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Solving Assignment Problem In Fuzzy Environment By Using New Ranking Technique In Trapezoidal Fuzzy Number

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ABSTRACT

The subject of assignment issues is fascinating, and it is constantly used to solve difficulties in engineering and management science. We suggested a novel ranking approach in trapezoidal fuzzy numbers to get the best answer in this research. The ranking system is entirely responsible for the best answer. Value costs are characterized by trapezoidal fuzzy numbers. When making decisions under uncertain environment, ranking of fuzzy numbers is extremely significant. This rating approach is the most dependable, easy to use, and applicable to all sorts of assignment difficulties. Our objective is to use a novel ranking algorithm to convert fuzzy data into crisp values. In assignment problem, we use crisp values. Our objective is to use a novel ranking algorithm to transform fuzzy data into crisp values. In the assignment issue, we use crisp values. The Fuzzy Hungarian Method is then used to find the best solution to the fuzzy assignment problem. It is clear from the numerical examples that the suggested ranking measure is simple to calculate and cost-effective for a fuzzy assignment issue, and it delivers much higher optimum value.

Keywords: Fuzzy set, Fuzzy number, trapezoidal fuzzy number, Fuzzy assignment problem, New Ranking Technique.

1. INTRODUCTION

A specific sort of linear programming issue is the Assignment Problem. The assignment problem is similar to how real-world problems are solved. Assignment issue is a fantastic subject that is constantly used to solve difficulties in engineering and management science, as well as being widely used in industrial and service systems. In a job assignment problem, n jobs must be assigned to n people based on their ability to complete the task. Cij signifies the cost of assigning the jth work to the ith individual in an assignment dilemma. We suppose that each individual can only do one job and that each person can only do one job.The aim is to select the smallest assignment possible such that the overall cost of all tasks is as low as possible or the total profit is as high as possible. In 1965, Zadeh [26] was the first to develop the fuzzy set as a mathematical expression of imprecision or indistinctness in everyday life. Various ways to solve FLP issues were offered by a number of authors. Researchers like F.Choobinesh and H.Li [10], R.R.Yager [25], S.H.Chen[8], R,Ramesh, and S.kumaraghuru [19] have studied ranking systems. Bortolan and Degani [1] evaluated and contrasted a few of these ranking systems. Chen and H Wang [6] evaluated the existing methods for ranking fuzzy numbers and found that each strategy had flaws in various areas, such as indiscrimination and difficult interpretation.The optimal solution of the transportation issue with fuzzy cost coefficients, according to S.Chanas and D.Kuchta [2]. The fuzzy variables in decision making are presented by Jain R [13]. Lee and Chen [15] investigated the various forms of fuzzy numbers and also their different aberrations. The ranking techniques of fuzzy numbers are revised by Wang YJ and H.S Lee [23]. We introduced a novel strategy for ranking trapezoidal fuzzy numbers in this study. To demonstrate this proposed strategy, instances are provided. Because the suggested ranking technique is straightforward and straightforward, it is simple to comprehend and use, making it simple to identify the fuzzy optimal solution to fuzzy assignment issues that arise in real-world situations.

2. PRELIMINARIES

2.1 Definition - Fuzzy Set

A fuzzy set $\tilde{A}$ is a subset of a universe of discourse *X*, which is characterized by a membership function $\mu_{\tilde{A}}(x)$ representing a mapping $\mu_{\tilde{A}}: X \to [0,1]$. The function value of $\mu_{\tilde{A}}(x)$ is called the membership value, which represents the degree of truth that *x* is an element of fuzzy set $\tilde{A}$.

2.2 Definition - Fuzzy Number

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number and its membership function $\tilde{A}: R \to [0,1]$ has the following characteristics,

(i) $\tilde{A}$ is convex.

$$\mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \quad \forall x \in [x_1, x_2], \lambda \in [0,1].$$

(ii) $\tilde{A}$ is normal if $\max \mu_{\tilde{A}}(x) = 1$.

(iii) $\tilde{A}$ is piecewise continuous.

