JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 1263-1270
https://publishoa.com
ISSN: 1309-3452

Determining the efficient optimal order quantity for an Inventory model with varying Fuzzy components

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ABSTRACT

In the field of applied mathematics, optimization techniques formulate to the maximizing and minimizing for an objective function. The purpose of the optimization problems plays a vital role in the field of inventory management. The aim is to minimize the total cost, which comprises many fluctuating costs such as shortage, ordering, and holding cost. In this paper, the defective items were under the classification synchronous and asynchronous under a rework strategy process. The rework strategy is separating and accumulating the imperfect items at the time of completion of the process. This study considered asynchronous defective items and tried to minimize the total cost incurred. The optimality of the non-linear programming was achieved by the Hessian matrix, which results in the minimization of the total cost incurred. Furthermore, the usage of hexagonal fuzzy numbers formulates many real-life problems that arise due to flawed knowledge. There might be several situations in decision-making problems where optimization techniques require six parameters or more. The inclusion of Python coding has further made numerical working simpler. Furthermore, sensitivity analysis is carried out.

Keywords:

Fuzzy, triangular fuzzy number, signed-distance, EOQ, Optimization

1 Introduction

Fuzzy sets were first introduced by Zadeh [3] in the year 1965. Henceforth there were major breakthrough in the field of fuzzy and its applications. It was in the year 1913, Harris [2] had proposed the Economic order quantity (EOQ) model. Zimmermann [4] introduced fuzzy sets in operations research studies. In 1987, Park [5] interpreted fuzzy sets in EOQ, where the order quantity and demand were considered as crisp quantities and order cost and holding cost were taken as fuzzy parameters. In 1996, Chen [6] studied a backorder fuzzy inventory model under function principle. Edward [19] systematically reviewed different inventory models and the applications of inventory management. In literature review, it is seen that demand was kept as a constant and Roy [7] studied an EOQ model under fuzzy environment considering demand dependent cost. Yao [8] studied a back-order inventory model with total cost fuzzified and used centroid and signed-distance defuzzification methods. Rosenblatt [10] had taken imperfect quality items under economic production quantity model. Further, studies in imperfect quality items were further considered in fuzzy EOQ models were done by Wang [13], Salameh [14]. In 1996, Vujosevic [15] the inventory cost was fuzzified in an EOQ model. In the year 1958, Wagner [16] had brought the dynamic version in economic lot size models.

Taha [17] in his book had developed several optimization techniques. Amran [1] investigated an inventory model with perishable items and applied Lagrangian method for optimizing the total cost. Optimization of economic order quantity was done many researchers [9, 12] and Kalaiarasi [11] applied Lagrangian optimization to evaluate the optimal order quantity. In 2011, Lagrangian method was adopted to the optimization of a two-stage integrated inventory models by

Ritha [20]. Vijayan and Ragavan [18] examined an inventory model with lost sales converted into fuzzy parameter and optimized using Lagrangian method.

Inventory management of any concern faces uncertainty and vagueness in forecasting the demand, due to that stockouts occur. In this paper, the economic order quantity was derived for the total cost and fuzzified by both triangular fuzzy numbers and for defuzzification, signed-distance method was applied. The demand and expected stockouts parameters are fuzzified using triangular fuzzy numbers and numerical analysis is done to compare the fuzzy and crisp values using sensitivity study.

This paper is organized such that section 2 exhibits the preliminaries needed. In section 3, the optimal order quantity of the inventory model is derived and followed by the fuzzy inventory model and optimization using Lagrangian method is done in Section 5. Further, section 6 has the numerical discussion and in section 6, conclusion is given.

2. Preliminaries

2.1 Definition: Fuzzy set [3]
Let X be a space of points (objects). A fuzzy set A in X is an object of the form A = {(x, μ_A(x)): x ∈ X} where $\mu_A: X \to [0,1]$ is called the membership function of the fuzzy set A.

