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Distinct-Congruent spectrum of graphs

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Abstract: Let A^*=A-{0}.A function f:E(G)→A^* is called a labeling of G. Any labeling induces a map f^+ : V(G)→A, defined by $f^+(v)=\sum f(u,v)$= distinct ,where u,v ϵ E(G).In this paper,weexamine,theDC-magic spectra of class of graphs such as cycle,star,path,k_1+P_3,$C_4@S_n$,$C_6@S_n$,$C_{10}@S_n$,$C_{12}@S_n$,where n is odd.The conjecture that the even cycle C_{2n} @S_n whose distinct magic spectrum is Z_n.

Keywords distinct congruent, DC graph, spectrum,

1.Introduction

Let G=(V,E) be a connected graph, without multiple edges or loops. For any abelian group A(written additively) .Let A^*=A-{0}.A function f:E(G)→A^* is called a labeling of G. Any labeling induces a map f^+ : V(G)→A, defined by $f^+(v)=\sum f(u,v)$ where u,v ϵ E(G).If there exists a labeling f which induces a distinct label c on V(G),we say that f is an spectrum of distinct magic labeling and that G is an distinct congruence magic graph.We denote by Z_n the group of integers (mod n).In this paper ,we are interested in determining for which values of k ≥3 a graph is DC-magic. The set{k: G is Z_n-magic,n≥3}is called the Distinct-Congruent spectrum of a graph G and is denoted by DC (G).In this paper,weexamine,the DC-magic spectra of class of graphs .

2.Definition

Let G=(V,E) be a connected graph, without multiple edges or loops. For any abelian group A(written additively) .Let A^*=A-{0}.A function f:E(G)→A^* is called a labeling of G. Any labeling induces a map f^+ : V(G)→A, defined by $f^+(v)=\sum f(u,v)$ where u,v ϵ E(G).If there exists a labeling f which induces a distinct label c on V(G),we say that f is an spectrum of distinct magic labeling and that G is an distinct-congruence magic graph.

In the first example the distinct congruence magic spectrum is Z_3 where as in the second DC[G]=Z_7

3.Some observations

Theorem 3.1

The odd cycle of order n whose distinct congruence magic spectrum is Z_n for all n≥3

Proof

Claim The cycle C_n, n is odd has DC Z_n

Construction

Label all the edges by 1,2,….n clockwise simultaneously. The sum at each vertex is distinct,they are 0,1,2,…n-1.Therefore DC[C_n]= Z_n

Verification

Let V(C_n) = {v_1,v_2,v_3,……v_n }

E(C_n)={v_iv_{i+1}/1≤ i ≤ n},v_{i+1}=v_1

Define f on E(C_n) by f(v_iv_{i+1})=i, 1≤ i ≤ n-1,then the induced vertex labeling f^+on V(C_n) by

$f^+(v_i)=f(v_iv_{i+1})+f(v_{i-1}v_i)$ (mod n)

= i+i-1(mod n)

=2i-1(mod n)

Therefore $f^+(v_i)$= 2i-1(mod n) for all $v_i \in V(C_n)$

Theorem 3.2

The star graph whose distinct congruence magic spectrum is Z_n for all n≥3

Proof

ClaimThe star S_n, n is odd has DC Z_n

Construction

Label the pendant edges by 1,2,……n-1,The sum at each vertex is corresponding to the edges 1,2,…..n-1 respectively.Thecentre vertex whose sum is zero

Verification

Let $V(C_n) = \{v_1,v_2,v_3,\ldots\ldots v_n\}$

$E(C_n)=\{v_nv_i/1\leq i \leq n-1\}$

Define f on $E(C_n)$ by $f(v_nv_i)=i$, $1\leq i \leq n-1$,then the induced vertex labeling f^+on $V(C_n)$ by

$f^+(v_i)= f(v_nv_i)=i$, $1\leq i \leq n-1$; $f^+(v_n)=\sum_{i=1}^{n-1} f(v_nv_i)=1+2+\ldots\ldots+n-1$

$=\frac{(n-1)n}{2}$(mod n)

$\equiv 0$ (mod n)

Theorem 3.3

The path graph whose distinct congruence magic spectrum is Z_n for all n≥3,n is odd.

