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Fixed Point Theorems In Generalized Fuzzy Symmetric Spaces Under Generalized Fuzzy Contractive Conditions

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Abstract: This work aims to present an extended work on fixed point theorem that carries over some existing fixed point results from fuzzy symmetric spaces to M-fuzzy metric spaces. The fuzzy contractive mapping, which was introduced by Gregori and Sapena [6], were considered to bring out the required results.

Key words: Fixed point, Fuzzy symmetric spaces, Fuzzy contractive.
Mathematics Subject Classification: 54H25, 47H10.

1. Introduction

In the year 1931, Wilson [8] came up with an idea of symmetric spaces. In the year 1965, Lofti A. Zadeh [11] brought out the idea of fuzzy sets. The concept of fuzzy contractive mapping was set up by Gregori and Sapena [5], and, they have also extended the Banach's fixed point theorem to fuzzy contractive mappings in a fuzzy metric space that is defined by Kramosil and Michalek [3]. George and Veeramani [1, 2] have also proved many fixed point theorems by modifying the definition of fuzzy metric space that was given by Kramosil and Michalek [3]. The aim here is to prove the existence and uniqueness of fixed points in M-fuzzy symmetric spaces under generalized fuzzy contractive conditions.

2. Preliminaries

Definition 2.1

A triplet $(X, M,*)$ is said to be a generalized fuzzy symmetric space [M-fuzzy symmetric spaces] if X is an arbitrary in nonempty set,$*$ is a continuous t- norm,

and M is a fuzzy set on $X^3(0,\infty)$, satisfying the following conditions:

For each x, y, z, a ∈ X and t, s > 0,

(FS-1) $M(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}) > 0$,

(FS-2) $M(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}) = 1$ if and only if $\mathrm{x} = \mathrm{y} = \mathrm{z}$, (FM-3) $M(\mathrm{x},\mathrm{y},\mathrm{y},\mathrm{t}) = M(\mathrm{y},\mathrm{x},\mathrm{x},\mathrm{t})$.

Remark 2.2

It is to be noted that every M- fuzzy metric space is an M- fuzzy symmetric space but the converse is not always true.

Example 2.3

Consider $X = [0,\infty)$, and $M(\mathrm{x},\mathrm{y},\mathrm{z},\mathrm{t}) = \frac{t}{t+|x-y|+|y-z|+|z-x|}$, $x \neq 0, y \neq 0$ and $z \neq 0$. Then $(X, M,*)$ is an M-fuzzy metric space.

Definition 2.4

A sequence $\{x_n\}$ in X converges to x if and only if $\lim_{n\to\infty}(\frac{1}{\mathcal{M}(x_n,x,x,t)} - 1) = 0$ for each t>0.

Definition 2.5

A sequence $\{x_n\}$ in X is said to be Cauchy if for t > 0 then there exists $n_0 \in$ N such that $\lim_{n\to\infty}(\frac{1}{M(x_n,x_n,x_m,t)} - 1) = 0$ and for all n, m $> n_0$.

Definition 2.6

An M - fuzzy symmetric space is said to be complete if in which every Cauchy sequence is convergent.

Preposition 2.7

In an M - fuzzy symmetric space (X, $\mathcal{M}$, $*$,), for x ,y ∈ X , $\lim_{n\to\infty}(\frac{1}{M(x_n,x,x,t)} - 1) = 0$, $\lim_{n\to\infty}(\frac{1}{M(x_n,y,y,t)} - 1) = 0$ and $\lim_{n\to\infty}(\frac{1}{M(x_n, z, z, t)} - 1) = 0$ imply that x = y= z .

Preposition 2.8

Let X be an M - fuzzy symmetric space. For $\{x_n\}$, $\{y_n\}$, x, y ∈ X, $\lim_{n\to\infty}(\frac{1}{M(x_n, x, x,t)} - 1) = 0$ and $\lim_{n\to\infty}(\frac{1}{M(x_n,y_n,y_n,t)} - 1) = 0$ imply that $\lim_{n\to\infty}(\frac{1}{M(x_n,y,yt)} - 1) = 0$.