2.2 Definition: Triangular fuzzy numbers (TFN)

A triangular fuzzy number $A(a_1, a_2 a_3)$ is said to have the following membership function

$$\mu_A(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0 & x > a_3 \end{cases}$$

2.3 Defuzzification Method

Let $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$ be triangular fuzzy numbers, its signed-distance formula is given by

$$\frac{a_1 + 2a_2 + a_3}{4}$$

3. The Inventory model

Parameters used
$Q \to$ order quantity
$I(\alpha) \to$ investment required to reduce the lost sales fraction
$\alpha \to$ annual fractional cost of capital investment
$S \to$ Expected stockout
$P \to$ Safety factor
$h \to$ holding cost per unit per year
$E \to$ Demand
$R \to$ Lead time per week
$E(X - W) \to$ expected shortage quantity at the end of the cycle
$C(l - t) \to$ Leadtime crashing cost

The total cost Biswajit [21] had been taken and the optimal order quantity was derived.

$$T_C = \alpha\, I(\alpha) + \frac{1}{Q}\,[S + C(l-t)] + h\left[P - ER + \frac{EQ}{2} + \alpha\, E(X-W)\right] \qquad -(1)$$

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Partially differentiating w.r.t 'Q'

$$\frac{\partial T_C}{\partial Q} = -\frac{1}{Q^2}\left[S + C(l-t)\right] + \frac{hE}{2} \qquad -(2)$$

Equating $\frac{\partial T_C}{\partial Q} = 0$,

$$\Rightarrow \frac{1}{Q^2}\left[S + C(l-t)\right] = \frac{hE}{2} \qquad -(3)$$

We obtain the optimal order quantity as

$$\Rightarrow Q = \sqrt{\frac{2(S + C(l-t))}{hE}} \qquad -(4)$$

4. Fuzzification process

The parameters $'S'$ and $'E'$ are fuzzified using triangular fuzzy numbers and signed-distance defuzzification method is applied to stabilize,
(S_1, S_2, S_3) and (E_1, E_2, E_3)

$$\widetilde{T_C} = \alpha\, I(\alpha) + \frac{1}{Q}\left[\tilde{S} + C(l-t)\right] + h\left[P - \tilde{E}R + \frac{\tilde{E}Q}{2} + \alpha\, E(X-W)\right] \qquad -(5)$$

$$\widetilde{T_C} = \alpha\, I(\alpha) + \frac{1}{Q}\left[(S_1, S_2, S_3) + C(l-t)\right] + h\left[P - (E_1, E_2, E_3)R + \frac{(E_1, E_2, E_3)Q}{2} + \alpha\, E(X-W)\right] \qquad -(6)$$

$$\widetilde{T_C} = \alpha\, I(\alpha) + \frac{1}{Q}\left[S_1 + C(l-t)\right] + h\left[P - E_1R + \frac{E_1Q}{2} + \alpha\, E(X-W)\right], \alpha\, I(\alpha) + \frac{1}{Q}\left[S_2 + C(l-t)\right] + h\left[P - E_2R + \frac{E_2Q}{2} + \alpha\, E(X-W)\right], \alpha\, I(\alpha) + \frac{1}{Q}\left[S_3 + C(l-t)\right] + h\left[P - E_3R + \frac{E_3Q}{2} + \alpha\, E(X-W)\right] \qquad -(7)$$

$$\Rightarrow \quad \frac{\partial \widetilde{T_C}}{\partial Q} = \frac{1}{4}\left\{\left(-\frac{1}{Q^2}\left[S_1 + C(l-t)\right] + h\frac{E_1}{2}\right) + 2\left(-\frac{1}{Q^2}\left[S_2 + C(l-t)\right] + h\frac{E_2}{2}\right) + \left(-\frac{1}{Q^2}\left[S_3 + C(l-t)\right] + h\frac{E_3}{2}\right)\right\} \qquad -(8)$$

Solving equation (8) we get,

$$\Rightarrow Q^* = \sqrt{\frac{2(S_1 + 2S_2 + S_3) + C(l-t)}{h(E_1 + 2E_2 + E_3)}} \qquad -(9)$$