Proof

ClaimThe path P_n, n is odd has DC Z_n

Construction

Label all the edges of path by 1,2,3,…..n,the sum at each vertex is distinct,such as 0,1,2,3,…..n-1.Therefore $DC[P_n]=Z_n$

Verification

$f(v_1,v_2)=1;f(v_{n-1},v_n)=n-1;f(v_i,v_{i+1})=i,1\leq i \leq n-1.$

The sum at each vertex is $f^+(v_1)=1,f^+(v_i)=f(v_i,v_{i+1})+f(v_{i-1},v_i)$

=i+i-1

=2i-1,(mod n) $1\leq i \leq n-1$

$f^+(v_n)=n-1$

Theorem 3.4

The cycle attached with star graph that is $DC(C_4@S_n)=Z_n$ where n is odd

Proof

Claim The graph $C_4@S_n$ has distinct magic spectrum.

Construction

Label the cycle edges by 1,n,2,n-1 and star edges by 3,4,5,…..n-2 in the clockwise direction.

Verification

$V(G)=\{v_1,v_2,\ldots..v_n\}$

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$E(G)=\{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_4,v_i)/,5\leq i \leq n\}$

Define f on E(G) by $f(v_1,v_2)=1$;

$f(v_2,v_3)=n$; $f(v_3,v_4)=2$;

$f(v_4, v_1)=n-1$; $f(v_4, v_i)=i$; $5\leq i \leq n$

$f^+(v_1)= f(v_1,v_2)+ f(v_4, v_1)=1+n-1=n\equiv 0(\text{mod } n)$

$f^+(v_2)= f(v_1,v_2)+ f(v_2,v_3)=1+n=n\equiv 1(\text{mod } n)$

$f^+(v_3)= f(v_2,v_3)+ f(v_3,v_4)=n+2\equiv 2(\text{mod } n)$

$f^+(v_4)= f(v_3,v_4) +f(v_4, v_1)+ f(v_4,v_i)$

$=2+n-1+3+4+\ldots.n-2=2+3+4+\ldots.n-1+n-2$

$=1+2+3+4+\ldots..n-2+n-2$

$=(n-2(n-1)/2)+n-2$

$=(n^2-3n+2/2) + n-2$

$\equiv 1 + n - 2(\text{mod } n)$

$\equiv n - 1(\text{mod } n)$

$f^+(v_i)=i-2,5\leq i \leq n$

Therefore the sum at each vertex is distinct. Therefore $DC(C_4@S_n)=Z_n$ where n is odd.

Theorem 3.5

The cycle attached with star graph that is $DC(C_6@S_n)=Z_n$ where n is odd

Proof

ClaimThe graph $C_6@S_n$ has distinct magic spectrum.

Construction

Label the cycle edges by 1,n-2,2,n-1,n,n-3 and star edges by 3,4,5,…..n-4 in the clockwise direction.

Verification

$V(G)=\{v_1,v_2,\ldots..v_n\}$

$E(G)=\{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_4,v_5),(v_5,v_n),(v_5,v_i) /,6\leq i \leq n - 1\}$

$f(v_1,v_2)=1$; $f(v_2,v_3)=n-2$; $f(v_3,v_4)=2$;

$f(v_4, v_5)=n-1$; $f(v_5, v_i)=i-3$; $5\leq i \leq n-1$;

$f(v_5,v_n)=n$;$f(v_n,v_1)=n-3$

$f^+(v_1)= f(v_1,v_2)+ f(v_n, v_1)=1+n-3\equiv n-2((\text{mod } n)$

$f^+(v_2)= f(v_1,v_2)+ f(v_2,v_3)=1+n-2 \equiv n -1(\text{mod } n)$

$f^+(v_3)= f(v_2,v_3)+ f(v_3,v_4)=n-2+2\equiv 0(\text{mod } n)$

$f^+(v_4)= f(v_3,v_4)+ f(v_4,v_5)=2+n-1+\equiv n+1\equiv 1 (\text{mod } n)$

$f^+(v_5)= f(v_4,v_5) +f(v_5, v_{13})+ f(v_5,v_i)$

$=n-1+n+i-3=n-1+n+3+4+5+\ldots\ldots.n-4$

$=3+4+5+\ldots\ldots.n-4+n-1+n$

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$=((n-4)(n-3)/2)-3+n-1+n$

$=((n^2-7n+12)/2)-3+ n-1+n$

$\equiv 6-3-1 (\mathrm{mod}\ n) \equiv 2 (\mathrm{mod}\ n)$

$f^+(v_i)=i-3, 6\leq i \leq n-1.$

Therefore the sum at each vertex is distinct. Therefore $DC(C_6@S_n)=Z_n$ where n is odd.