3. Main Result

Theorem 3.1:

Let (X, M,$*$) be a complete M - fuzzy symmetric space and the following generalized contraction conditions satisfied:

$$\left(\frac{1}{M(fx,fy,fz,t)} - 1\right) \le \phi\big(\Delta(x,y,z)\big) + c\, min \left\{ \begin{matrix} \left(\left(\frac{1}{M(x,fx,fx,t)} - 1\right), \left(\frac{1}{M(y,fy,fy,t)} - 1\right), \left(\frac{1}{M(z,fz,fz,t)} - 1\right)\right. \\ \left., \left(\frac{1}{M(x,fy,fy,t)} - 1\right), \left(\frac{1}{M(y,fz,fz,t)} - 1\right), \left(\frac{1}{M(z,fx,fx,t)} - 1\right)\right) \end{matrix} \right\} \quad (3.1.1)$$

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Where $\Delta(x,y,z) = max\left\{\begin{matrix}\left(\frac{1}{M(x,y,z,t)}-1\right), \frac{\left(\frac{1}{M(x,fx,fx,t)}-1\right)\left(\left(\frac{1}{M(y,fy,fy,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}, \\ \frac{\left(\frac{1}{M(y,fy,fy,t)}-1\right)\left(\left(\frac{1}{M(z,fz,fz,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}, \frac{\left(\frac{1}{M(z,fz,fz,t)}-1\right)\left(\left(\frac{1}{M(x,fx,fx,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}\end{matrix}\right\}$

for all x ,y, z ∈ X , $\beta \geq 0$, and ψ : [0,∞) → [0,∞) be a continuous ,non- decreasing function and $\lim_{n\to\infty} \psi^n(t) = 0$ for all t > 0. Then f has a unique fixed point.

Proof:

Suppose x_0 is an arbitrary point in X and set x_n = f x_{n-1} for all n ∈ N. If there exists m ∈ N such that $x_m = x_{m+1}$, then $x_m = x_{m+1}$ = f x_m, and x_m is a fixed point. Suppose, on the contrary, that $x_{n+1} \neq x_n$ for all n ∈ N.

That is, $\lim_{n\to\infty}\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)} - 1\right) > 0$.

Now ,

$$\left(\frac{1}{M(fx_n,fx_{n+1},fx_{n+1},t)} - 1\right) \leq \psi\left(\Delta(x_n, x_{n+1}, x_{n+1})\right) + c\, min\left\{\begin{matrix}\left(\frac{1}{M(x_n,fx_n,fx_n,t)}-1\right), \left(\frac{1}{M(x_{n+1},fx_{n+1},fx_{n+1},t)}-1\right), \left(\frac{1}{M(x_{n+1},fx_{n+1},fx_{n+1},t)}-1\right) \\ , \left(\frac{1}{M(x_n,fx_{n+1},fx_{n+1},t)}-1\right), \left(\frac{1}{M(x_{n+1},fx_{n+1},fx_{n+1},t)}-1\right), \left(\frac{1}{M(x_{n+1},fx_n,fx_n,t)}-1\right)\end{matrix}\right\}$$

$$\left(\frac{1}{M(fx_n,fx_{n+1},fx_{n+1},t)} - 1\right) \leq \psi\left(\Delta(x_n, x_{n+1}, x_{n+1})\right) + c\, min\left\{\begin{matrix}\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right), \left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right), \left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right) \\ , \left(\frac{1}{M(x_n,x_{n+2},x_{n+2},t)}-1\right), \left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right), \left(\frac{1}{M(x_{n+1},x_{n+1},x_{n+1},t)}-1\right)\end{matrix}\right\} \quad (3.1.2)$$

