zero Pythagorean fuzzy costs(decagonal number)to form a balanced one.

Step 3: Find the rank of each cell C_{ij} of the chosen Pythagorean fuzzy cost matrix by using

the ranking function as mentioned.

Step 4: Apply Zero Cost Method, (i.e) If demand is greater than supply then the respective cell cost will be zero. After complete this process then apply VAM method.

Step 5: Proceed by the VAM method to find the initial basic feasible solution and if m+n-1 =

No. of allocations, then proceeds by Modified Distribution method to obtain the optimal solution.

Step 6: Add the optimal Pythagorean fuzzy cost using Pythagorean decagonal fuzzy addition mentioned to optimize the cost.

4.1: Numerical Example:

The input data for Decagonal fuzzy transportation problem is given bellow. The optimal solution of the process will be minimizing the transportation cost and maximizing the profit.

Fuzzy transportation problem with decagonal numbers

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	D1	D2	D3	D4	Supply
S1	**(-5,−4,-3,-2,1, 2,3,4,5,6)**	**(−7,-6,-5,0,2, 3,4,5,6,7)**	**[-18,-16,-14,-11,0, 8,11,14,16,18)**	**(-13,-10,-9,-7,1, 2,7,9,10,13]**	**(-1,0,1,2,3, 8,17,29,40,51)**
S2	**(-13,-12,-11,-9,0,5, 9,11,12,13)**	**(−4,−3,−2,−1,0, 1,2,4,5,6)**	**(-11,-10-9,-7,2, 5,7,9,10,11)**	**(-13.-12,-11,-9,0, 6,9,11,12,13)**	**(-1,1,5,20,25, 30,35,40,45,50)**
S3	**(-11,-10,-9,-8,0, 6,8,9,10,11)**	**(-13,-12,-11,-9,2, 5,9,11,12,13)**	**(-21,-20,-18,-16,0, 4,16,18,20,21)**	**(-16,-14,-13,-8,1,7 ,8,13,14,16)**	**(-10,-7,12,17,18, 19,21,22,33,35)**
Demand	**(-4,0,4,5,10, 16,17,19,20,23)**	**(0,1,2,3,4, 5,7,8,9,11)**	**(−23,-21,13,19,21, 23,30,35,36,37)**	**(−1,0,10,13,15, 23,30,35,50,55)**	

Solution:

Step 1: The Decagonal fuzzy numbers transportation problem is converted into Pythagorean fuzzy transportation problem using the ranking method.

	D1	D2	D3	D4	Supply
S1	**(0.7,0.3)**	**(0.9,0.1)**	**(0.8,0.2)**	**(0.3,0.7)**	**15**
S2	**(0.5,0.5)**	**(0.8,0.2)**	**(0.7,0.3)**	**(0.6,0.4)**	**25**
S3	**(0.6,0.4)**	**(0.7,0.3)**	**(0.4,0.6)**	**(0.8,0.2)**	**16**
Demand	**11**	**5**	**17**	**23**	

Step 2: Using Score function to convert PFTP into Crisp number

	D1	D2	D3	D4	Supply
S1	**0.21**	**0.09**	**0.16**	**0.21**	**15**
S2	**0.25**	**0.16**	**0.21**	**0.24**	**25**
S3	**0.24**	**0.21**	**0.24**	**0.16**	**16**
Demand	**11**	**5**	**17**	**23**	

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Step 3: After apply the zero cost method the matrix will be

	D1	D2	D3	D4	Supply
S1	**0.21**	**0.09**	**0**	**0**	**15**
S2	**0.25**	**0.16**	**0.21**	**0.24**	**25**
S3	**0.24**	**0.21**	**0**	**0**	**16**
Demand	**11**	**5**	**17**	**23**	

IBFS = (11×0.25) + (5×0.16) + (9 ×0.21) + (8×0) + (15×0) + (8×0)

= Rs. 5.44

After apply MODI method, All $\Delta_{ij} \geq 0$, so the optimal value is 5.44.