2.3 Definition - Non-Negative Fuzzy Number

A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only $\mu_{\tilde{A}}(x) = 0, \ \forall \ x < 0$

2.4 Definition - Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A} = (a, b, c, d)$ is called a trapezoidal fuzzy number, if its membership function $\mu_{\tilde{A}}$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if\ x < a \\ \frac{x-a}{b-a}, & if\ a < x \leq b \\ 1, & if\ b < x < c \\ \frac{d-x}{d-c} & if\ c \leq x < d \\ 0\ , & if d > d \end{cases}$$

The trapezoidal fuzzy number $\tilde{A}$ is denoted by quadruplet $\tilde{A} = (a, b, c, d)$ and has the shape of a trapezoid.

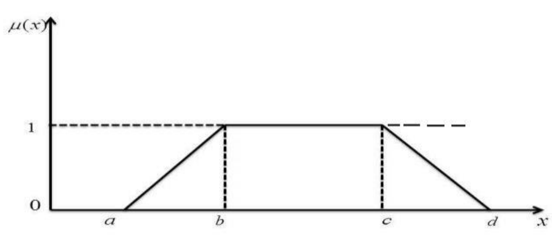

Figure: 1 Membership function of Trapezoidal fuzzy number $\tilde{A}$

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2.5 Definition

A trapezoidal fuzzy number $\tilde{A}=(a,b,c,d)$ is said to be non-negative (non positive) trapezoidal fuzzy number. i.e. $\tilde{A} \geq 0$ $(\tilde{A} \leq 0)$ if and only if $a \geq 0$ $(c \leq 0)$. A trapezoidal fuzzy number is said to be positive (negative) trapezoidal fuzzy number

i.e. $\tilde{A} > 0$ $(\tilde{A} < 0)$ if and only if $a > 0 (c < 0)$.

2.6 Definition

Two trapezoidal fuzzy number $\tilde{A}_1=(a,b,c,d)$ and $\tilde{A}_2=(e,f,g,h)$ are said to be equal. i.e. $\tilde{A}_1=\tilde{A}_2$ if and only if a=e, b=f, c=g, d=h.

2.7 Definition α–cut

An α -cut of a fuzzy set A is a Crisp set A_a that has all the elements of the entire set X that has a membership grade in A greater than or equal to specified value of α .

Thus $A=\left\{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\right\}$, $0 \leq \alpha \leq 1$.

2.8 Definition - Arithmetic Operations of Trapezoidal Fuzzy Numbers

Let $\tilde{A}=(a_1,a_2,a_3,a_4)$ and $\tilde{B}=(b_1,b_2,b_3,b_4)$ be two non-negative trapezoidal fuzzy number then

(i) $$\tilde{A} \oplus \tilde{B}=(a_1,a_2,a_3,a_4) \oplus (b_1,b_2,b_3,b_4)=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$$

(ii) $$\tilde{A}-\tilde{B}=(a_1,a_2,a_3,a_4)-(b_1,b_2,b_3,b_4)=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_2)$$

(iii) $$-\tilde{A}=-(a_1,a_2,a_3,a_4)=(-a_4,-a_3,-a_2,-a_1)$$

(iv) $$\tilde{A} \otimes \tilde{B}=(a_1,a_2,a_3,a_4) \otimes (b_1,b_2,b_3,b_4)=(a_1b_1,a_2b_2,a_3b_3,a_4b_4)$$

(v). $$\frac{1}{\tilde{A}} \cong \left(1/a_4 , 1/a_3 , 1/a_2 , 1/a_1 \right)$$

3. PROPOSED RANKING PROCEDURE

We proposed a novelranking for trapezoidal fuzzy numbers in this chapter. Ranking algorithms convert a fuzzy number into a crisp number. The below are some of the processes in the mathematical model for getting an optimal solution:

3.1 Proposed Ranking Function

This following membership function is used by us.

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} \frac{x-a}{b-a}, & if\ a < x \leq b \\ 1, & if\ b < x \leq c \\ \frac{d-x}{d-c} & if\ c \leq x < d \end{cases}$$

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By using $\alpha - cut$, where $\alpha \in [0,1]\, and\, 0 \le \alpha \le 1, then$

$\alpha = \frac{x-a}{b-a}$ and $\alpha = \frac{d-x}{d-c}$

x = a + $\alpha(b-a)$ = $A_L\alpha$ and x = d - $\alpha(d-c)$ = $A_U\alpha$

$$R(\tilde{A}_\alpha) = \int_0^1 m(A_\alpha)\, d\alpha$$

$$where\, m(A_\alpha) = \frac{A_L\alpha + A_U\alpha}{2}$$

$$R(\tilde{A}_\alpha) = \int_0^1 \left(\frac{a + \alpha(b-a) + d - \alpha(d-c)}{2} \right) d\alpha$$

$$= \frac{1}{2}\left(a(\alpha)_0^1 + (b-a)(\alpha^2)_0^1 + d(\alpha)_0^1 - (d-c)(\alpha^2)_0^1 \right)$$

$$= \frac{1}{2}\left(a + b + c + d \right)$$

4. THE GENERAL ASSIGNMENT PROBLEM

The general assignment problem can be represented mathematically as follows:

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = 1 \quad for \quad i = 1,2,3,\ldots\ldots,n$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad for \quad j = 1,2,3,\ldots\ldots,n$$

and $x_{ij} = 0\, or\, 1$

$$x_{ij} = \begin{cases} 1 & if\ the\ i^{th}\ person\ is\ assigned\ \ j^{th}\ job \\ 0 & otherwise \end{cases}$$

4.1 Fuzzy Assignment Problem

The general fuzzy assignment problem can be mathematically stated as follows:

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Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = 1 \quad for \quad i = 1,2,3,\ldots\ldots,n$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad for \quad j = 1,2,3,\ldots\ldots,n$$

and $x_{ij} = 0 \; or \; 1$

$$x_{ij} = \begin{cases} 1 & if\ the\ i^{th}\ person\ is\ assigned\ \ j^{th}\ job \\ 0 & otherwise \end{cases}$$

4.2. Hungarian Assignment Algorithm

Step 1: Check whether the problem is balanced or unbalanced.

(i) If the number of rows and columns are equal, the problem is said to be balanced.
(ii) Otherwise, unbalanced.

Step 2: If the problem is balanced, then move to Step 6. If the problem is unbalanced, add dummy variable whose entries are zeros and make it balanced, then move to Step 6.
Step 3: Use the following Hungarian method to solve the assignment problem
Hungarian Algorithm:

Step (i): Choose the smallest element in each row and subtract with each element in the corresponding row.
Step (ii): Choose the smallest element in each column and subtract with each element in the corresponding column.
Step (iii): Cover all the zero entries of the cost matrix by drawing minimum number of lines through appropriate rows and columns.
Step (iv): To check optimality,

- The problem is solved if the minimum number of lines covering zero elements in the cost matrix (square matrix) equal to *n* (the number of rows or columns)
- If the minimum number of lines is less than *n*, then proceed to Step (v).

Step (v): Determine the smallest element which is not covered by any line. Add that element with the intersected elements by lines and subtract with uncovered elements by lines. Go to Step (iii) until the optimality is reached.

Step 4: Optimum allocation is determined for the problem.

4.3 Numerical Example

A,B,C,D are the four sources of people in a corporation, while I,II,III,IV are the four destinations of jobs. The cost matrix[Cij] is presented, using trapezoidal fuzzy integers as its entries. The goal is to determine the best assignment such that the overall cost of the work allocated is as low as possible.

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Persons	Jobs			
	I	II	III	IV
A	(3,5,7,9)	(5,8,11,12)	(9,10,11,14)	(5,8,11,12)
B	(7,8,10,11)	(3,5,7,9)	(6,8,10,12)	(5,8,9,10)
C	(3,4,5,8)	(5,7,11,13)	(8,12,13,15)	(4,6,8,10)
D	(6,8,10,12)	(2,5,6,7)	(2,3,5,6)	(5,6,10,11)

To covert crisp value numbers, the suggested ranking algorithm for trapezoidal fuzzy numbers was used, $R(\tilde{A}_\alpha) = \frac{1}{2}(a+b+c+d)$

For ranking technique for trapezoidal fuzzy number is (3,5,7,9) to covert crisp value is 6

$$R(\tilde{A}_\alpha) = \frac{1}{2}(3+5+7+9) = 6$$

FUZZY ASSIGNMENT PROBLEM

Persons	Jobs			
	I	II	III	IV
A	6	9	11	9
B	9	6	9	8
C	5	9	12	7
D	9	5	4	8

Applying Hungarian method, we get the optimal solution.

A→I, B→II, C→IV, D→III

Minimum Total Cost = Rs.(6+6+4+7) = Rs 23.

