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Object Scheduling and Optimization Techniques Using Hesitancy Fuzzy Network Diagram

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ABSTRACT:
This paper proposed a new approaches for project scheduling via Hesitancy Fuzzy network diagram. This study helps to planning, controlling and managing the project. It is an essential tool to estimate the project completion or cost. In this paper we introduce the concept of Hesitancy Fuzzy Network diagram with algorithm to analyze the optimization techniques such as PERT and Assignment problems with real life examples.

Conclusions: The Project evaluation and review techniques plays a vital role in applied Mathematical modeling. It is useful to assign work to the and determination of the time schedule for the activities of a construction project. In this paper, we construct the Hesitancy Fuzzy networks to find out the project completion time and also useful to assign work to the relevant persons. This algorithm is used for the following situations like Minimal spanning tree, Shortest path problem, Maximal flow and Critical path problems.

Keywords: Hesitancy Fuzzy Network diagram, Hesitancy fuzzy optimization.

1. Introduction

The theory of fuzzy set was investigated by Zadeh [11], it contains degree of membership. After the establishment of the fuzzy sets, it has enhanced the many research field in numerous disciplines. Atanassov [1], introduced the concept of intuitionistic fuzzy set theory and it includes degree of membership and degree of non membership. In the real world, maximum number of problems become apparent due to ambivalence. So in the different perspective, Torra [9], was established the Hesitant Fuzzy set. It has enormous applications in different fields and it received good attention in graph theory, operation research,decision making problems and data mining. This Hesitancy Fuzzy set have categorized by membership, hesitancy membership and non membership. Using Hesitancy element concept to aggregation in decision making problems proposed by Xia and Xu [10]. Rosenfeld [8], investigate fuzzy graphs in fuzzy sets and established some applications to cognitive and decision processes. Zhu et al. [12], analyze and inculcate the Hesitancy Fuzzy geometric in multi criteria decision making problems. A new type of algorithm developed by Parimala et al [6] via Hesitancy Fuzzy digraph, it is helped to identify the shortest path for any types of digraphs. The concept of Intutionistic fuzzy and Neutrosophic environment to find out a shortest path problems by Parimala [5]. Using Intutionistic fuzzy graph method for findout the shortest path in networks by Karunambigai et al [4]. Dubois [3], studied the concept of fuzzy sets and systems.

In this paper, we confer the basic definition of Hesitancy Fuzzy set. This is very essential for further discussion.

Definition 2.1. Let us consider a Fuzzy set U, a Hesitancy Fuzzy set on U is in the form of a function that U is a subset of [0, 1]. It is defined by $H = \{\langle u, h_H(u)\rangle : u \in U\}$, where $h_H(u)$ is denoting the possible membership degrees of the element $u \in U$ to the set H.

Definition 2.2. Let us consider U be a non empty set finite hesitancy fuzzy set, $N_1 = \langle U, M_i, H_i, N_i\rangle$ a hesitancy fuzzy set of U, where the membership function is $M_i : U \to [0,1]$, the hesitancy membership function is $H_i : U \to [0,1]$, non membership function is $N_i : U \to [0,1]$ and $M_i + H_i + N_i = 1$. Let $N_2 = \langle U \times U, M_{i,j}, H_{i,j}, N_{i,j}\rangle$ be a hesitancy fuzzy relation on U, where the membership function is $M_{i,j} : U \times U \to [0,1]$, the hesitancy membership function is $H_{i,j} : U \times U \to [0,1]$, non membership function is $N_{i,j} : U \times U \to [0,1]$ and $0 \leq M_i + H_i + N_i \geq 1$. Here

$M_{i,j} \le \min\{u_i, u_j\}$, $N_{i,j} \le \max\{u_i, u_j\}$ and $M_{i,j} \le \min\{u_i, u_j\}$. Then the ordered pair $G = (N_1, N_2)$ is called hesitancy fuzzy digraph.

Definition 2.3. Let us consider two hesitancy fuzzy number as $A_1 = \langle M_1, H_1, N_1 \rangle$ and $A_2 = \langle M_2, H_2, N_2 \rangle$. Then,

1. $A_1 \oplus A_2 = \langle M_1 + M_2 - M_1M_2, H_1 + H_2 - H_1H_2, N_1N_2 \rangle$.
2. $A_1 \otimes A_2 = \langle M_1M_2, H_1H_2, N_1 + N_2 - N_1N_2 \rangle$.
3. $\lambda A_1 = \left\langle \left(1 - (1 - M_1)^{\lambda}\right), \left(1 - (1 - H_1)^{\lambda}\right), N_1^{\lambda} \right\rangle$.
4. $A_1^{\lambda} = \left\langle M_1^{\lambda}, H_1^{\lambda}, \left(1 - (1 - N1)^{\lambda}\right) \right\rangle$.

