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On new generalized closed sets in nano ideal topological spaces

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ABSTRACT :

Inthispaper,weintroducenano idealM-closedsetsandtheconceptsof nano idealM-open setsinnano idealtopologicalspacesisintroduceandstudy.Therelationshipsof nano idealM-closedsetswithvariousothersetsareinvestigatedandfurtherinthispaper,therelationships ofnano idealW-g-closedsetswithvariousothersetsarediscussed.

Keywords: nanoopen,nanog-closed, nano idealW-g-closed, nano idealM-g-closed and nano idealR-g-closed

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1.Introduction

AnidealI [10]onaspace(X,τ)isanon-emptycollectionofsubsetsofX whichsatisfiesthefollowingconditions.

(1)$A\in I$and$B\subset A$imply$B\in I$and

(2)$A\in I$and$B\in I$imply$A\cup B\in I$.

Givenaspace(X,τ)withanidealI onX if$\wp(X)$isthesetofallsubsetsofX,asetoperator$(.)^*:\wp(X)\rightarrow\wp(X)$,calledalocalfunctionofAwithrespecttoτandI isdefinedasfollows:for$A\subset X$, $A^*(I,\tau)=\{x\in X : U\cap A\notin I \text{ for every } U\in\tau(x)\}$where$\tau(x)=\{U\in\tau : x\in U\}$[2].Theclosureoperatordefinedby$cl^*(A)=A\cup A^*(I,\tau)$[11]isaKuratowskiclosureoperatorwhichgeneratesatopology$\tau^*(I,\tau)$calledthe*-topologywhichisfinerthenτ.Wewillsimplywrite$A^*$for$A^*(I,\tau)$and$\tau^*$for$\tau^*(I,\tau)$.IfI isanidealonX,then(X,τ,I)iscalledanidealtopologicalspaceoranidealspace.

Inthispaper,weintroducenano idealM-closedsetsandtheconceptsofnano idealM-open setsinnano idealtopologicalspacesisintroduceandstudy.Therelationshipsofnano idealM-closedsetswithvariousothersetsareinvestigatedandfurtherinthispaper,therelationships ofnano idealW-g-closedsetswithvariousothersetsarediscussed.

2.Preliminaries

Definition2.1.[8]LetUbeanon-emptyfinitesetofobjectscalledtheuniverseandRbean equivalencerelationonUnamedasthein discernibilityrelation.Elementsbelongingtothesame equivalenceclassaresaidtobeindiscerniblewithoneanother.Thepair(U,R)issaidtobethe approximationspace.Let$X\subseteq U$.

(1) ThelowerapproximationofXwithrespecttoRisthesetofallobjects,whichcanbefor certainclassifiedasXwithrespecttoRanditisdenotedby$L_R(X)$.Thatis,$L_R(X)=\bigcup_{x\in U}\{R(x):R(x)\subseteq X\}$,whereR(x)denotestheequivalenceclassdeterminedbyx.

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(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = {}_{x\in U}\{R(x): R(x) \cap X = \varphi\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [3] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then R(X) satisfies the following axioms:

(1) U and $\varphi \in \tau_R(X)$,

(2) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$,

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nanotopological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a nano open set is called nano closed.

Throughout paper, we denote a nano topological space by NTS. The nano-interior and nano-closure of a subset C of U are denoted by $N\,int(C)$ and $N\,cl(C)$, respectively.

A nano topological space NTS with an ideal I on U is called [6] nano ideal topological space and is denoted by NITS. $H_n(x) = \{H_n \mid x \in H_n, H_n \in \tau_R(X)\}$, denotes [6] the family of nano open sets containing x.

Definition 2.3. [6] Let NITS be a space with an ideal I on U. Let $(.)^*$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $C \subseteq U$, $A^{*N}(I, \tau_R(X)) = \{x \in U : H_n \cap C \notin I, \text{for every } G_n \in G_n(x)\}$ is called the nano local function (briefly, nano local function) of C with respect to I and $\tau R(X)$. We will simply write C^{*N} for $C^{*N}(I, \tau_R(X))$.

