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On some new sets in nano ideal topological spaces

[1,a]D. Arumugakani and M. Raja kalaivanan

[1]Research Scholar (Part time),School of Mathematics,Madurai Kamaraj University,Madurai - 625 021, Tamilnadu, India.
[a]Department of Mathematics, Sri Meenakshi Government Arts College for Women(A),
Madurai - 625 002, Tamilnadu, India.

E-mail :kanimaths82@gmail.com.

Department of Mathematics, PasumponMuthuramalinga Thevar College, Usilampatti,
Madurai - 625532, Tamilnadu, India.

E-mail :rajakalaivanan@yahoo.com.

ABSTRACT: weintroducetheconceptsofan ideal nano K-set, an ideal nano O-set, an nano ideal K_α-set and an nano ideal O_α-set are investigate and deal with an nano ideal topologicalspaces.

Keywords: an nano ideal K-set an nano ideal O-set an nano ideal K_α-set and an nano ideal O_α-set.

1. Introduction

An idealI[7]onaspace(X,τ)isanon-emptycollectionofsubsetsofXwhichsatisfiesthefollowingconditions.

(1) A∈IandB⊂AimplyB∈Iand

(2) A∈IandB∈IimplyA∪B∈I.

Givenaspace(X,τ)withan idealIonXifP(X)isthesetofallsubsetsofX,asetoperator(.)*:P(X)→P(X),calledalocalfunctionofAwithrespecttoτa ndIisdefinedasfollows:forA⊂X,A* (I,τ)={x∈X:U∩A∉IforeveryU∈τ(x)}whereτ(x)={U∈τ:x∈U }[1]. The closure operator defined by cl*(A) = A ∪ A* (I, τ) [8] is a Kuratowski closure operator whichgenerates a topologyτ* (I, τ) called the *-topology which is finer then τ. We will simply write A*forA*(I,τ)andτ*forτ* (I,τ).IfIisan idealonX,then(X,τ,I)iscalledan idealtopologicalspaceoran idealspace.

Parimalaetal.[4,3]introducedanewnotionsofnano ideal topologicalspaces.

Inthispaper,weintroducetheconceptsofnano ideal K-set,nano ideal O-set,nano ideal K_α-setandnano ideal O_α-setareinvestigateanddeal withnano ideal topologicalspaces.

2. Preliminaries

Definition2.1.[5]

LetUbeanon-emptyfinitesetofobjectscalledtheuniverseandRbeanequivalence relation on U named as the indiscernibility relation.Elements belonging to the sameequivalence class are said to be indiscernible with one another.The pair (U, R) is said to be theapproximationspace.LetX⊆U.

(1) The lower approximation of X with respect to R is the set of all objects, which can be forcertain classifiedasXwithrespecttoRanditisdenotedby$L_R(X)$.Thatis,$L_R(X)=\bigcup_{x\in U}\{R(x):R(x)\subseteq X\}$,whereR(

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x)denotestheequivalenceclassdeterminedbyx.

(2) TheupperapproximationofXwithrespecttoRisthesetofallobjects,whichcanbe possiblyclassifiedasXwithrespecttoRanditisdenotedbyU$_R$(X).Thatis,$U_R(X)=\bigcup_{x\in U}\{R(x):R(x)\cap X\neq \varphi\}$.

(3) TheboundaryregionofXwithrespecttoRisthesetofallobjects,whichcanbeclassifiedneitherasX norasnot-X withrespecttoR and it is denoted by $B_R(X)$. That is,$B_R(X)=U_R(X)-L_R(X)$.

Definition2.2. [2]Let U be the universe, R be anequivalence relation on U and $\tau_R(X)=\{U,\varphi,L_R(X),U_R(X),B_R(X)\}$whereX$\subseteq$U.ThenR(X)satisfiesthefollowingaxioms:

(1) Uand$\varphi\in\tau_R(X)$,

(2) Theunionoftheelementsofanysubcollectionof$\tau_R(X)$isin$\tau_R(X)$,

(3) Theintersectionoftheelementsofanyfinitesubcollectionof$\tau_R(X)$isin$\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on Ucalled the nano topology with respect to X and (U, $\tau_R(X)$) is calledthe nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets).Thecomplementofananoopensetiscallednanoclosed.

Throughout paper,we denote a nano topological space by NTS. The nano-interior and nano-closureofasubsetCofUaredenotedbyNint(C)andNcl(C),respectively.