Definition 2.4. Let $A = \langle M, H, N \rangle$ be a hesitancy fuzzy number. Then, the hesitancy score function $s_h(A)$ is defined by $s_h(A) = \dfrac{1 + (M + 2H - N)(2 - M - N)}{2}$.

3. Network Diagram Representation and Algorithm

A network Diagram is a graphical representation of project scheme. In general Network diagram is used to analysis specific techniques such as planning, control of projects and Management. Consider the directed network with Hesitancy Fuzzy finite set of nodes represented by N = {1, 2, ...i} and Hesitancy Fuzzy finite set of edges represented by $E \subseteq N \times N$. Every node represents some activities such as predeccessor node and successor. If node i is directly connected with node i+1 is said to be successor node of,node i .

In this section let us consider the Project evaluation and review technique in Hesitancy Fuzzy number with network diagram, which is an essential tool to analysis project network, planning and estimation of time/cost. This techniques has three types of estimation. That is optimistic time, pessimistic time and most likely time estimation. The brief explanation of the above estimations are, the shortest possible time time to complete an activity is called optimistic time. It is denoted by O_t. The shortest time conceivable for an activity is known as pessimistic time. It is denoted by P_t. The time estimation with the normal conditions of activity. It is denoted by M_t

3.1. Algorithm :

1. Illustrate the Hesitancy Fuzzy value of O_t, P_t and M_t for each activities by using Hesitancy score function.

2. Calculate the value of Hesitancy Fuzzy expected time $E_t = \dfrac{O_t + 4M_t + P_t}{6}$, Hesitancy Fuzzy variance $S_n^2 = \left(\dfrac{P_t - O_t}{6}\right)^2$ and Hesitancy Fuzzy normal variable $N_h = \dfrac{X - \overline{X}}{S_n}$, where X is the given project duration and $\overline{X}$ is the critical path duration.

3. Construct the Hesitancy Fuzzy network diagram using Hesitancy Fuzzy expected time E_t.

4. Calculate a Hesitancy Fuzzy Earliest start time and Hesitancy Fuzzy Latest finish time for each node.

5. Identify the critical path.

6. Find out the Hesitancy Fuzzy standard deviation by using critical path.

7. Estimate the probability of completing the project duration time.

3.2. Illustrate with Real life example :

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A publisher has a contract with an author to publish a textbook. The author submits a hard copy and a computer file of the manuscript. The activities associated with the production of the textbook are summarized in the following **Table 1** and **Table 2**. Draw the project network and find the probability that the project is completed in one week.

Activity	Description	Predecessors
A	Manuscript prepared by author	-
B	Sample pages and artwork preparation	-
C	Book formatting	-
D	Manuscript proof reading by Editor	A
E	Author's approval of edited sample pages and artwork	B
F	Book design	C
G	Author's review of formatted pages	C
H	Author's review of Book design	E, F
I	Production of printing plates	E, F
J	Book Production	D,H
K	Book Printing	G, I

Table 1: Each activities with Predecessors

Activity	Optimistic time	Most likely time	Pessimistic time
A	(0.5, 0.4, 0.1)	(0.2, 0.6, 0.2)	(0.3, 0.4, 0.3)
B	(0.3, 0.5, 0.2)	(0.6, 0.3, 0.1)	(0.5, 0.3, 0.2)
C	(0.7, 0.1, 0.2)	(0.5, 0.3, 0.1)	(0.7, 0.2, 0.1)
D	(0.2, 0.5, 0.3)	(0.8, 0.1, 0.1)	(0.4, 0.3, 0.3)
E	(0.6, 0.2, 0.2)	(0.3, 0.4, 0.3)	(0.2, 0.7, 0.1)
F	(0.5, 0.3, 0.2)	(0.2, 0.5, 0.3)	(0.1, 0.6, 0.3)
G	(0.4, 0.5, 0.1)	(0.7, 0.2, 0.1)	(0.3, 0.5, 0.2)
H	(0.4, 0.4, 0.1)	(0.6, 0.2, 0.2)	(0.1, 0.5, 0.4)
I	(0.5, 0.3, 0.2)	(0.4, 0.5, 0.1)	(0.5, 0.2, 0.3)
J	(0.3, 0.4, 0.3)	(0.4, 0.4, 0.2)	(0.3, 0.4, 0.3)
K	(0.3, 0.6, 0.1)	(0.2, 0.4, 0.4)	(0.2, 0.6, 0.2)