Theorem 2.4. [6] Let NITS be a space and A and B be subsets of U. Then

(1) $C \subseteq B \Rightarrow C^{*N} \subseteq B^{*N}$,

(2) $C^{*N} = N\,cl(C^{*N}) \subseteq N\,cl(C)$ (C^{*N} is a nano closed subset of Ncl(C)),

(3) $(C^{*N})^{*N} \subseteq C^{*N}$,

(4) $(C \cup B)^{*N} = C^{*N} \cup B^{*N}$,

(5) $V \in \tau_R(X) \Rightarrow V \cap C^{*N} = V \cap (V \cap C)^{*N} \subseteq (V \cap C)^{*N}$,

(6) $J \in I \Rightarrow (C \cup J)^{*N} = C^{*N} = (C - J)^{*N}$.

Theorem 2.5. [6] Let NITS be a space with an ideal I and $C \subseteq C^{*N}$, then $C^{*N} = N\,cl(C^{*N}) = N\,cl(C)$.

Definition 2.6. [6] Let NITS. The set operator $N\,cl^*$ called a nano*-closure is defined by $N\,cl^*(C) = C \cup C^{*N}$ for $C \subseteq X$.

It can be easily observed that $N\,cl^*(C) \subseteq N\,cl(C)$.

Theorem 2.7. [7] In a space NITS, if C and B are subsets of U, then the following results are true for the set operator $N\,cl^*$.

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(1) $C \subseteq Ncl^*(C)$,

(2) $Ncl^*(\varphi) = \varphi$ and $Ncl^*(U) = U$,

(3) If $C \subset B$, then $Ncl^*(C) \subseteq Ncl^*(B)$,

(4) $Ncl^*(C) \cup Ncl^*(B) = Ncl^*(C \cup B)$.

(5) $Ncl^*(Ncl^*(C)) = Ncl^*(C)$.

Definition 2.8. A subset C of a space NTS and NITS, is called a

(1) nano nowhere dense [4] if $N int(N cl(C)) = \varphi$

(2) nano g-closed [1] if $N cl(C) \subseteq G$, whenever $C \subseteq G$ and G is nano open.

(3) nano ideal pre*-closed [9] if $N cl^*(N int(C) \subseteq C$.

(4) nano ideal L-I-closed set [9] if $C = N cl^*(N int(C))$.

(5) nano*-closed [6] if $C^{*N} \subseteq C$.

(6) nano ideal g-closed [5] if $C^{*N} \subseteq G$ whenever $C \subseteq G$ and G is nano open.

Theorem 2.9. [5] In a space (U, N, I), each nano *-closed set is nano ideal g-closed.

3. Some new types of generalized closed sets in nano ideal spaces

Definition 3.1. A subset C of a space NITS, is called

1) nano ideal W-g-closed if $(N int(C))^{*N} \subseteq G$ whenever $C \subseteq G$ and G is nano open.
2) nano ideal M-g-closed if $(N int(C))^{*N} \subseteq G$ whenever $C \subseteq G$ and G is nano g-open. The complement of a nano ideal M-g-open set is called nano ideal M-g-closed.
3) nano ideal R-g-closed if $C^{*N} \subseteq G$ whenever $C \subseteq G$ and G is nano-open.

Example 3.2. Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{1\}, \{4\}, \{2, 3\}\}$ and $X = \{1, 3\}$. Then $\tau_R(X) = \{\varphi, \{1\}, \{2, 3\}, \{1, 2, 3\}, U\}$ and $I = \{\varphi, \{3\}\}$.

Hence the set $\{4\}$ is nano ideal W-g-closed, nano ideal M-g-closed and nano ideal R-g-closed.

Theorem 3.3. In a space NITS, a subset C is nano ideal M-g-closed iff $(N int(C))^{*N} \subseteq C$.

Proof. If $(N int(C))^{*N} \not\subset C$, there exists $1 \in U$ such that $1 \in (N int(C))^{*N} - C$. Then $1 \in (N int(C))^{*N} - C \subseteq U - C$ and so $C \subseteq U - \{1\}$ where $U - \{1\}$ is nano g-open being nano open. Thus $C \subseteq U - \{1\}$ where $U - \{1\}$ is nano g-open. But $(N int(C))^{*N} \not\subset U - \{1\}$ since $1 \in (N int(C))^{*N}$. This implies that C is not nano ideal M-g-closed which proves the necessary part.