Solving the unconstraint problem,

$$\Rightarrow \frac{1}{4}\left\{\left(\alpha\, I(\alpha) + \frac{1}{Q_3}\left[S_1 + C(l-t)\right] + h\left[P - E_1R + \frac{E_1Q_1}{2} + \alpha\, E(X-W)\right]\right) + 2\left(\alpha\, I(\alpha) + \frac{1}{Q_2}\left[S_2 + C(l-t)\right] + h\left[P - E_2R + \frac{E_2Q_2}{2} + \alpha\, E(X-W)\right]\right) + \alpha\, I(\alpha) + \frac{1}{Q_1}\left[S_3 + C(l-t)\right] + h\left[P - E_3R + \frac{E_3Q_3}{2} + \alpha\, E(X-W)\right]\right\} \qquad -(10)$$

now we partially differentiating w.r.t Q_1, Q_2, Q_3 respectively,

$$\Rightarrow \quad \frac{\partial T_C}{\partial Q_1} = \frac{1}{4}\left[\frac{hE_1}{2} - \frac{1}{Q_1^2}\left[S_3 + C(l-t)\right]\right]$$

Letting $\frac{\partial T_C}{\partial Q_1} = 0$.

$$Q_1 = \sqrt{\frac{2(S_3 + C(l-t))}{hE_1}} \qquad -(11)$$

$$\Rightarrow \qquad \frac{\partial T_C}{\partial Q_2} = \frac{1}{4}\left[-\frac{2}{Q_2^2}[S_3 + C(l-t)] + \frac{hE_2}{2}\right]$$

Letting $\frac{\partial T_C}{\partial Q_2} = 0.$

$$Q_2 = \sqrt{\frac{4(S_2 + C(l-t))}{hE_2}} \qquad -(12)$$

Letting $\frac{\partial T_C}{\partial Q_3} = 0.$

$$\Rightarrow \qquad \frac{\partial T_C}{\partial Q_3} = \frac{1}{4}\left[-\frac{1}{Q_3^2}[S_1 + C(l-t)] + \frac{E_3 h}{2}\right]$$

$$Q_3 = \sqrt{\frac{2(S_1 + C(l-t))}{hE_3}} \qquad -(13)$$

5. Optimization by Lagrangian Method

The above results show that $Q_1 > Q_2 > Q_3$ but contrastingly we have $0 < Q_1 \leq Q_2 \leq Q_3$. Hence w and set k=1 and we convert the inequality constraint by

Optimizing the total cost subject to Lagrangian method subject to $Q_2 - Q_1 = 0$

$$L(Q_1, Q_2, Q_3, \lambda) = P(T_C(Q)) - \lambda(Q_2 - Q_1) \qquad -(14)$$

Now taking the partial derivatives w.r.t Q_1, Q_2, Q_3 and λ and the minimize $L(Q_1, Q_2, Q_3, \lambda)$

$$\frac{\partial L}{\partial Q_1} = 0$$

$$\Rightarrow \quad \frac{1}{4}\left[\frac{hE_1}{2} - \frac{1}{Q_1^2}[S_3 + C(l-t)]\right] + \lambda = 0 \qquad -(15)$$

$$\frac{\partial L}{\partial Q_2} = 0$$

$$\Rightarrow \qquad \frac{1}{4}\left[-\frac{2}{Q_2^2}[S_3 + C(l-t)] + \frac{hE_2}{2}\right] - \lambda = 0 \quad -(16)$$

$$\frac{\partial L}{\partial Q_3} = 0$$

$$\Rightarrow \qquad \frac{1}{4}\left[-\frac{1}{Q_3^2}[S_1 + C(l-t)] + \frac{E_3 h}{2}\right] = 0 \qquad -(17)$$

$$\Rightarrow \qquad \frac{\partial L}{\partial \lambda} = -(Q_2 - Q_1) \qquad -(18)$$

$$Q_1 = Q_2 = \sqrt{\frac{2\big(S_1 + C(l-t)\big) + 4\,\big(S_2 + C(l-t)\big)}{h(E_1 + E_2)}} \qquad -(19)$$