Table 2: Hesitancy Fuzzy values for each activities

SOLUTION:

Step 1 : The following steps are used to solve the above problem. Illustrate the Hesitancy Fuzzy value of O_t, P_t and M_t for each activities by using Hesitancy score function and also calculate as shown in the following **Table 3.**

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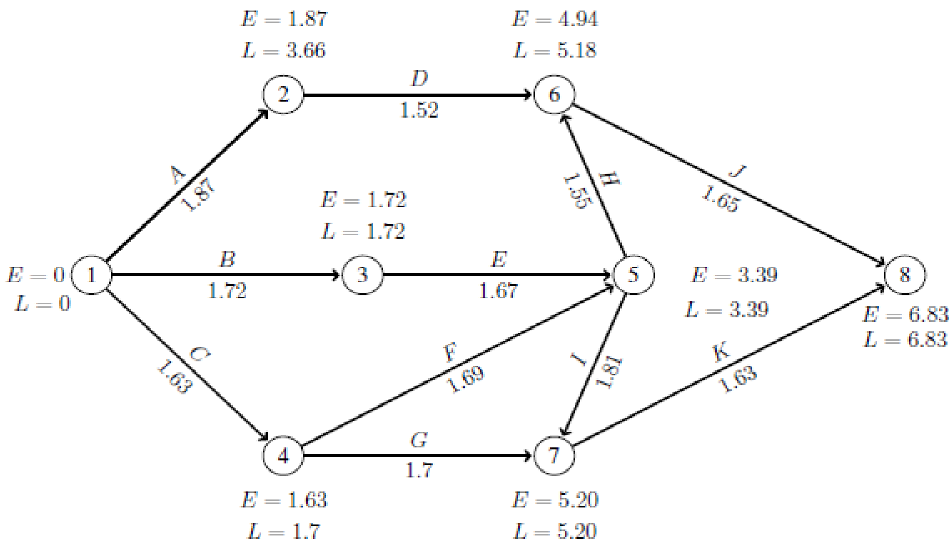
Activity	**O_t**	**M_t**	**P_t**	**E_t**	S_n^2
A	1.84	1.96	1.56	1.87	0.0022
B	1.83	1.72	1.59	1.72	0.0016
C	1.39	1.70	1.60	1.63	0.0012
D	1.68	1.50	1.46	1.52	0.0014
E	1.48	1.56	2.28	1.76	0.0178
F	1.59	1.68	1.80	1.69	0.0013
G	1.98	1.60	1.83	1.7	0.0006
H	1.83	1.48	1.53	1.55	0.0025
I	1.59	1.98	1.36	1.81	0.0013
J	1.56	1.70	1.56	1.65	0.0
K	2.12	1.42	1.96	1.63	0.0007

Table 3: Hesitancy fuzzy Expected time and Variance

Step 2: Draw the Hesitancy Fuzzy network diagram using Hesitancy Fuzzy expected time E_t.



Step 3: To identify the critical path.

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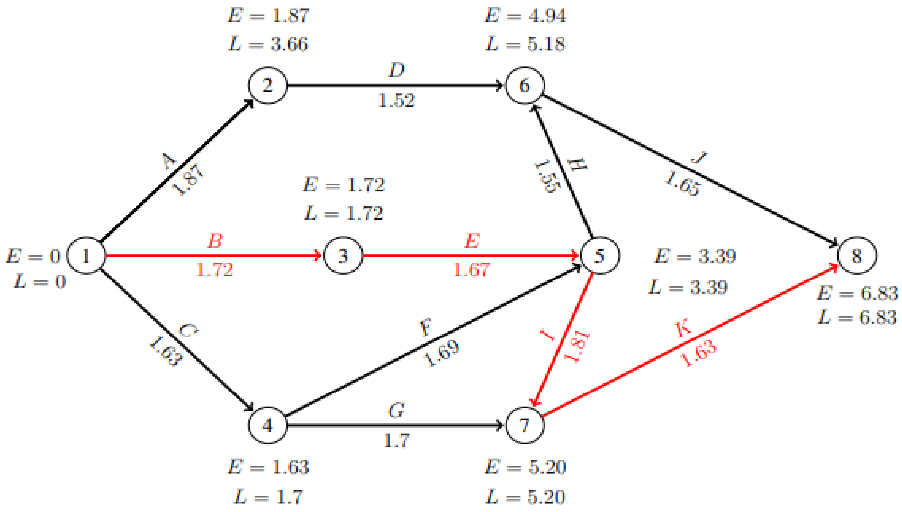

The critical path is **B - E - I - K.**

The project completion time is 6.83 days.