Conversely, Let $(N int(C))^{*N} \subseteq C$ and G be any nano g-open subset such that $C \subseteq G$. Then $(N int(C))^* \subseteq C \subseteq G$. This implies that C is nano ideal M-g-closed which proves the suficiency part.

Theorem 3.4. In a space NITS, a subset C is nano ideal W-g-closed iff $(N int(C))^{*N} \subseteq C$.

Proof. If $(N int(C))^{*N} \not\subset C$, there exists $1 \in U$ such that $1 \in (N int(C))^{*N} - C$. Then $1 \in (N int(C))^{*N} - C \subseteq U - C$ and so $C \subseteq U - \{1\}$ where $U - \{1\}$ is nano open. Thus $C \subseteq U - \{1\}$ where $U - \{1\}$ is nano open. But $(N int(C))^{*N} \not\subset U - \{1\}$ since $1 \in (N int(C))^{*N}$. This implies that C is not nano ideal W-g-closed.

Conversely,Let$(N\,int(C))^{*N}\subseteq C$andGbeanynanoopensetsuchthat$C\subseteq G$.Then$(N\,int(C))^{*N}\subseteq C\subseteq G$.Thisimpliesthat Cisnano idealW-g-closed.

Theorem3.5. InaspaceNITS,asubsetCisnano idealM-g-closediffCisnano ideal W-g-closed.

Proof. ProoffollowsbyTheorem3.3andTheorem3.4.

Theorem3.6. InaspaceNITS,asubsetCisnano idealR-g-closediff$C^{*N}\subseteq C$.

Proof. If$C^{*N}\not\subset C$,thereexists$1\notin U$suchthat$a\in C^{*N}-C$.Then$1\notin C^{*N}-C\subseteq U-C$andso $C\subseteq U-\{1\}$where$U-\{1\}$isnanog-openbeingnanoopen.Thus$C\subseteq U-\{1\}$where$U-\{1\}$isnanog-open.But$C^{*N}\not\subset U-\{1\}$since$1\notin C^{*N}$.ThisimpliesthatCisnotnano ideal R-g-closed.

Conversely,Let$C^{*N}\subseteq C$andGbeanynanog-opensetsuchthat$C\subseteq G$.Then$C^{*N}\subseteq C\subseteq G$.ThisimpliesthatCisnano idealR-g-closed.

Theorem3.7. InaspaceNITS,asubsetCisnano idealg-closediff$C^{*N}\subseteq C$.

Proof. If$C^{*N}\not\subset C$,thereexists$1\notin U$suchthat$1\notin C^{*N}-C$.Then$1\in C^{*N}-C\subseteq U-C$andso $C\subseteq U-\{1\}$where$U-\{1\}$isnanoopen.Thus$C\subseteq U-\{1\}$where$U-\{1\}$isnanoopen.But$C^{*N}\not\subset U-\{1\}$since$1\in C^{*N}$.ThisimpliesthatCisnotnano idealg-closed.

Conversely,Let$C^{*N}\subseteq C$andGbeanynanoopensetsuch that$C\subseteq G$.Then$C^{*N}\subseteq C\subseteq G$.ThisimpliesthatCisnano idealg-closed.

Theorem3.8. InaspaceNITS,asubsetCisnano idealR-g-closediffCisnano idealg-closed.
Proof. ProoffollowsfromTheorem3.6andTheorem3.7.

Proposition3.9. InaspaceNITS,

(1)eachnano idealg-closedsetisnano idealW-g-closed.

(2)eachnano idealR-g-closedsetisnano idealM-g-closed.

Proof. Obvious.

Remark3.10. TheconversesofProposition3.9arenottrueasshowninthefollowingExample.

Example3.11. InExample3.2,then$\{2\}$isnano idealW-g-closedsetandnano idealM-g-closedsetbutnotnano idealg-closedandnano idealR-g-closed.

Theorem3.13. InaspaceNITS,forasubsetCofU,thefollowingpropertiesareequivalent.

(1)Cisnano idealM-g-closed,
(2)$(N\,int(C))^{*N}-C=\varphi$,
(3)$N\,cl^{*}(N\,int(C))-C=\varphi$,
(4)$N\,cl^{*}(N\,int(C))\subseteq C$,
(5)Cisnano idealpre*-closed.

