

DOM-CHROMATIC NUMBER OF CERTAIN LADDER GRAPHS

Joice Punitha M.¹, Beulah Angeline E. F.², Helda Mercy M.³

¹Department of Mathematics,
Bharathi Women's College, Chennai, Tamilnadu-600108, India

E-mail : joicepunithabwcmath@gmail.com

²Department of Mathematics,
Nazareth College of Arts and Science, Chennai, Tamilnadu-600062, India

E-mail : efbeulahenry@gmail.com

³Department of Mathematics,
Department of Information Technology, Panimalar Engineering College,
Tamilnadu

E-mail : mercy_hilda@yahoo.co.in

Abstract:

The dom-coloring set of a graph G is a non-empty subset S of $V(G)$ such that S is a dominating set of G , which contains atleast one vertex from each color class of G . The minimum cardinality of a dom-coloring set is called the dom-chromatic number of the graph G and is denoted by $\gamma_{dc}(G)$. In this paper, we have designed an algorithm called the dom-coloring algorithm and also determined the dom-chromatic number of certain ladder graphs like Mobius ladder graphs and circular ladder graphs.

Keywords and Phrases: Dominating set, Chromatic number, Dom-coloring set, Dom-chromatic number.

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1. Introduction and Preliminaries

The basic idea of graphs were first introduced in the 18th century by the Swiss Mathematician Leonhard Euler. A graph $G = (V, E)$ consists of a set of objects $V = \{v_1, v_2, \dots, v_p\}$ called vertices and another set $E = \{e_1, e_2, \dots, e_q\}$, whose elements are called edges, such that each edge e_i is identified with an unordered pair

of vertices (v_i, v_j) . Graph colouring and domination play an equally important role in Graph Theory. Both concepts have a number of applications in real life which includes scheduling, assignment of radio frequencies, separating combustible chemical combinations and computer optimization [8]. The study of domination in graphs was developed in 1958 by Claude Berge [1]. Oystein Ore in 1962 introduced the terms “Dominating set” and “Domination number” [2]. In 1964, A. M. Yaglom and I. M. Yaglom made a detailed study on domination and ended up in solutions to some problems for rooks, knights, kings and bishops [3]. After a decade, Cockayne and Hedetniemi published a survey paper in which the notation $\gamma(G)$ was first used for domination number of a graph [4]. Today several research papers have been published and an extensive study has been carried on in the area of domination.

Graph coloring originated with the four color conjecture in 1852. Later in 1880, P.G.Tait, Professor of Natural philosophy in Edinburgh, gave further proofs of the four color conjecture. Finally in 1976, the four-color problem was settled by K. Appel and W. Haken, and further developed by proving Tait’s result that the edges of every cubic map can indeed be colored using only three colors. Today graph coloring is popular and extensively researched subject in the field of Graph Theory.

The combination of the two broad concepts give rise to a new problem called the dom-chromatic problem, which deals with the determination of the dom-chromatic number. In 2011, Janakiraman and Poobalaranjani introduced the *dom-chromatic number* $\gamma_{ch}(G)$, by defining the dom-chromatic set S , which is a dominating set with the property that $\chi(< S >) = \chi(G)$ [5]. Later, Chaluvvaraju and Appajigowda in 2016, defined the *dom-chromatic number* $\gamma_{dc}(G)$, by using the dom-coloring set S , which is a dominating set with atleast one vertex from each color class of G . Among the above definitions on dom-chromatic number, the latter one yields an optimal value in most of the graphs and was found to be equal to the domination number in certain classes of graphs [13]. For example, let $G = P_5$ be a path. Clearly $\chi(G) = 2$ and $\gamma(G) = \gamma_{dc}(G) = 2$. But $\gamma_{ch}(G) \neq 2$, since $< S >$ has isolated vertices and hence $\chi(< S >) \neq \chi(G)$. Hence $\gamma_{ch}(G) = 3$. In this paper we consider finite, connected and undirected graphs without loops and multiple edges.

Definition 1.1. *A non-empty subset S of the vertex set V of a graph $G(V, E)$ is said to be a dominating set if for every vertex v in $V(G) - S$, there is a vertex u in S such that u is adjacent to v . This set S is said to be a minimum if it has the*

least number of vertices dominating the graph G .

The cardinality of a minimum dominating set is called the domination number of G and is denoted by $\gamma(G)$ [10,11].