A nano topological space NTS with an ideal I on U is called [4] an nano ideal topological spaceandisdenotedbyINTS.$H_n(x)=\{H_n|x\in H_n,H_n\in\tau_R(X)\}$,denotes[4]thefamilyofnanoopensetscontainingx.

Definition 2.3. [4] Let INTS be a space with an idealI on U . Let $(.)^*$be a set operator from P(U)toP(U)(P(U) is the set of all subsets of U). For a subset C $\subseteq$ U , A^{*N} (I, $\tau_R(X)$) = {x $\in$ U :$H_n\cap C\notin I$,foreveryG$_n\in$G$_n$(x)}iscalledthenanolocalfunction(briefly,nanolocalfunction) ofCwithrespecttoIand$\tau_R(X)$.WewillsimplywriteC^{*N}forC^{*N}(I,$\tau_R(X)$).

Theorem2.4.[4]LetINTSbeaspaceandCandBbesubsetsofU.Then

(1) $C\subseteq B\Rightarrow C^{*N}\subseteq B^{*N}$,

(2) C^{*N}= Ncl(C^{*N})$\subseteq$Ncl(C)(C^*isananoclosedsubsetofNcl(C)),

(3)$(C^{*N})^{*N}\subseteq C^{*N}$,

(4)$(C\cup B)^{*N}=C^{*N}\cup B^{*N}$,

(5)$V\in\tau_R(X)\Rightarrow V\cap C^{*N}=V\cap(V\cap C)^{*N}\subseteq(V\cap C)^{*N}$,

(6)$J\in I\Rightarrow(C\cup J)^{*N}=C^{*N}=(C-J)^{*N}$.

Theorem2.5.[4]LetINTSbeaspacewithan idealIandC$\subseteq C^{*N}$,the C^{*N}=Ncl(C^{*N})=Ncl(C).

Definition2.6. [4] Letan INTS. The setoperator Ncl*called a nano *-closure is defined by

$Ncl^*(C)=C\cup C^{*N}$forC$\subseteq$X.

ItcanbeeasilyobservedthatNcl*(C)$\subseteq$Ncl(C).

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Theorem2.7.[3] InaspaceINTS,ifCandBaresubsetsofU,thenthefollowingresultsaretrueforthesetoperatorNcl^*.

(1) $C\subseteq Ncl^*(C)$,
(2) $Ncl^*(\varphi)=\varphi$ and $Ncl^*(U)=U$,
(3) If $C\subset B$,then $Ncl^*(C)\subseteq Ncl^*(B)$,
(4) $Ncl^*(C)\cup Ncl^*(B)=Ncl^*(C\cup B)$.
(5) $Ncl^*(Ncl^*(C))=Ncl^*(C)$.

Definition2.8.[6] AsubsetCofspaceINTSissaidtobe

(1) an nano ideal α-open if $C\subset Nint(Ncl^*(Nint(C)))$,
(2) an nano ideal pre-open if $C\subset Nint(Ncl^*(C))$.

3. OnK-setsandnanoO-setsin nano ideal spaces

Definition3.1. AsubsetCofaspaceINTSiscalled
(1) nano ideal K-setif $Nint(C)=Nint(Ncl^*(C))$,
(2) nano ideal O-setif $C=Q\cap P$,whereQisnanoopenandPisnano ideal K-set,
(3) nano ideal K_α-setif $Nint(C)=Nint(Ncl^*(Nint(C)))$,
(4) nano ideal O_α-setif $C=Q\cap P$,whereQisnanoopenandPisnano ideal K_α-set.

Example3.2. Let $U=\{1,2,3,4\}$ with $U/R=\{\{2\},\{4\},\{1,3\}\}$ and $X=\{3,4\}$. Then $\tau_R(X)=\{\varphi,U,\{4\},\{1,3\},\{1,3,4\}\}$ and $I=\{\varphi,\{3\}\}$.

(1) theset{2}isnano ideal K-set.
(2) theset{4}isnano ideal K_α-set.
(3) theset{2,3}isnano ideal O-set
(4) theset{1,3}isnano ideal O_α-set

Remark3.3.InaINTS,

(1) eachnanoopensetisnano ideal O-set.
(2) eachnano ideal K-setisnano ideal O-set.

Remark3.4. The converses of Remark3.3 are not true as shown in the next Examples.

Example3.5. InExample3.2,

(1) theset{2}isnano ideal O-setbutnotnanoopenset.
(2) theset{1,3,4}isnano ideal O-setbutnotnano ideal K-set.