Proof.(1)$\Longleftrightarrow$(2):Cisnano idealM-g-closed$\Longleftrightarrow(N\,int(C))^{*N}\subseteq C$byTheorem3.3$\Longleftrightarrow(N\,int(C))^{*N}-C=\varphi$.

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(2)$\Longleftrightarrow$(3):$(N\,int(C))^{*N}- C=\varphi \Longleftrightarrow N\,cl^*(N\,int(C)) - C= ((N\,int(C))^{*N}\cup N\,int(C)) - C= ((N\,int(C))^{*N}- C)\cup (N\,int(C)- C)= N\,int(C)- C= \varphi$.

(3)$\Longleftrightarrow$(4):$N\,cl^*(N\,int(C))- C=\varphi \Longleftrightarrow N\,cl^*(N\,int(C))\subseteq C$.

(4)$\Longleftrightarrow$(5):$N\,cl^*(N\,int(C))\subseteq C \Longleftrightarrow$ Cisnano idealpre*-closed.

Theorem3.14. InaspaceNITS,ifCisnano idealM-g-closed,then$C\cup(U- (N\,int(C))^{*N})$ isnano idealM-g-closed.

Proof. SinceCisnano idealM-g-closed,$(N\,int(C))^{*N}\subseteq C$byTheorem3.3.
Then$U- C\subseteq U- (N\,int(C))^{*N}andC\cup(U- C)\subseteq C\cup(U- (N\,int(C))^{*N})$.
Thus$U\subseteq C\cup(U- (N\,int(C))^{*N})$andso$C\cup(U- (N\,int(C))^{*N})= U$.
Hence$C\cup(U- (N\,int(C))^{*N})$isnano idealM-g-closed.

Theorem3.15. InaspaceNITS,thefollowingpropertiesareequivalent:

(1)Cisnano*-closedandnanoopenset,

(2)Cis nano idealL-closedandnanoopenset,

(3)Cis nano idealM-g-closedandnanoopenset.

Proof.(1)$\Rightarrow$(2):SinceCisnano*-closedandnanoopen,$C= N\,cl^*(C)$and$C= N\,int(C)$.Thus$C= N\,cl^*(N\,int(C))$and$C= N\,int(C)$.HenceCis nano idealL-closedandnanoopen.

(2)$\Rightarrow$(3):SinceCis an ideal nanoL-closed and nano open,$C= N\,cl^*(N\,int(C))= (N\,int(C))^{*N}\cup N\,int(C)= (N\,int(C))^{*N}\cup C$.Thus$(N\,int(C))^{*N}\subseteq C$.ByTheorem3.3,Cisnano idealM-g-closedandnanoopen.

(3)$\Rightarrow$(1):SinceCisnano idealM-g-closed,$(N\,int(C))^{*N}\subseteq C$byTheorem3.3.AgainCisnanoopenimplies$C^{*N}= (N\,int(C))^{*N}\subseteq C$.ThusCisnano*-closedandnanoopen.

Theorem3.16. InaspaceNITS,ifCisnano idealM-g-closedandGissubsetsuchthat$C\subseteq G\subseteq N\,cl^*(N\,int(C))$,thenGisnano idealM-g-closed.

Proof. SinceCisnano idealM-g-closed,$N\,cl^*(N\,int(C))\subseteq C$by(4)ofTheorem4.1.Thusbyassumption,$C\subseteq G\subseteq N\,cl^*(N\,int(C))\subseteq C$.Then$C= G$andsoGisnano idealM-g-closed.

Corollary3.17. LetNITSbeaspace.IfCisanano idealM-g-closedsetandnanoopenset,then$N\,cl^*(C)$isnano idealM-g-closed.

Proof. SinceCisnanoopeninU,$C\subseteq N\,cl^*(C)= N\,cl^*(N\,int(C))$.Cisnano idealM-g-closedimplies$N\,cl^*(C)$isnano idealM-g-closedbyTheorem4.15.

Theorem3.18. InaspaceNITS,nanonowheredensesubsetisnano idealM-g-closed.