Where $\Delta(x_n, x_{n+1}, x_{n+1}) = max\left\{\begin{matrix}\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right), \\ \frac{\left(\frac{1}{M(x_n,fx_n,fx_n,t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},fx_{n+1},fx_{n+1},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1}x_{n+1},t)}-1\right)+1\right)}, \\ \frac{\left(\frac{1}{M(x_{n+1},fx_{n+1},fx_{n+1},t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},fx_{n+1},fx_{n+1},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)} \\ , \frac{\left(\frac{1}{M(x_{n+1},fx_{n+1},fx_{n+1},t)}-1\right)\left(\left(\frac{1}{M(x_n,fx_n,fx_n,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)}\end{matrix}\right\}$

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$$= max\left\{\begin{array}{c}\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right),\\ \frac{\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1}x_{n+1},t)}-1\right)+1\right)},\\ \frac{\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)},\\ \frac{\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,\ x_{n+1},x_{n+1},t)}-1\right)+1\right)}.\end{array}\right\}$$

Consider the following cases,

If $\Delta(x_n, x_{n+1}, x_{n+1}) = \left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)$ then from (3.1.2),

$$\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right) \leq \psi\ \left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)\right) < \left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)\ . \qquad (3.1.3)$$

If $\Delta(x_n, x_{n+1}, x_{n+1}) = \left\{\frac{\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1}x_{n+1},t)}-1\right)+1\right)}\right\}$, then from (3.1.2),

$$\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right) \leq\ \psi\ \left\{\frac{\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1}x_{n+1},t)}-1\right)+1\right)}\right\}$$

$$< \left\{\frac{\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1}x_{n+1},t)}-1\right)+1\right)}\right\}.$$

Hence , $\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right) < \left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)$. (3.1.4)

If $\Delta(x_n, x_{n+1}, x_{n+1}) = \left(\frac{\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)\left(\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)}\right)$, from (3.1.2),

$\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right) < \left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2}t)}-1\right)$ and this is absurd.

If $\Delta(x_n, x_{n+1}, x_{n+1}) = \left(\frac{\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,\ x_{n+1},x_{n+1},t)}-1\right)+1\right)}\right)$

$$\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right) \leq\ \psi\ \left(\frac{\left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2},t)}-1\right)\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,\ x_{n+1},x_{n+1},t)}-1\right)+1\right)}\right)$$

$< \left(\frac{1}{M(x_{n+1},x_{n+2},x_{n+2}t)}-1\right)$ which is absurd . (3.1.5)

In any case, it is proved (3.1.3) holds. Since $\{\left(\frac{1}{M(x_{n+2},x_{n+2},x_{n+1},t)}-1\right)\}$ is decreasing. Thus, it converges to a nonnegative number, $\beta\ \geq 0$. If $\beta > 0$, then letting $n \to +\infty$ in (3.1.2),

$\Longrightarrow\ \beta \leq\ \psi(max\{\ \beta,\ \beta, \frac{\beta(1+\beta)}{1+\beta}, \frac{\beta(1+\beta)}{1+\beta}\}) = \psi(\beta) < \beta,$

$\Longrightarrow\ \beta = 0$

That is, $\lim_{n\to\infty}\left(\frac{1}{M(x_{n+2},x_{n+2},x_{n+1},t)} - 1\right) = 0$.

Hence, $\{x_n\}$ is a Cauchy sequence, and (X , M , $*$) is complete, there exists u ∈ X such that

$\lim_{n\to\infty}\left(\frac{1}{M(x_n,u,u,t)} - 1\right) = 0$.