Definition 1.2. *A proper coloring of a graph G is the assignment of colors to the vertices of G such that no two adjacent vertices receive the same color. The least number of colors needed to color the graph in such a manner is called the chromatic number of G and is denoted by $\chi(G)$. The set of all vertices with any one color is independent and is called a color class.*

Definition 1.3. *A dom-coloring set is a non-empty dominating set S of $V(G)$ containing atleast one vertex from each color class of G . The minimum cardinality of a dom-coloring set is called the dom-chromatic number of the graph G and is denoted by $\gamma_{dc}(G)$ [6].*

2. Main Results

In this section we determine the dom-coloring set of Mobius and circular ladder graphs by designing an algorithm called the dom-coloring algorithm which gives a proper coloring to the graph G with the dominating set having atleast one vertex from each color class of G called the dom-coloring set.

Theorem 2.1. [6] *For any graph G , $\max\{\gamma(G), \chi(G)\} \leq \gamma_{dc}(G) \leq \gamma(G) + \chi(G) - 1$. The bounds are sharp.*

2.1 Dom-chromatic number of Mobius ladder graphs

Definition 2.1. *A Mobius ladder graph M_n is a cubic graph with $2n$ vertices and $3n$ edges obtained from the ladder $P_n \times P_2$ by joining together the opposite end points of the two copies of P_n [7].*

In other words, a Mobius ladder graph M_n is obtained by the union of two paths $P_{n_1} : u_1, u_2, \dots, u_n$ and $P_{n_2} : v_1, v_2, \dots, v_n$ with additional edges $(u_1, v_n), (u_n, v_1)$ and $(u_i, v_i), i = 1, 2, 3, \dots, n$ [12].

2.1.1 Dom-coloring algorithm of Mobius ladder graphs

Here, we construct a dom-coloring algorithm for Mobius ladder graph.

Algorithm 1 Dom-coloring algorithm of Mobius ladder graphs

Require: Mobius ladder graph M_n with $2n$ vertices and $3n$ edges, $n \geq 3$

Ensure: Dom-coloring set of $M_n, n \geq 3$

Check the value of n

if n is even **then**

$i \leftarrow 1, j \leftarrow 2$

while $(i \leq n \text{ and } j \leq n - 2)$ **do**

Color $u_i = 1, u_{i+1} = 2, v_j = 1, v_{j+1} = 2$

$i \leftarrow i + 2, j \leftarrow j + 2$

end while

Color $v_1 = v_n = 3, u_n = 2, u_{n-1} = 1$

if $n \equiv 0 \pmod{4}$ **then**

$i, j \leftarrow 1$

while $(i, j \leq \frac{n}{4})$ **do**

$D_1 = \{u_{4i-1}, v_{4j-3}\}$

$i \leftarrow i + 1, j \leftarrow j + 1$

end while

$D = D_1 \cup \{v_n\}$

end if

if $n \equiv 2 \pmod{4}$ **then**

$i, j \leftarrow 1$

while $(i \leq \lfloor \frac{n+1}{4} \rfloor \text{ and } j \leq \lfloor \frac{n}{4} \rfloor + 1)$ **do**

$D_1 = \{u_{4i-1}, v_{4j-3}\}$

$i \leftarrow i + 1, j \leftarrow j + 1$

end while

$D = D_1 \cup \{v_{n-1}\}$

end if

else

for $(i = 1 ; i < n ; i = i + 2)$ **do**

Color $u_i = 1, u_{i+1} = 2, v_i = 2, v_{i+1} = 1$

end for

Color $u_n = 1, v_n = 2$

$i, j \leftarrow 1$

while $(i \leq \lfloor \frac{n}{4} \rfloor + 1 \text{ and } j \leq \lfloor \frac{n+1}{4} \rfloor)$ **do**

$D_1 = \{u_{4i-3}, v_{4j-1}\}$

$i \leftarrow i + 1, j \leftarrow j + 1$

end while

$D = D_1 \cup \{u_n\}$

end if

Proof of Correctness: The proposed algorithm gives a proper coloring of M_n for any n . The set of vertices in D form a dominating set of M_n , as it dominates the remaining vertices of M_n . Since D contains atleast one vertex from each color class of M_n , D is a dom-coloring set of M_n .