Proof. IfCisnanonowheredensesubsetinUthen$N\,int(N\,cl(C))= \varphi$.Since$N\,int(C)\subseteq N\,int(N\,cl(C))$,$N\,int(C)= \varphi$.Hence$(N\,int(C))^{*N}= \varphi^{*N}= \varphi\subseteq C$.Thus,Cisnano ideal M-g-closedbyTheorem3.3.

Remark3.19. TheconverseofTheorem4.17isnottrueasshowninthefollowingExample.

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Example3.20. InExample3.2,

Thentheset{1, 3, 4}isnano idealM-g-closedsetbutnotnanonowheredense.

Remark3.21. InaspaceNITS,theintersectionoftwonano idealM-g-closedsubsetsisnano idealM-g-closed.

Proof. LetCandG be an ideal nanoM-g-closed subsets.Then $(N\,int(C))^{*N} \subseteq C$ and $(N\,int(G))^{*N} \subseteq G$ byTheorem3.3.Also$(N\,int(C\cap G))^{*N} \subseteq (N\,int(C))^{*N} \cap (N\,int(G))^{*N} \subseteq C\cap G$.Thisimpliesthat $C\cap G$ isnano idealM-g-closedbyTheorem3.3.

Example3.22. InExample3.2,

ThenthesetsP= {1, 4}andQ= {2, 4}isnano idealM-g-closedsets.ButP∩Q= {4}is nano idealM-g-closed.

Remark3.23. InaspaceNITS,theunionoftwonano idealM-g-closedsubsetsbutnotnano idealM-g-closed.

Example3.24. InExample3.2,

ThenthesetsP= {2} andQ= {3} isnano idealM-g-closedsets.ButP∪Q= {2, 3}isnot nano idealM-g-closed.

Theorem3.25. InaspaceNITS,asubsetCisnano idealM-g-openifandonlyif$C\subseteq N\,int^{*}(N\,cl(C))$.

Proof. Cisnano idealM-g-open$\Longleftrightarrow$ U− Cisnano idealM-g-closed$\Longleftrightarrow$ U− Cisnano ideal pre*-closedby(5)ofTheorem4.1$\Longleftrightarrow$ Cisnano idealpre*-open$\Longleftrightarrow$ $G\subseteq N\,int^{*}(N\,cl(C))$.

Theorem3.26. InaspaceNITS,ifthesubsetCisnano idealM-g-closed,then$N\,cl^{*}(N\,int(C)) - C$isnano idealM-g-open.

Proof. SinceCisnano idealM-g-closed,$N\,cl^{*}(N\,int(C))- C= \varphi$by(3)ofTheorem4.1.Thus$N\,cl^{*}(N\,int(C))- C$isnano idealM-g-open.

Theorem3.27. InaspaceNITS,ifCisnano ideal M-g-open,then$N\,int^{*}(N\,cl(C))\cup(U-C)= U$.

Proof. SinceCisnano ideal M-g-open,$C\subseteq N\,int^{*}(N\,cl(C))$byTheorem4.24.So$(U-C)\cup C\subseteq (U- C)\cup N\,int^{*}(N\,cl(C))$whichimplies$U= (U- C)\cup N\,int^{*}(N\,cl(C))$.

Theorem3.28. InaspaceNITS,ifCisnano idealM-g-openandGisasubsetsuchthat$N\,int^{*}(N\,cl(C))\subseteq G\subseteq C$,thenGisnano idealM-g-open.

Proof. SinceCisnano idealM-g-open,$C\subseteq N\,int^{*}(N\,cl(C))$byTheorem4.24.Byassumption$N\,int^{*}(N\,cl(C))\subseteq G\subseteq C$.Thisimplies$C\subseteq N\,int^{*}(N\,cl(C))\subseteq G\subseteq C$.ThusC= GandsoGisnano idealM-g-open.

Corollary3.29. InaspaceNITS,ifCisnano idealM-g-opensetandnanoclosedset,then$N\,int^{*}(C)$isnano idealM-g-open.

Proof. LetCbe an ideal nanoM-g-open set and nano closed set.Then$N\,int^{*}(N\,cl(C))= N\,int^{*}(C)\subseteq N\,int^{*}(C)\subseteq C$.Thus,byTheorem4.27,$N\,int^{*}(C)$isnano idealM-g-open.