Now converting the inequality constraints $Q_2 - Q_1 \geq 0$, $Q_3 - Q_2 \geq 0$ into equality constraints $Q_2 - Q_1 = 0 \; and \; Q_3 - Q_2 = 0$. Optimizing

$$L(Q_1, Q_2, Q_3, \lambda_1, \lambda_2) = P(T_C(Q)) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2) \qquad -(20)$$

$$\frac{\partial L}{\partial Q_1} = 0$$

$$\Rightarrow \quad \frac{1}{4}\left[\frac{hE_1}{2} - \frac{1}{Q_1^2}[S_3 + C(l-t)]\right] + \lambda_1 = 0 \qquad -(21)$$

$$\frac{\partial L}{\partial Q_2} = 0$$

$$\Rightarrow \qquad \frac{1}{4}\left[-\frac{2}{Q_2^2}[S_3 + C(l-t)] + \frac{hE_2}{2}\right] - \lambda_1 + \lambda_2 = 0 \qquad -(22)$$

$$\frac{\partial L}{\partial Q_3} = 0$$

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Volume 13, No. 3, 2022, p. 1263-1270
https://publishoa.com
ISSN: 1309-3452

$$\Rightarrow \quad \frac{1}{4}\left[-\frac{1}{Q_3^2}[S_1 + C(l-t)] + \frac{E_3 h}{2}\right] - \lambda_2 = 0 \qquad -(23)$$

$$\Rightarrow \quad \frac{\partial L}{\partial \lambda_1} = -(Q_2 - Q_1), \quad \frac{\partial L}{\partial \lambda_2} = -(Q_3 - Q_2) \qquad -(24)$$

$$Q_1 = Q_2 = Q_3 = \sqrt{\frac{2\big(S_1 + C(l-t)\big) + 4\big(S_2 + C(l-t)\big) + 2\big(S_3 + C(l-t)\big)}{h(E_1 + E_2 + E_3)}} \qquad -(25)$$

Therefore, $\tilde{Q} = (Q_1, Q_2, Q_3)$ satisfies all the inequality constraints and we obtain the optimum solution for the problems.

Let $Q_1 = Q_2 = Q_3 = Q^*$ Then Optimal fuzzy EOQ is given by

$$Q^* = \sqrt{\frac{2\big((S_1 + 2S_2 + S_3) + 4C(l-t)\big)}{h(E_1 + E_2 + E_3)}} \qquad -(26)$$

6. Numerical Analysis and Discussion

The numerical values for the parameters are given by S =70, = 2.5, $h = 5$, l =13, $E = 10$, t =10. Table 1 shows the sensitivity analysis for the triangular fuzzy numbers were compared with crisp values. It is clearly visible that after fuzzification using the above fuzzy numbers and defuzzification using signed distance method, the fuzzy values remain the same. There are slight variations between the crisp and fuzzy values.

Optimal value in Crisp sense is $\boldsymbol{Q = 1.717556}$ and fuzzy optimal quantity values is $\boldsymbol{Q^* = 1.684488}$. The comparison between the two optimal values are shown in Fig.1. A sensitivity analysis tabulation is done between the crisp and fuzzy values in Table 1. The sensitivity graph Fig.2 compares the variations between the two quantities.