$$\left(\frac{1}{M(fu,u,u,t)} - 1\right) = \lim_{n\to\infty}\left(\frac{1}{M(x_{n+1},fu,fu,t)} - 1\right) = \lim_{n\to\infty}\left(\frac{1}{M(fx_n,fu,fu,t)} - 1\right)$$

$$\leq \lim_{n\to\infty}\left[\psi\big(\Delta(x_n,u,u)\big) + c\ min\left\{\begin{matrix}\left(\frac{1}{M(x_n,fx_n,fx_n,t)} - 1\right),\left(\frac{1}{M(u,fu,fu,t)} - 1\right),\left(\frac{1}{M(u,fu,fu,t)} - 1\right)\\ ,\left(\frac{1}{M(x_n,fu,fu,t)} - 1\right),\left(\frac{1}{M(u,fu,fu,t)} - 1\right),\left(\frac{1}{M(u,fx_n,fx_n,t)} - 1\right)\end{matrix}\right\}\right]$$

$$= \lim_{n\to\infty}\left[\psi\big(\Delta(x_n,u,u)\big) + c\ min\left\{\begin{matrix}\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)} - 1\right),\left(\frac{1}{M(u,fu,fu,t)} - 1\right),\left(\frac{1}{M(u,fu,fu,t)} - 1\right)\\ ,\left(\frac{1}{M(x_n,fu,fu,t)} - 1\right),\left(\frac{1}{M(u,fu,fu,t)} - 1\right),\left(\frac{1}{M(u,x_{n+1},x_{n+1},t)} - 1\right)\end{matrix}\right\}\right]$$

$$= \lim_{n\to\infty}\left[\psi\big(M(x_n\ ,u,u,)\big)\right] \leq \left(\frac{1}{M(fu,u,u,t)} - 1\right), \text{ where}$$

$$\Delta(x_n,u,u) = max\left\{\begin{matrix}\left(\frac{1}{M(x_n,u,u,t)} - 1\right),\frac{\left(\frac{1}{M(x_n,fx_n,fx_n,t)}-1\right)\left(\left(\frac{1}{M(u,fu,fu,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,u,u,t)}-1\right)+1\right)},\\ \frac{\left(\frac{1}{M(u,fu,fu,t)}-1\right)\left(\left(\frac{1}{M(u,fu,fu,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,u,u,t)}-1\right)+1\right)},\frac{\left(\frac{1}{M(u,fu,fu,t)}-1\right)\left(\left(\frac{1}{M(x_n,fx_n,fx_n,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,u,u,t)}-1\right)+1\right)}\end{matrix}\right\}$$

$$= max\left\{\begin{matrix}\left(\frac{1}{M(x_n,u,u,t)} - 1\right),\frac{\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)\left(\left(\frac{1}{M(u,fu,fu,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,u,u,t)}-1\right)+1\right)},\\ \frac{\left(\frac{1}{M(u,fu,fu,t)}-1\right)\left(\left(\frac{1}{M(u,fu,fu,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,u,u,t)}-1\right)+1\right)},\frac{\left(\frac{1}{M(u,fu,fu,t)}-1\right)\left(\left(\frac{1}{M(x_n,x_{n+1},x_{n+1},t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x_n,u,u,t)}-1\right)+1\right)}\end{matrix}\right\}$$

$$= \left(\frac{1}{M(fu,u,u,t)} - 1\right) \text{ as } n \to \infty.$$

This leads to a contradiction. Thus, $\left(\frac{1}{M(fu,u,u,t)} - 1\right) = 0$, fu = u.

To prove the uniqueness, Suppose v be another fixed point of f such that fu = u and fv = v.

$$\left(\frac{1}{M(u,v,v,t)} - 1\right) = \left(\frac{1}{M(fu,fv,fv,t)} - 1\right)$$

$$\leq \psi\left(max\left\{\begin{matrix}\left(\frac{1}{M(u,v,v,t)} - 1\right),\frac{\left(\frac{1}{M(u,fu,fu,t)}-1\right)\left(\left(\frac{1}{M(v,fv,fv,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(u,v,v,t)}-1\right)+1\right)},\\ \frac{\left(\frac{1}{M(v,fv,fv,t)}-1\right)\left(\left(\frac{1}{M(v,fv,fv,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(u,v,v,t)}-1\right)+1\right)},\frac{\left(\frac{1}{M(v,fv,fv,t)}-1\right)\left(\left(\frac{1}{M(u,fu,fu,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(u,v,v,t)}-1\right)+1\right)}\end{matrix}\right\}\right)$$