Theorem 2.2. Let G be a Mobius ladder graph M_n with $2n$ vertices and $3n$ edges. Then the dom-chromatic number of $M_n, n \geq 3$ is given by

$$\gamma_{dc}(M_n) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor & \text{if } n \text{ is odd} \\ \frac{n}{2} + 1 & n \equiv 0 \pmod{4} \\ \frac{n}{2} & n \equiv 2 \pmod{4} \end{cases}$$

Proof. Let G be a Mobius ladder graph M_n with $2n$ vertices and $3n$ edges, $n \geq 3$. We claim the dom-chromatic number of M_n in the following cases.

Case 1: When n is odd ($n = 2k + 1, k = 1, 2, 3, \dots$)

Let $D = \{u_{4i-3}, v_{4j-1}\}$ for $i = 1, 2, 3, \dots, \left\lfloor \frac{n}{4} \right\rfloor + 1$ and $j = 1, 2, 3, \dots, \left\lfloor \frac{n+1}{4} \right\rfloor$. Choose any arbitrary vertex of M_n , say u_2 which dominates the three adjacent vertices u_1, v_2 and u_3 . This comprises a maximum of 4 vertices namely $\{u_1, u_2, u_3, v_2\}$, as the degree of each vertex of M_n is 3. Proceeding in this manner, the domination number of M_n is $\gamma(M_n) = \left\lceil \frac{2n}{4} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$. Thus D yields a minimum dominating set that contains atleast one vertex from each color class of G . Hence D is a dom-coloring set.

Therefore the cardinality of D is $\gamma_{dc}(M_n) = \left\lfloor \frac{n}{4} \right\rfloor + 1 + \left\lfloor \frac{n+1}{4} \right\rfloor = \left\lfloor \frac{2k+1}{4} \right\rfloor + 1 + \left\lfloor \frac{2k+2}{4} \right\rfloor = \left\lfloor \frac{4k+3}{4} \right\rfloor + 1 = k + 1 = \left\lceil \frac{k+1}{2} \right\rceil = \left\lceil \frac{2k+1}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$.

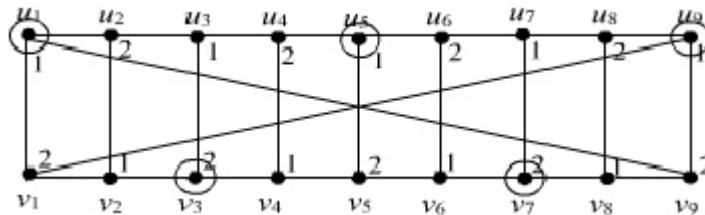


Fig. 1: Mobius ladder M_9

Case 2: When n is even.

Subcase (i): $n \equiv 0 \pmod{4}, n = 4k, k = 1, 2, \dots$

Let $D = \{u_{4i-1}, v_{4j-3}\} \cup \{v_n\}$ for $i, j = 1, 2, 3, \dots, \frac{n}{4}$. Choose any arbitrary vertex of M_n , say u_3 which dominates the three adjacent vertices u_2, v_3 and u_4 . This comprises maximum of 4 vertices namely $\{u_2, u_3, u_4, v_3\}$, as the degree of each vertex of M_n is 3. Proceeding in this manner, the domination number of M_n is $\gamma(M_n) = \left\lceil \frac{2n}{4} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$. Thus D yields a minimum dominating set that contains atleast one vertex from each color class of G . Hence D is a dom-coloring set. Therefore the cardinality of D is $\gamma_{dc}(M_n) = \frac{n}{4} + \frac{n}{4} + 1 = \frac{4k}{4} + \frac{4k}{4} + 1 = 2k + 1 = \frac{4k}{2} + 1 = \frac{n}{2} + 1$.