Theorem 3.6

$C_{12}@S_n$ graph whose distinct magic spectrum is Z_n

Proof

Let $C_{\mathbf{n}}$ be the Cycle graph.

Where n=12

To prove that $C_{12}@S_n$ graph has magic spectrum.

Next, let us take n=12, we get C_{12} graph.

$C_{12}@S_n$ is a distinct congruence magic graph.

Let the integer set be $Z_{22}=\{0,1,2,3,4,5,…. ….21\}$

Construction:-

Since by using the definition of labeling, we label the edges by 1,2,3,4,5,…21 from Z_{22}

Verification:-

$f(v_1 v_2)=3$, $f(v_2 v_3)=18$, $f(v_3 v_4)=4$, $f(v_4 v_5)=19$, $f(v_5 v_6)=5$,$f(v_6 v_7)=20$,

$f(v_7 v_8)=21$,$f(v_8 v_9)=15$,$f(v_9 v_{10})=1$,$f(v_{10} v_{11})=16$,$f(v_{11}v_{12})=2$,$f(v_{12} v_1)=17$,

$f(v_7 v_{13})=6$,$f(v_7 v_{14})=7$,$f(v_7 v_{15})=8$,$f(v_7 v_{16})=9$,$f(v_7 v_{17})=10$,$f(v_7 v_{18})=11$,

$f(v_7 v_{19})=12$,$f(v_7 v_{20})=13$,$f(v_7 v_{21})=14$

$f^+(v_1)=20$, $f^+(v_2)=21$, $f^+(v_3)=22$,$f^+(v_4)=23$,

$f^+(v_5)=24$, $f^+(v_6)=25$, $f^+(v_7)=131$,$f^+(v_8)=36$

$f^+(v_9)=16$, $f^+(v_{10})=17$, $f^+(v_{11})=18$,$f^+(v_{12})=19$

$f^+(v_{13})=6$, $f^+(v_{14})=7$, $f^+(v_{15})=8$,$f^+(v_{16})=9$

$f^+(v_{17})=10$, $f^+(v_{18})=11$, $f^+(v_{18})=12$,$f^+(v_{20})=13$

$f^+(v_{21})=14$,

Therefore all are distinct

Congruency:-

20 ≡ **<u>20</u>**(mod 21)

21 ≡ **<u>0</u>** (mod 21)

22 ≡ **<u>1</u>** (mod 21)

23 ≡ **<u>2</u>** (mod 21)

24 ≡ **<u>3</u>**(mod 21)

25 ≡ **<u>4</u>** (mod 21)

$131 \equiv \underline{\mathbf{5}}$ (mod 21)

$36 \equiv \underline{\mathbf{15}}$ (mod 21)

$16 \equiv \underline{\mathbf{16}}$(mod 21)

$17 \equiv \underline{\mathbf{17}}$ (mod 21)

$18 \equiv \underline{\mathbf{18}}$ (mod 21)

$19 \equiv \underline{\mathbf{19}}$(mod 21)

$6 \equiv \underline{\mathbf{6}}$(mod 21)

$7 \equiv \underline{\mathbf{7}}$ (mod 21)

$8 \equiv \underline{\mathbf{8}}$ (mod 21)

$9 \equiv \underline{\mathbf{9}}$ (mod 21)

$10 \equiv \underline{\mathbf{10}}$ (mod 21)

$11 \equiv \underline{\mathbf{11}}$ (mod 21)

$12 \equiv \underline{\mathbf{12}}$ (mod 21)

$13 \equiv \underline{\mathbf{13}}$(mod 21)

$14 \equiv \underline{\mathbf{14}}$ (mod 21)

Therefore the sum at each vertex is distinct. Therefore $DC(C_{12}@S_n)=Z_n$ where n is odd.

Theorem 3.7

$C_{10}@S_n$ graph whose distinct magic spectrum is Z_n

Proof

Let $C_{\mathbf{n}}$ be the Cycle graph.

Where n=10

To prove that $C_{10}@S_n$ graph has magic spectrum.

Next, let us take n=10, we get C_{10} graph.

$C_{2n}@S_n$ is a distinct congruence magic graph.