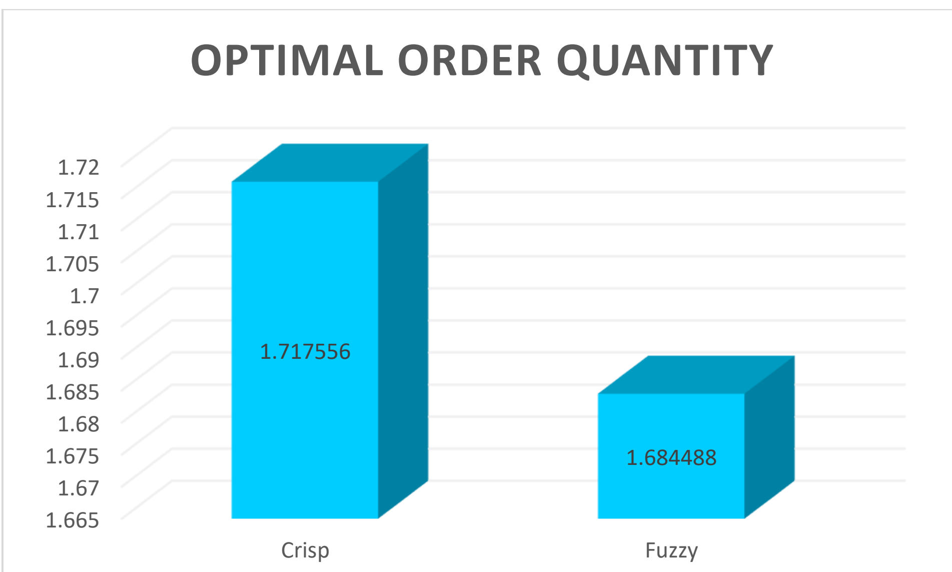

Fig.1 Chart exhibiting Crisp value against the Fuzzy value

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Volume 13, No. 3, 2022, p. 1263-1270
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ISSN: 1309-3452

Sensitivity Variations	***S*** **(TFN)**	***E*** **(TFN)**	**Crisp Values** ***Q***	**Fuzzy Values** Q^*
-50%	35 $(25, 35, 45)$	5 $(2.5, 5, 7.5)$	1.760682	1.695582
-25%	52.5 $(42.5, 52.5, 62.5)$	7.5 $(2.5, 7.5, 12.5)$	1.732051	1.681940
No Variations	70 $(60, 70, 80)$	10 $(5, 10, 15)$	1.717556	1.684488
$+25\%$	87.5 $(77.5, 87.5, 97.5)$	12.5 $(7.5, 12.5, 17.5)$	1.708801	1.682260
$+50\%$	105 $(95, 105, 115)$	15 $(10,15, 20)$	1.702939	1.680774

Table 1: Sensitivity Analysis using Triangular fuzzy numbers

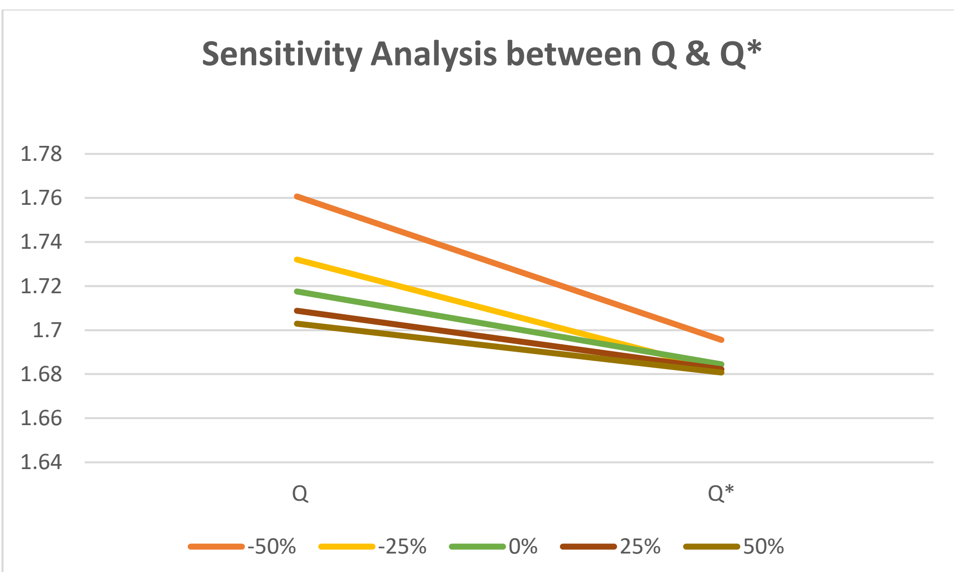

Fig.2: Comparison between the optimal values based on sensitivity analysis

7 Conclusion

The optimal order quantity for an inventory model was derived and fuzzified using triangular fuzzy numbers. The two sensitive parameters of any inventory model, the demand and the stockouts are fuzzified using triangular fuzzy numbers. In Defuzzification method was done using signed-distance method. Fig.1 shows the results of the fuzzy outputs compared with crisp values. Further, the results can be studied and compared using various fuzzy numbers and defuzzification methods. The difference between the crisp and fuzzy values are shown in Tab.1. It is observed that, the fuzzy values are significantly lesser when compared to the crisp values.

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