Comparison Table:

S.No.	NWCM	Optimal Solution	LCM	Optimal Solution	VAM	Optimal Solution	Proposed Method	Optimal Solution
1	**10.64**	**10.64**	**10.51**	**10.51**	**10.51**	**10.51**	**5.44**	**5.44**

Conclusion:

[7] This paper has proposed a new ranking for Pythagorean decagonal fuzzy numbers. The proposed ranking is applied to explicate Pythagorean decagonal fuzzy transportation problem. Further, a numerical example is explained whose costs are taken as Pythagorean decagonal fuzzy numbers .The proficiency of the proposed technique is shown in the comparison table. As a future extension, the proposed algorithm may be used to solve, Pythagorean fuzzy Assignment problem (any number) and Pythagorean fuzzy interval valued fuzzy assignment and Pythagorean fuzzy transportation problems with any number.

References:

1. L. A. Zadeh, "Fuzzy Sets", Information and Computation, vol.8, 338-353, 1965.
2. Atanassov K T, "Intuitionistic fuzzy sets", Fuzzy sets Syst., 20, 87-96, 1986.
3. Yager R R, "Pythagorean fuzzy subsets", In: Proceedings of the joint IFS world congress NAFIPS annual meeting, 57-61, 2013.
4. Yager R R, "Pythagorean membership grades in multicriteria decision making", In: Technical report MII-3301, Machine Intelligence Institute, Iona College, New Rochelle, 2013.
5. Kaur A, Kumar A, "A new method for solving fuzzy transportation problems using ranking function", Appl Math Model 35:5652-5661, 2011.
6. S.Krishna Prabha , S. Sangeetha , P. Hema , Muhammed Basheer, and G. Veeramala, "Geometric Mean with Pythagorean Fuzzy Transportation Problem", Turkish Journal of Computer and Mathematics Education Vol.12 No.7 (2021),1171-1176.
7. R. M. Umamageswari and G.Uthra , " A Pythagorean Fuzzy Approach to Solve Transportation Problem", Adalya Journal Volume 9, Issue 1, 2020.
8. K.Jeyalakshmi,L.Chitra,G.Veeramalai,S.Krishna Prabha and S. Sangeetha, "Pythagorean Fuzzy Transportation Problem via Monalisha Technique", Annals of R.S.C.B.Vol. 25, Issue 3, 2021, Pages. 2078 – 2086, 2021.

9. G.Charles Rabinson and R.Chandrasekaran, "A Method for Solving a Pentagonal Fuzzy Transportation Problem via Ranking Technique and ATM",Journal of Research in Engineering, IT and Social Sciences, Impact Factor: 6.565, Volume 09 Issue 04, , Page 71-75, 2019.
10. D. Santhoshkumar and G.Charles Rabinson, "A New Proposed Method to Solve Fully FuzzyTransportation Problem Using Least Allocation Method", International Journal of Pure and Applied Mathematics, Volume 119 No. 15 2018, 159-166.
11. K. Nandhini and G. Charles Rabinson, "Socratic Technique to Solve the Bulk Hexadecagonal Fuzzy Transportation Problem Using Ranking Method", Advances in Mathematics: Scientific Journal 10 (2021), no.1, 331–337.
12. M Pachamuthu and G Charles Rabinson, "Neutrosophic Fuzzy Transportation Problem for finding optimal solution Using Nanogonal Number", Annals of R.S.C.B. Vol. 25, Issue 6, 2021, Pages. 18672 – 18676.
13. J.Jon Arockiaraj1 and N.Sivasankar, "A Decagonal Fuzzy Number and its Vertex Method", International Journal of Mathematics And its Applications, Volume 4, Issue 4 (2016), 283–292.
14. D. Gurukumaresan, C. Duraisamy and R. Srinivasan, "Optimal Solution of Fuzzy Transportation Problem Using Octagonal Fuzzy Numbers", Computer Systems Science & Engineering, 2021, vol.37, no.3, pages: 415-421.

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