Let the integer set be Z_{18}={0,1,2,3,4,5,…. ….17}

Construction:-

Since by using the definition of labeling, we label the edges by 1,2,3,4,5,…17 from Z_{18}

Verification:-

$f(v_1 v_2)=14$, $f(v_2 v_3)=3$, $f(v_3 v_4)=15$, $f(v_4 v_5)=4$, $f(v_5 v_6)=16$,$f(v_6 v_7)=17$,

$f(v_7 v_8)=12$,$f(v_8 v_9)=1$,$f(v_9 v_{10})=13$,$f(v_{10} v_1)=2$,$f(v_6 v_{11})=5$,$f(v_6 v_{12})=6$,

$f(v_6 v_{13})=7$,$f(v_6 v_{14})=8$,$f(v_6 v_{15})=9$,$f(v_6 v_{16})=10$,$f(v_7 v_{17})=11$,

$f^+(v_1)=16$, $f^+(v_2)=17$, $f^+(v_3)=18$,$f^+(v_4)=19$,

$f^+(v_5)=20$, $f^+(v_6)=89$, $f^+(v_7)=29$,$f^+(v_8)=13$

$f^+(v_9)=14$, $f^+(v_{10})=15$, $f^+(v_{11})=5$,$f^+(v_{12})=6$

$f^+(v_{13})=7$, $f^+(v_{14})=8$, $f^+(v_{15})=9$,$f^+(v_{16})=10$

$f^{+}(v_{17})=11$

Therefore all are distinct

Congruency:-

16 ≡ **16**(mod 17)

17 ≡ **0** (mod 17)

18 ≡ **1** (mod 17)

19 ≡ **2** (mod 17)

20≡ **3**(mod 17)

89 ≡ **4** (mod 17)

29 ≡ **12** (mod 17)

13 ≡ **13** (mod 17)

14≡ **14**(mod 17)

15 ≡ **15** (mod 17)

5 ≡ **5** (mod 17)

6 ≡ **6**(mod 17)

7≡ **7**(mod 17)

8≡ **8** (mod 17)

9≡**9** (mod 17)

10 ≡ **10** (mod 17)

11 ≡ **11**(mod 17)

Therefore the sum at each vertex is distinct. Therefore $DC(C_{12}@S_n)=Z_n$ where n is odd.

Next,LetC_n be the Cycle graph.

Where n=8

To prove that $C_8@S_n$ graph has magic spectrum.

Next, let us take n=8, we get C_8 graph.

$C_8@S_n$ is a distinct congruence magic graph.

Let the integer set be $Z_{14}=\{0,1,2,3,4,5,…. ….13\}$

Construction:-

Since by using the definition of labeling, we label the edges by 1,2,3,4,5,…13 from Z_{14}

Verification:-

$f(v_1 v_2)= 2$, $f(v_2 v_3)=11$, $f(v_3 v_4)=3$, $f(v_4 v_5)=12$, $f(v_5 v_6)=13$,$f(v_6 v_7)=9$,

$f(v_7 v_8)=1$,$f(v_8 v_1)=10$,$f(v_5 v_9)=4$,$f(v_5 v_{10})=5$,$f(v_5 v_{11})=6$,

$f(v_5 v_{12})=7$,$f(v_5 v_{13})=8$

$f^{+}(v_1)=12$, $f^{+}(v_2)=13$, $f^{+}(v_3)=14$,$f^{+}(v_4)=15$,

$f^{+}(v_5)=55$, $f^{+}(v_6)=22$, $f^{+}(v_7)=10$,$f^{+}(v_8)=11$

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$f^+(v_9)=4$, $f^+(v_{10})=5$, $f^+(v_{11})=6$,$f^+(v_{12})=7$

$f^+(v_{13})=8$

Therefore all are distinct

Congruency:-

12 ≡ **12**(mod 13)

13 ≡ **0** (mod 13)

14 ≡ **1** (mod 13)

15 ≡ **2** (mod 13)

55≡ **3**(mod 13)

22 ≡ **9** (mod 13)

10 ≡ **10** (mod 13)

11 ≡ **11** (mod 13)

4≡ **4**(mod 13)

5 ≡ **5** (mod 13)

6 ≡ **6** (mod 13)

7 ≡ **7**(mod 13)

8≡ **8**(mod 13)

Therefore the sum at each vertex is distinct. Therefore $DC(C_8@S_n)=Z_n$ where n is odd.

Theorem 3.8

The conjecture that the even cycle C_{2n} $@S_n$ whose distinct magic spectrum is Z_n.

Conclusion

The graphs whose distinct magic spectrum is Z_n . More general graphs like stars ,cycles,paths,etc., are also discussed.Main applications of this labeling is network theory,coding theory,etc.The conjecture that the even cycle C_{2n} attached with stars whose distinct magic spectrum is Z_n.

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