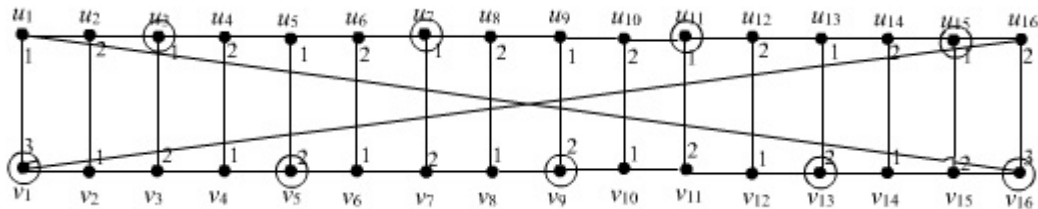


Fig. 2: Mobius ladder M_{16}

Subcase (ii): $n \equiv 2 \pmod{4}, n = 4k + 2, k = 1, 2, \dots$

Let $D = \{u_{4i-1}, v_{4j-3}\}$ for $i = 1, 2, 3, \dots, \left\lfloor \frac{n+1}{4} \right\rfloor$ and $j = 1, 2, 3, \dots, \left\lfloor \frac{n}{4} \right\rfloor + 1$. Choose any arbitrary vertex of M_n , say u_3 which dominates the three adjacent vertices u_2, v_3 and u_4 . This comprises maximum of 4 vertices namely $\{u_2, u_3, u_4, v_3\}$, as the degree of each vertex of M_n is 3. Proceeding in this manner, the domination number of M_n is $\gamma(M_n) = \left\lceil \frac{2n}{4} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$. Thus D yields a minimum dominating set that contains atleast one vertex from each color class of G . Hence D is a dom-coloring set. Therefore the cardinality of D is $\gamma_{dc}(M_n) = \left\lfloor \frac{n+1}{4} \right\rfloor + \left\lfloor \frac{n}{4} \right\rfloor + 1 = \left\lfloor \frac{4k+3}{4} \right\rfloor + \left\lfloor \frac{4k+2}{4} \right\rfloor + 1 = \left\lfloor k + \frac{3}{4} \right\rfloor + \left\lfloor k + \frac{1}{2} \right\rfloor + 1 = k + k + 1 = 2k + 1 = \frac{4k+2}{2} = \frac{n}{2}$.

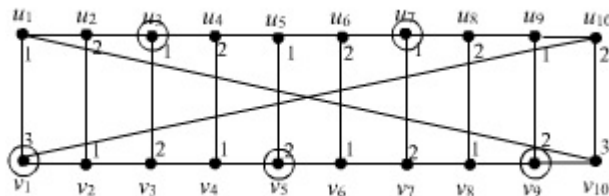


Fig. 3: Mobius ladder M_{10}

□

2.2 Dom-chromatic number of circular ladder graphs

Definition 2.2. A circular ladder graph $CL(n)$ is defined as the cartesian product $C_n \times K_2$ where K_2 is the complete graph on two vertices and C_n is the cycle graph on n vertices [9].

In other words, a circular ladder graph $CL(n)$ is defined as the union of an outer cycle $u_1, u_2, \dots, u_n, u_1$ and inner cycle $v_1, v_2, \dots, v_n, v_1$ with additional edges $(u_i, v_i), i = 1, 2, 3, \dots, n$

2.2.1 Dom-coloring algorithm of circular ladder graphs

Algorithm 2 Dom-coloring algorithm of circular ladder graphs

Require: Circular ladder graph $CL(n)$ with $2n$ vertices and $3n$ edges, $n \geq 4$