Proposition 3.6. Let P and Q be subsets of a space INTS. If P and Q are nano ideal K-sets,then $P\cap Q$ isnano ideal K-set.

Proof. LetPandQbenano ideal K-sets. Thenwehave $Nint(P\cap Q)\subset Nint(Ncl^*(P\cap Q))\subset Nint(Ncl^*(P)\cap Ncl^*(Q))=Nint(Ncl^*(P))\cap Nint(Ncl^*(Q))=Nint(P)\cap Nint(Q)=Nint(P\cap Q)$.
Then $Nint(P\cap Q)=Nint(Ncl^*(P\cap Q))$ andhence $P\cap Q$ isnano ideal K-set.

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Example3.7. InExample3.2,

P={1,3}andQ={3,4}isnano ideal K-set.ButP∩Q={3}isnano ideal K-set.

Proposition3.8. ForasubsetCofaspaceINTS,the next conditions are be quivalent:

(1) Cisnanoopen,

(2) Cisnano ideal pre-openandnano ideal O-set.

Proof.(1)⇒ (2):LetCbenano open. Then C= Nint(C) ⊂ Nint(Ncl*(C)) and C isnano ideal pre-open.AlsobyRemark3.3Cisnano ideal O-set.

(2) ⇒(1):Given Cisnano ideal O-set.SoC=H∩KwhereHisnanoopenandNint(K)=Nint(Ncl(K)). Then C ⊂ H = Nint(H). Also, C is ideal anano pre-open implies C ⊂Nint(Ncl(C))⊂ Nint(Ncl*(K)) = Nint(K) by assumption. Thus C ⊂ Nint(H) ∩ Nint(K) =Nint(H∩K)=Nint(C)andhenceCisnanoopen.

(3)

Remark 3.9. In a space INTS, nano ideal pre-open sets and nano ideal O-sets are independent.

Example3.10. InExample3.2,

(1) theset{1,4}isnano ideal pre-openbutnotnano ideal O-set.

(2) theset{2}isnano ideal O-setbutnotnano ideal pre-open.

Remark3.11. Inaspace,

(1) each nano ideal K_α-setis nano ideal O_α-set.

(2) eachnanoopensetis nano ideal O_α-set.

Example3.12. InExample3.2,

(1) theset{1,3,4}is nano ideal O_α-setbutnotnano ideal K_α-set.

(2) theset{1}is nano ideal O_α-setbutnotnanoopenset.

Proposition3.13. IfPandQare nano ideal K_α-setsofaspaceINTS,thenP∩Qisa nano ideal K_α-set.

Proof. Let P and Q be nano ideal K_α-sets.Then we have Nint(P ∩Q) ⊂ Nint(Ncl*(Nint(P ∩Q)))⊂Nint[Ncl*(Nint(P))∩Ncl*(Nint(Q))]=Nint(Ncl*(Nint(P)))∩Nint(Ncl*(Nint(Q)))=Nint(P)∩Nint(Q)=Nint(P∩Q).ThenNint(P∩Q)=Nint(Ncl*(Nint(P∩Q)))andhenceP∩Qisnano ideal K_α-set.

Example3.14. InExample3.2,

theset{2,3} andtheset{1,2}isnano ideal K_α-set.Butintersectionofbothsetsfortheset{2}is nano ideal K_α-set.

Proposition3.15. ForasubsetCofaspaceINTS,thenextconditionsareequivalent:

(1) Cisnanoopen.

(2) Cisnano ideal α-openandnano ideal O_α-set.

Proof.(1)⇒ (2):LetCbenanoopen.TheNc=Nint(C)⊂Ncl*(Nint(C))andC=Nint(C)⊂ Nint(Ncl*(Nint(C))).

ThereforeCisnano ideal α-open.AlsobyRemark3.11(1),Cisnano ideal O_α-set.

(2)⇒(1):Given Cisnano ideal O_α-set. SoC=H∩KwhereHisnanoopenandNint(K)=

Nint(Ncl*(Nint(K))).Then C⊂ H= Nint(H).AlsoCisideal nano α-open impliesC⊂Nint(Ncl*(Nint(H)))⊂Nint(Ncl*(Nint(K)))=Nint(K)byassumption.
ThusC⊂Nint(H)∩Nint(K)=Nint(H∩K)=Nint(C)andCisnanoopen.

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