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$$+ \quad c\, min \left\{ \begin{array}{l} \left(\frac{1}{M(u,fu,fu,t)} - 1\right), \left(\frac{1}{M(v,fv,fv,t)} - 1\right), \left(\frac{1}{M(v,fv,fv,t)} - 1\right) \\ , \left(\frac{1}{M(u,fv,fv,t)} - 1\right), \left(\frac{1}{M(v,fv,fv,t)} - 1\right), \left(\frac{1}{M(v,fu,fu,t)} - 1\right) \end{array} \right\}$$

$$= \psi\left(\left(\frac{1}{M(u,v,v,t)} - 1\right)\right) < \left(\frac{1}{M(u,v,v,t)} - 1\right)$$

$$\Rightarrow \left(\frac{1}{M(u,v,v,t)} - 1\right) = 0.$$

Thus u = v and f has a unique fixed point.

Theorem 3.2:

Let (X , M ,$*$) be a complete M - fuzzy symmetric space and the following generalized contraction conditions satisfied:

$$\left(\frac{1}{M(fx,fy,fz,t)} - 1\right) \leq$$

$$\psi\left(max \left\{ \begin{array}{c} \left(\frac{1}{M(x,y,z,t)} - 1\right), \frac{\left(\frac{1}{M(x,fx,fx,t)}-1\right)\left(\left(\frac{1}{M(y,fy,fy,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}, \\ \frac{\left(\frac{1}{M(y,fy,fy,t)}-1\right)\left(\left(\frac{1}{M(z,fz,fz,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}, \frac{\left(\frac{1}{M(z,fz,fz,t)}-1\right)\left(\left(\frac{1}{M(x,fx,fx,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)} \end{array} \right\} \right)$$

For all x ,y, z $\in$ X ,$\beta \geq 0$ and ψ: $[0,\infty) \to [0,\infty)$ be a continuous ,non- decreasing function and $\lim_{n\to\infty} \psi^n(t) = 0$, for all t > 0. Then f has a unique fixed point.

Proof:

In the above theorem, take $\beta = 0$.

Corollary 3.3:

Let (X , M ,$*$) be a complete M -fuzzy symmetric space and the following generalized contraction conditions satisfied:

$$\left(\frac{1}{M(fx,fy,fz,t)} - 1\right) \leq \psi\left(\frac{\left(\frac{1}{M(x,fx,fx,t)}-1\right)\left(\left(\frac{1}{M(y,fy,fy,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}\right) ,$$

for all x ,y $\in$ X , $\beta \geq 0$, and ψ: $[0,\infty) \to [0,\infty)$ be a continuous ,non- decreasing function and $\lim_{n\to\infty} \psi^n(t) = 0$ for all t > 0. Then f has a unique fixed point.

Proof:

In theorem (3.1), take $\beta = 0$ and

$$max\left\{\begin{array}{c}\left(\frac{1}{M(x,y,z,t)}-1\right),\\ \frac{\left(\frac{1}{M(x,fx,fx,t)}-1\right)\left(\left(\frac{1}{M(y,fy,fy,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)},\\ \frac{\left(\frac{1}{M(y,fy,fy,t)}-1\right)\left(\left(\frac{1}{M(z,fz,fz,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)},\\ \frac{\left(\frac{1}{M(z,fz,fz,t)}-1\right)\left(\left(\frac{1}{M(x,fx,fx,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}\end{array}\right\}=\frac{\left(\frac{1}{M(x,fx,fx,t)}-1\right)\left(\left(\frac{1}{M(y,fy,fy,t)}-1\right)+1\right)}{\left(\left(\frac{1}{M(x,y,z,t)}-1\right)+1\right)}.$$

Conclusion

The work done here have shown that a function defined on an $\mathcal{M}$- fuzzy symmetric space must have a unique fixed point under the stated generalized contractive conditions. The same result can also be further easily extended over generalized spaces like intuitionistic fuzzy metric spaces and neutrosophic metric spaces.

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