Ensure: Dom-coloring set of $CL(n), n \geq 4$

Check the value of n

if n is odd **then**

$i \leftarrow 1$

if $n \equiv 0 \pmod{3}$ **then**

while $(i < n)$ **do**

Color $u_i = v_{i+2} = 1, u_{i+1} = v_i = 2, u_{i+2} = v_{i+1} = 3$

$i \leftarrow i + 3$

end while

end if

if $n \equiv 1 \pmod{3}$ **then**

while $(i < n)$ **do**

Color $u_i = v_{i+2} = 1, u_{i+1} = v_i = 2, u_{i+2} = v_{i+1} = 3$

$i \leftarrow i + 3$

end while

Color $u_n = 2$ and $v_n = 3$

end if

if $n \equiv 2 \pmod{3}$ **then**

while $(i < n - 1)$ **do**

Color $u_i = v_{i+2} = 1, u_{i+1} = v_i = 2, u_{i+2} = v_{i+1} = 3$

$i \leftarrow i + 3$

end while

Color $u_{n-1} = 1, u_n = v_{n-1} = 2, v_n = 3$

end if

end if

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if  $n$  is even then
     $i \leftarrow 1$ 
    while  $(i < n)$  do
        Color  $u_i = 1, u_{i+1} = 2, v_i = 2, v_{i+1} = 1$ 
         $i \leftarrow i + 2$ 
    end while
end if
if  $n \equiv 2 \pmod{4}$  then
     $i, j \leftarrow 1$ 
    while  $(i \leq \lfloor \frac{n}{4} \rfloor \text{ and } j \leq \lfloor \frac{n}{4} \rfloor)$  do
         $D_1 = \{u_{4i-3}, v_{4j-1}\}$ 
         $i \leftarrow i + 1, j \leftarrow j + 1$ 
    end while
     $D = D_1 \cup \{u_{n-1}, v_n\}$ 
end if
if  $n \equiv 0 \pmod{4}$  or  $n \equiv 3 \pmod{4}$  then
     $i, j \leftarrow 1$ 
    while  $(i \leq \lfloor \frac{n}{4} \rfloor \text{ and } j \leq \lfloor \frac{n-2}{4} \rfloor + 1)$  do
         $D = \{u_{4i-3}, v_{4j-1}\}$ 
         $i \leftarrow i + 1, j \leftarrow j + 1$ 
    end while
end if
if  $n \equiv 1 \pmod{4}$  then
     $i, j \leftarrow 1$ 
    while  $(i \leq \lfloor \frac{n}{4} \rfloor \text{ and } j \leq \lfloor \frac{n}{4} \rfloor)$  do
         $D_1 = \{u_{4i-3}, v_{4j-1}\}$ 
         $i \leftarrow i + 1, j \leftarrow j + 1$ 
    end while
     $D = D_1 \cup \{u_n\}$ 
end if

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Proof of Correctness: The proposed algorithm gives a proper coloring of $CL(n)$ for any n . The set of vertices in D form a dominating set of $CL(n)$, as it dominates the remaining vertices of $CL(n)$. Since D contains atleast one vertex from each color class of $CL(n)$, D is a dom-coloring set of $CL(n)$.

Theorem 2.3. Let G be a circular ladder graph $CL(n)$ with $2n$ vertices and $3n$ edges. Then the dom-chromatic number of $CL(n), n \geq 4$ is given by

$$\gamma_{dc}(G) = \begin{cases} \frac{n}{2} & n \equiv 0 \pmod{4} \\ \left\lceil \frac{n}{2} \right\rceil & n \not\equiv 2 \pmod{4} \\ \frac{n}{2} + 1 & n \equiv 2 \pmod{4} \end{cases}$$

Proof. Let G be a circular ladder graph $CL(n)$ with $2n$ vertices and $3n$ edges, $n \geq 4$. We claim the dom-chromatic number of $CL(n)$ in the following cases.

Case 1: $n \equiv k \pmod{4}, k \neq 2$

Let $D = \{u_{4i-3}, v_{4j-1}\}$ for $i = 1, 2, 3, \dots, \left\lceil \frac{n}{4} \right\rceil$ and $j = 1, 2, 3, \dots, \left\lfloor \frac{n-2}{4} \right\rfloor + 1$. Choose any arbitrary vertex of $CL(n)$, say u_2 which dominates the three adjacent vertices u_1, v_2, u_3 . This comprises maximum of 4 vertices namely $\{u_1, u_2, u_3, v_2\}$ as the degree of each vertex of $CL(n)$ is 3. Proceeding in this manner the domination number of $CL(n)$ is $\gamma(CL(n)) = \left\lceil \frac{2n}{4} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$. Thus D yields a minimum dominating set that contains atleast one vertex from each color class of G . Hence D is a dom-coloring set.

Subcase (i): $n \equiv 0 \pmod{4}, n = 4k, k = 1, 2, \dots$

The dom-chromatic number $\gamma_{dc}(CL(n))$ is given by the cardinality of D .

$$\gamma_{dc}(CL(n)) = \left\lceil \frac{n}{4} \right\rceil + \left\lfloor \frac{n-2}{4} \right\rfloor + 1 = \left\lceil \frac{4k}{4} \right\rceil + \left\lfloor \frac{4k-2}{4} \right\rfloor + 1 = k + \left\lfloor k - \frac{1}{2} \right\rfloor + 1 = k + k - 1 + 1 = 2k = \frac{4k}{2} = \frac{n}{2}.$$

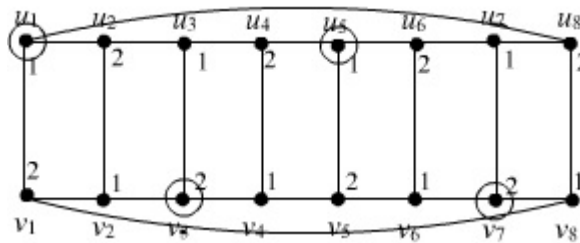


Fig. 4: Circular ladder $CL(8)$

Subcase (ii): $n \equiv 1 \pmod{4}, n = 4k + 1, k = 1, 2, \dots$

The dom-chromatic number $\gamma_{dc}(CL(n))$ is given by the cardinality of D .