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4. Properties of generalized closed sets in ideal spaces

Theorem 4.1. In a space NITS, for a subset C of U, the following properties are equivalent.

(1) C is nano ideal W-g-closed;
(2) $(N\operatorname{int}(C))^{*N} - C = \varphi$;
(3) $Ncl^{*}(N\operatorname{int}(C)) - C = \varphi$;
(4) $Ncl^{*}(N\operatorname{int}(C)) \subseteq C$;
(5) C is nano ideal pre*-closed.

Proof. (1) $\Leftarrow\Rightarrow$ (2): C is nano ideal W-g-closed $\Leftarrow\Rightarrow (N\operatorname{int}(C))^{*N} \subseteq C$ by Theorem 3.4 $\Leftarrow\Rightarrow (N\operatorname{int}(C))^{*N} - C = \varphi$.

(2) $\Leftarrow\Rightarrow$ (3): $(N\operatorname{int}(C))^{*N} - C = \varphi \Leftarrow\Rightarrow Ncl^{*}(N\operatorname{int}(C)) - C = ((N\operatorname{int}(C))^{*N} \cup N\operatorname{int}(C)) - C = [(N\operatorname{int}(C))^{*N} - C] \cup [N\operatorname{int}(C) - C] = N\operatorname{int}(C) - C = \varphi$.

(3) $\Leftarrow\Rightarrow$ (4): $Ncl^{*}(N\operatorname{int}(C)) - C = \varphi \Leftarrow\Rightarrow Ncl^{*}(N\operatorname{int}(C)) \subseteq C$.

(4) $\Leftarrow\Rightarrow$ (5): $Ncl^{*}(N\operatorname{int}(C)) \subseteq C \Leftarrow\Rightarrow$ C is nano ideal pre*-closed.

Theorem 4.2. In a space NITS, if C is nano ideal W-g-closed, then $C \cup (U - (N\operatorname{int}(C))^{*N})$ is nano ideal W-g-closed.

Proof. Since C is nano ideal W-g-closed, $(N\operatorname{int}(C))^{*N} \subseteq C$ by Theorem 3.4. Then $U - C \subseteq U - (N\operatorname{int}(C))^{*N}$ and $C \cup (U - C) \subseteq C \cup (U - (N\operatorname{int}(C))^{*N})$. Thus $U \subseteq C \cup (U - (N\operatorname{int}(C))^{*N})$ and so $C \cup (U - (N\operatorname{int}(C))^{*N}) = U$. Hence $C \cup (U - (N\operatorname{int}(C))^{*N})$ is nano ideal W-g-closed.

Theorem 4.3. In a space NITS, the following properties are equivalent:

(1) C is nano*-closed and nano open set,

(2) C is nano ideal L-closed and nano open set,

(3) C is nano ideal W-g-closed and nano open set.

Proof. (1) $=\Rightarrow$ (2): Since C is nano*-closed and nano open, $C = Ncl^{*}(C)$ and $G = N\operatorname{int}(C)$. Thus $C = Ncl^{*}(N\operatorname{int}(C))$ and $C = N\operatorname{int}(C)$. Hence C is nano ideal L-closed and nano open.

(2) $=\Rightarrow$ (3): Since C is nano ideal L-closed and nano open, $C = Ncl^{*}(N\operatorname{int}(C)) = (N\operatorname{int}(C))^{*} \cup N\operatorname{int}(C) = (N\operatorname{int}(C))^{*N} \cup C$. Thus $(N\operatorname{int}(C))^{*} \subseteq C$. By Theorem 3.4, C is nano ideal W-g-closed and nano open.

(3) $=\Rightarrow$ (1): Since C is nano ideal W-g-closed, $(N\operatorname{int}(C))^{*N} \subseteq C$ by Theorem 3.4. Again C is nano open implies $C^{*N} = (N\operatorname{int}(C))^{*N} \subseteq C$. Thus C is nano*-closed and nano open.

Theorem 4.4. In a space NITS, a subset C is nano*-closed $\Leftarrow\Rightarrow$ nano ideal g-closed.

Proof. Let C be a nano*-closed subset of NITS. If G is any nano open set such that $C \subseteq G$ and C is nano*-closed, then $C^{*N} \subseteq C \subseteq G$ and hence C is nano ideal g-closed.