Step 4: The project completion time on or before one week is

$$P(x \leq 7) = P\left(Z \leq \frac{7-6.83}{0.146}\right) = P(Z \leq 1.16) = 0.5 + 0.3770 = 0.877.$$

Therefore the probability of completing the project on or before one week is 88%.

4. Application of Hesitancy Fuzzy number with an Assignment Problem

Assignment problem plays a vital role to allocate works in any kind of practical problems. It is an essential tool to fulfill the demand of any type of problems. The main purpose of this Assignment method is to minimize time/cost to complete the tasks. The following table shows that Hesitancy fuzzy service time of each counter by using the score function.

In this section illustrate an Assignment problem by using Hesitancy Fuzzy number the following example is used.

In a car company four salesmen A, B, C and D are available to handle four counters stock Management, Enquiry section, Product sales, cash counter. Each salesmen can handle any counter. How should the salesmen be allocated to appropriate counters so as to minimize the total service time.

Counters	A	B	C	D
C1 Stock Management	(0.3, 0.5, 0.2)	(0.3, 0.4, 0.3)	(0.5, 0.3, 0.2)	(0.5, 0.3, 0.2)
C_2 Enquiry Section	(0.5, 0.4, 0.1)	(0.4, 0.3, 0.3)	(0.6, 0.3, 0.1)	(0.7, 0.1, 0.2)
C_3 Product Sales	(0.6, 0.2, 0.2)	(0.7, 0.2, 0.1)	(0.3, 0.6, 0.1)	(0.5, 0.2, 0.3)
C_4 Cash counter	(0.4, 0.4, 0.2)	(0.2, 0.7, 0.1)	(0.4, 0.3, 0.1)	(0.3, 0.5, 0.2)

Table 4: Each counter hesitancy Fuzzy time

Solution:

Let us consider the Hesitancy Fuzzy service time of each counters when manned by each salesmen is given in the following **Table 5.**

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Counters	A	B	C	D
C1	1.83	1.56	1.59	1.59
C_2	1.84	1.46	1.72	1.28
C_3	1.48	1.60	2.12	1.36
C_4	1.70	2.28	1.68	1.83

Table 5: Hesitancy Fuzzy time score value

Now let us consider row difference method from each row:

Counters	A	B	C	D
C1	0.27	0	0.03	0.03
C_2	0.56	0.18	0.44	0
C_3	0.12	0.24	0.76	0
C_4	0.02	0.60	0	0.15

Table 6: Row difference

Now let us consider column difference method from each column:

Counters	A	B	C	D
C1	0.25	0	0.03	0.03
C_2	0.54	0.18	0.44	0
C_3	0.10	0.24	0.76	0
C_4	0	0.60	0	0.15

Table 7: Column difference

Here we can't assign each salesmen with one counter, so the following table shows that minimum number of straight lines cover all zeros.

Counters	A	B	C	D

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C1	~~0.25~~	~~0~~	~~0.03~~	~~0.03~~
C_2	0.54	0.18	0.44	~~0~~
C_3	0.10	0.24	0.76	~~0~~
C_4	~~0~~	~~0.60~~	~~0~~	~~0.15~~

Table 8:Minimum Number of Straight Lines

Now we assign salesman for each counter as follows:

Counters	A	B	C	D
C1	0.25	[0]	0.03	0.13
C_2	0.44	0.08	0.34	[0]
C_3	[0]	0.14	0.66	0
C_4	0	0.60	[0]	0.25

Table 9: Schedule

5. Results

Hence the optimum Assignment schedule is :

Salesman	Counter	Hesitancy Fuzzy Service Time
A	Product Sales	1.48 hrs
B	Stock Management	1.56 hrs
C	Cash Counter	1.68 hrs
D	Enquiry Section	1.28 hrs

Hence the corresponding minimum Hesitancy Fuzzy Service time is 6 hrs.

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