$$\gamma_{dc}(CL(n)) = \left\lceil \frac{n}{4} \right\rceil + \left\lfloor \frac{n-2}{4} \right\rfloor + 1 = \left\lceil \frac{4k+1}{4} \right\rceil + \left\lfloor \frac{4k+1-2}{4} \right\rfloor + 1 = \left\lceil k + \frac{1}{4} \right\rceil + \left\lfloor \frac{4k-1}{4} \right\rfloor + 1 = k + 1 + \left\lfloor k - \frac{1}{4} \right\rfloor = k + 1 + (k - 1) + 1 = 2k + 1 = \left\lceil 2k + \frac{1}{2} \right\rceil = \left\lceil \frac{4k+1}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil.$$

Subcase (iii): $n \equiv 3 \pmod{4}, n = 4k + 3, k = 1, 2, \dots$

The dom-chromatic number $\gamma_{dc}(CL(n))$ is given by the cardinality of D .

$$\gamma_{dc}(CL(n)) = \left\lceil \frac{n}{4} \right\rceil + \left\lfloor \frac{n-2}{4} \right\rfloor + 1 = \left\lceil \frac{4k+3}{4} \right\rceil + \left\lfloor \frac{4k+3-2}{4} \right\rfloor + 1 = \left\lceil k + \frac{3}{4} \right\rceil + \left\lfloor k + \frac{1}{4} \right\rfloor + 1 = k + 1 + k + 1 = 2k + 2 = \left\lceil 2k + \frac{3}{2} \right\rceil = \left\lceil \frac{4k+3}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil.$$

Case 2: $n \equiv 2 \pmod{4}, n = 4k + 2, k = 1, 2, \dots$

Let $D = \{u_{4i-3}, v_{4j-1}\}$ for $i = 1, 2, 3, \dots, \left\lfloor \frac{n}{4} \right\rfloor$ and $j = 1, 2, 3, \dots, \left\lfloor \frac{n-2}{4} \right\rfloor + 1$. Choose any arbitrary vertex of $CL(n)$, say u_2 which dominates the three adjacent vertices u_1, v_2 and u_3 . This comprises maximum of 4 vertices namely $\{u_1, u_2, u_3, v_2\}$ as the degree of each vertex of $CL(n)$ is 3. Proceeding in this manner the domination number of $CL(n)$ as $\gamma(CL(n)) = \left\lceil \frac{2n}{4} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$. Thus D yields a minimum dominating set that contains atleast one vertex from each color class of G . Hence D is a dom-coloring set. The dom-chromatic number $\gamma_{dc}(CL(n))$ is given by the cardinality of D .

$$\gamma_{dc}(CL(n)) = \left\lceil \frac{n}{4} \right\rceil + \left\lfloor \frac{n}{4} \right\rfloor + 1 = \left\lceil \frac{4k+2}{4} \right\rceil + \left\lfloor \frac{4k+2}{4} \right\rfloor + 1 = \left\lceil k + \frac{1}{2} \right\rceil + \left\lfloor k + \frac{1}{2} \right\rfloor + 1 = k + 1 + k + 1 = 2k + 2 = (2k + 1) + 1 = \frac{4k+2}{2} + 1 = \frac{n}{2} + 1. \quad \square$$

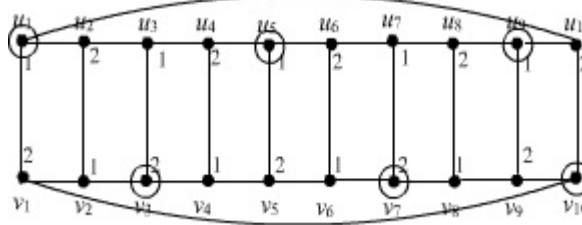


Fig. 5: Circular ladder $CL(10)$

3. Conclusion

The concepts of domination number and chromatic number together yield the dom-chromatic number, has been discussed in this paper. The dom-chromatic number of various types of graphs like Mobius ladder graphs and circular ladder graphs have been determined. It would be quite interesting to find the dom-chromatic number for other special graphs like Butterfly and Benes graphs etc.

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