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Conversely, if C is not nano*-closed, let $1 \in C^{*N} - C$. Then $1 \in C^{*N} - C \subseteq U - C$ and $C \subseteq U - \{1\}$ where $U - \{1\}$ is nano open. But $C^{*N} \nsubseteq U - \{1\}$ which means that C is not nano ideal g-closed. This proves that C is nano ideal g-closed $\Longrightarrow$ C is nano*-closed.

Theorem 4.5. In a space NITS, C is nano closed $\Longrightarrow$ C is nano ideal W-g-closed.

Proof. If C is nano closed, then C is nano*-closed and nano ideal g-closed by Theorem 4.4. By Proposition 3.9, C is nano ideal W-g-closed.

Remark 4.7. The converse of Theorem 4.5 is not true follows from the following Example.

Example 4.8. Let $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{1, 3\}$. Then $\tau_R(X) = \{\varphi, \{2, 3\}, U\}$ and ideal $I = \{\varphi, \{2\}\}$. Then the set $\{1, 2\}$ is nano ideal W-g-closed but not nano closed.

Remark 4.9. The following Example shows that the concepts of nano ideal g-closedness and nano ideal W-g-closedness are independent of each other.

Example 4.10. In Example 4.8.

Then the set $\{2\}$ is nano ideal g-closed set but not nano ideal W-g-closed.

Example 4.11. Let $U = \{1, 2, 3, 4\}$ with $U/R = \{\{2\}, \{4\}, \{1, 3\}\}$ and $X = \{3, 4\}$. Then $\tau_R(X) = \{\varphi, \{4\}, \{1, 3\}, \{1, 3, 4\}, U\}$. Ideal be $I = \{\varphi, \{3\}\}$.

Then the set $\{1\}$ is nano ideal W-g-closed but not nano ideal g-closed.

Definition 4.12. A subset C of a space NITS is called a nano I^*-B-set if $C = H \cup K$ where H is nano closed and K is nano ideal pre*-open.

Example 4.13. In Example 4.8.

Then the set $\{2, 3\}$ is nano I^*-B-set.

Theorem 4.14. In a space NITS, C^{*N} is always nano ideal W-g-closed for each subset C of U.

Proof. Since $(C^{*N})^{*N} \subseteq C^{*N}$ by Theorem 4.8 of (3), C^{*N} is nano*-closed. Hence C^{*N} is an ideal nano g-closed by Theorem 4.4 and nano ideal W-g-closed by Proposition 3.9.

Theorem 4.15. In a space NITS, if C is nano ideal W-g-closed and G is a subset such that $C \subseteq G \subseteq Ncl^*(Nint(C))$, then G is nano ideal W-g-closed.

Proof. Since C is nano ideal W-g-closed, $Ncl^*(Nint(C)) \subseteq C$ by (4) of Theorem 4.1. Thus by assumption, $C \subseteq G \subseteq Ncl^*(Nint(C)) \subseteq C$. Then $C = G$ and so G is nano ideal W-g-closed.

Corollary 4.16. Let NITS be a space. If C is nano ideal W-g-closed set and nano open set, then $Ncl^*(C)$ is nano ideal W-g-closed.

Proof. Since C is nano open in U, $C \subseteq Ncl^*(C) = Ncl^*(Nint(C))$. C is nano ideal W-g-closed implies $Ncl^*(C)$ is nano ideal W-g-closed by Theorem 4.15.

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Theorem 4.17. In a space NITS, a nano nowhere dense subset is nano ideal W-g-closed.

Proof. If C is nano nowhere dense subset in U then $N\,int(N\,cl(C)) = \varphi$. Since $N\,int(C) \subseteq N\,int(N\,cl(C))$, $N\,int(C) = \varphi$. Hence $(N\,int(C))^{*N} = \varphi^{*N} = \varphi \subseteq C$. Thus, C is nano ideal W-g-closed by Theorem 3.4.

Remark 4.18. The converse of Theorem 4.17 is not true as shown in the following example.

Example 4.19. In Example 4.8.

Then the set $\{1, 2\}$ is nano ideal W-g-closed but not nano nowhere dense.