5. CONCLUSION

In this research, a novel ranking methodology in trapezoidal fuzzy numbers is used to propose a straightforward way of tackling fuzzy assignment problems. We concluded that the findings were good, which was supported by numerical examples. Each ranking system reflects a unique perspective on ambiguous numbers. The majority of the time, selecting a method is a question of personal preference. We find that the Ranking function approach is a very effective way to solve fuzzy situations. The technique suggested in this research offers system managers and practitioners with useful information.

REFERENCES

[1]. Bortolan G, Degani R, A review of some methods for ranking fuzzy subsets, Fuzzy Set and Systems, 15 (1985), 1-19.

[2]. Chanas. S and Kuchta. D, A concept of the optimal solution of the transportation problem with fuzzy cost

coefficients, Fuzzy Sets and Systems, 82 (1996), 299-305.
[3]. Chanas. S and Kuchta. D, Fuzzy integer transportation problem, Fuzzy Sets and Systems, 98 (1998), 291-298.
[4]. Chanas. S, Kolodziejczyk. W and Machaj.A, A fuzzy approach to the transportation problem, Fuzzy Sets and Systems, 13 (1984), 211-221.
[5]. ChenC. B and Kleinc. M, A simple approach to ranking a group of aggregated fuzzy utilities, IEEETrans, Syst, Man, Cybern.B, SMC-27 (1997),26-35.
[6]. Chensj,H and Wangcl, Fuzzy multiple attribute decision making, Berlin,Springer,1992.
[7]. Chen. M. S, On A fuzzy assignment problem, Tamkang J, 22 (1985), 407-411.
[8]. Chen.S.H,Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, 17 (1985), 113-129.
[9]. Chi-Jen Lin and Ue-PyngWen, A labeling algorithm for the fuzzy assignment problem, Fuzzy Sets and Systems, 142 (2004),373-391.
[10]. Choobineh. F and Li.H, An index for ordering fuzzy numbers, Fuzzy Sets and Systems, 54 (1993),287-294.
[11]. Dubois. D and Fortemps. P, Computing improved optimal solutions to max-min flexible constraint satisfaction problems, European. j Operations Research,118(1999),95-126.
[12]. Fortemps. P and Roubens. M, Ranking and defuzzification methods based on are a compensation, Fuzzy Sets and Systems, 82 (1996), 319-330.
[13]. Jain R. Decision making in the presence of fuzzy variables, IEEE Transactions on Systems, Man and Cybernetics, 6, (1976), 698 -703.
[14]. Klir. G. J and Yuan. B, Fuzzy Sets and Fuzzy Logic, Theory and Applications, Prentice-Hall, International Inc,1995.
[15]. LeeL W, Chen SM. Fuzzy risk analysis based on fuzzy numbers with different shapes and different deviations, Experts Systems with Applications,34,(2008), 63-71.
[16]. M.OLhELigeartaigh, A fuzzy transportation algorithm, Fuzzy Sets and Systems, 8(1982), 235-243.
[17]. Nirmala G and Anju R, Cost Minimization assignment problem using fuzzy quantifier, International Journal of Computer Science and Information Technologies, 5(6), (2014), 7948-7950.
[18]. Phani Bushan Rao P and Ravishankar N, Ranking fuzzy numbers with a distance method using circum center of centroids and an Index of Modality, Advances in Fuzzy System,10,(2011),1155-1161.
[19]. Ramesh R and Kumaraghuru S, Optimal Performance Measures of Retrial Queueing Model with Fuzzy Parameters using Robust Ranking Technique, International Journal of Advanced and Innovative Research, 3,(4), (2014), 62-66.
[20]. Sakawa. M, Nishizaki. I and Uemura. Y, Interactive fuzzy programming for two level linear and linear fractional production and assignment problems, a case study, European J.OperationsResearch, 135 (2001), 142-157.
[21]. Tada. M and Ishii. H, An integer fuzzy transportation problem, Comput, Math, Appl, 31(1996),71-87.
[22]. Valvise, A new linear ordering of fuzzy numbers on sub sets of F(R), Fuzzy Optimization and Decision Making.8,(2009), 141-163.
[23]. Wang YJ, Lee HS, The revised method of ranking fuzzy numbers with an area bet ween the centroid and original points, Computer and Mathematics with Applications, 55 (2008), 2033-2042.
[24]. Wang. X, Fuzzy optimal assignment problem, Fuzzy Mathematics, 3 (1987), 101-108.
[25]. Yager. R. R, A procedure for ordering fuzzy sub sets of the unit interval, Information Sciences,24 (1981), 143-161.