Remark 4.20. In a space NITS, the intersection of two nano ideal W-g-closed subsets is nano ideal W-g-closed.

Proof. Let C and G be nano ideal W-g-closed subsets in NITS. Then $(N\,int(C))^{*N} \subseteq C$ and $(N\,int(B))^{*N} \subseteq G$ by Theorem 3.4. Also $[N\,int(C \cap G)]^{*N} \subseteq [N\,int(C)]^{*N} \cap [N\,int(G)]^{*N} \subseteq C \cap G$. This implies that $C \cap G$ is nano ideal W-g-closed by Theorem 3.4.

Example 4.21. In Example 4.8.

Then the set $H = \{1\}$ and $K = \{1, 3\}$ are nano ideal W-g-closed sets.
But $C = H \cap K = \{1\}$ is nano ideal W-g-closed set.

Remark 4.22. In a space NITS, the union of two nano ideal W-g-closed subsets but not nano ideal W-g-closed.

Example 4.23. In Example 4.11.

Then $H = \{1\}$ and $K = \{3\}$ are nano ideal W-g-closed sets. But $C = H \cup K = \{1, 3\}$ is not nano ideal W-g-closed set.

Theorem 4.24. In a space NITS, a subset C is nano ideal W-g-open $\Longleftrightarrow C \subseteq N\,int^{*}(N\,cl(C))$.

Proof. Let C is nano ideal W-g-open $\Longleftrightarrow U - C$ is nano ideal W-g-closed $\Longleftrightarrow U - C$ is nano ideal pre*-closed by (5) of Theorem 4.1 $\Longleftrightarrow$ C is nano ideal pre*-open $\Longleftrightarrow C \subseteq N\,int^{*}(N\,cl(C))$.

Theorem 4.25. In a space NITS, if the subset C is nano ideal W-g-closed, then $N\,cl^{*}(N\,int(C)) - C$ is nano ideal W-g-open.

Proof. Since C is nano ideal W-g-closed, $N\,cl^{*}(N\,int(C)) - C = \varphi$ by (3) of Theorem 4.1. Thus $N\,cl^{*}(N\,int(C)) - C$ is nano ideal W-g-open.

Theorem 4.26. In a space NITS, if C is nano ideal W-g-open, then $N\,int^{*}(N\,cl(C)) \cup (U - C) = U$.

Proof. Since C is nano ideal W-g-open, $C \subseteq N\,int^{*}(N\,cl(C))$ by Theorem 4.24. So $(U - C) \cup C \subseteq (U - C) \cup N\,int^{*}(N\,cl(C))$ which implies $U = (U - C) \cup N\,int^{*}(N\,cl(C))$.

Theorem 4.27. In a space NITS, if C is nano ideal W-g-open and G is a subset such that $N\,int^{*}(N\,cl(C)) \subseteq G \subseteq C$, then G is nano ideal W-g-open.

Proof. Since C is nano ideal W-g-open, C⊆N int*(N cl(C)) by Theorem 4.24. By assumption N int*(N cl(C))⊆G⊆C. This implies C⊆N int*(N cl(C))⊆G⊆C. Thus C= G and so G is nano ideal W-g-open.

Corollary 4.28.
In a space NITS, if C is nano ideal W-g-open set and a nano closed set, then N int*(C) is nano ideal W-g-open.

Proof. Let C be a nano ideal W-g-open set and a nano closed set. Then N int*(N cl(C))= N int*(C)⊆ N int*(C)⊆C. Thus, by Theorem 4.27, N int*(C) is nano ideal W-g-open.

Proposition 4.29. In a space NITS, each nano ideal pre*-open (resp. nano closed) set is a nano I*-B-set.

Remark 4.30. The converses of Proposition 4.29 is not true as shown in the following example.

Example 4.31.

In Example 4.11.

Then the set {1, 2} is nano I*-O-set but not nano ideal pre*-open. Also {3} is nano I*-B-set but not nano closed.

Remark 4.32.

In a space the family of nano closed sets and the family of nano ideal pre*-open sets are independent.

Example 4.33.

In Example 4.11,

(1) then the set {1} is a nano ideal pre*-open set but not nano closed.
(2) then the set {2} is an nano closed set but not nano ideal pre*-open.

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