

Beta Generalized Closed Sets in Topological Spaces

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ABSTRACT

The this paper, we introduce a new class of closed sets called Beta generalized closed set in topological spaces is to describe and analyze the basics of the set of closed sets, especially Beta generalized closed set in topological spaces. We also study their properties with few existing examples of closed sets, along with their characterizations.

Keywords – Closed set, Open set, β g- Closed set, β g-Open set, g-closed set, gs-closed set.

INTRODUCTION

One of the widely studied areas of Mathematics is Topology which was studied in detail by the famous Mathematical scientist Henri Poincare in the 19th century. Levine (1960) introduced and studied the concepts of semi-open sets in topological spaces and investigated generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semi pre-open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets. From the latest studies, we presume that several topologists are interested in the study of generalized types of closed sets. The study of generalized closed sets was initiated by Levine to further expand other properties of closed sets to a larger family of sets. The researchers in topology studied some of g-open sets and also made changes to the continuous functions that stand as the main concept of topology. Further studies

by some of the pioneers in the subject of generalized closed set opened new avenues and scope of further studies to the current researchers.

2.1 PRELIMINARIES

Definition 2.1 [2] Let (X, τ) be a topological space. The intersection of all closed sets containing A is called closure of A and is denoted by $\text{cl}(A)$. The union of all open sets contained in A is called interior of A and is denoted by $\text{int}(A)$.

Definition 2.2 [3] Let (X, τ) be a topological space. A subset A of (X, τ) is called

α -open set if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

Semi open set if $A \subseteq \text{cl}(\text{int}(A))$ and semi closed set if $\text{int}(\text{cl}(A)) \subseteq A$.

Pre open set if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.

β -open set if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and semi-pre closed set if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

Regular open set, if $A = \text{int}(\text{cl}(A))$ and regular closed set if $A = \text{cl}(\text{int}(A))$.

The complements of the above mentioned sets are known as semi-open set, α -open set, pre-open set, semi pre open set and regular open set respectively.

The intersection of all semi-closed (resp. α -closed set, pre-closed set, semi pre closed set and regular closed set) subsets of (X, τ) containing A is called the semi-closure (resp. α -closure, pre-closure, semi pre closure and regular closure) of A and is denoted by $scl(A)$ (resp. $\alpha cl(A)$, $pcl(A)$, $spcl(A)$ and $rcl(A)$). A subset A of (X, τ) is called clopen if it is both open set and closed set in (X, τ) .

Definition 2.3 [2]: The union of all β -open sets of X contained in A is called the beta-interior of A and it is denoted by $\beta \text{ int}(A)$ and the intersection of all β -closed sets of X containing A is called the beta-closure of A and it is denoted by $\beta cl(A)$.

Definition 2.4 [6] Let (X, τ) be a topological space. A subset A of X is called as Generalized closed set (briefly g -closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X .

Definition 2.5 [7] Let (X, τ) be a topological space. A subset A of X is called as Semi-generalized closed set (briefly sg -closed set) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open set in X .

Definition 2.6 [8] Let (X, τ) be a topological space. A subset A of X is called as Generalized semi closed set (briefly gs -closed set) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X .

Definition 2.7 [6] Let (X, τ) be a topological space. A subset A of X is said to be Generalized α -closed set (briefly $g\alpha$ -closed set) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open set in X .

Definition 2.8 [6] Let (X, τ) be a topological space. A subset A of X is said to be α -Generalized closed set (briefly αg -closed set) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X .

Definition 2.9 [7] Let (X, τ) be a topological space. A subset A of X is called as Generalized semi-pre closed set (briefly gsp -closed set) if $Spcl(A) \subseteq U$ Whenever $A \subseteq U$ and U is open set in X .

Definition 2.10 [8] Let (X, τ) be a topological space. A subset A of X is called as Regular generalized closed set (briefly rg -closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open set in X .

Definition 2.11 [9] Let (X, τ) be a topological space. A subset A of X is called as Generalized pre closed set (briefly gp -closed set) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X .

Definition 2.12 [10] Let (X, τ) be a topological space. A subset A of X is said to be Generalized pre regular closed set (briefly gpr -closed set) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open set in X .

Definition 2.13 [11] Let (X, τ) be a

topological space. A subset A of X is said to be Weakly closed (briefly w -closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .

Definition 2.14 [6] Let (X, τ) be a topological space. A subset A of X is said to be Weakly generalized closed set (briefly wg -closed set) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open set in X .

Definition 2.15 [1-7] Let (X, τ) be a topological space. A subset A of X is said to be Strongly generalized closed set (briefly g^* -closed set) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open set in X .

Definition 2.16 [12-14] Let (X, τ) be a topological space. A subset A of X is said to be generalized star pre-closed set (briefly g^*p -closed set) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .

Definition 2.17 [6] Let (X, τ) be a topological space. A subset A of X is said to be Regular weakly generalized closed set (briefly rwg -closed set) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open set in X .

Definition 2.18 [6] Let (X, τ) be a topological space. A subset A of X is said to be A subset A of a space X is called regular generalized weakly closed (briefly rgw -closed) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X .

The complements of the above mentioned closed set sets are their respective open set sets.

3. BETA GENERALIZED (βg) CLOSED SETS IN TOPOLOGICAL SPACES

In this section, we define βg - closed sets, βg - neighbourhoods, βg - closure and βg -interior operators and investigate some of their basic properties.

Definition 3.1 A subset A of a topological space (X, τ) is called beta generalized closed (briefly, βg -closed) set if $\beta \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open set in X .

Theorem 3.2 Let A be βg -closed set in (X, τ) . Then $\beta \text{cl}(A) - A$ does not contain any non-empty g -closed set.

Proof. Suppose that F is a g -closed subset of $\beta \text{cl}(A) - A$. Since $X - F$ is an g -open set with $A \subseteq X - F$ and A is βg -closed set, $\beta \text{cl}(A) \subseteq X - F$. Thus implies $F \subseteq (X - \beta \text{cl}(A)) \cap (\beta \text{cl}(A) - A) \subseteq (X - \beta \text{cl}(A)) \cap \beta \text{cl}(A) = \emptyset$. Therefore $F = \emptyset$.

Corollary 3.3 Let A be a βg -closed set in (X, τ) . Then $\beta \text{cl}(A) \setminus A$ does not contain any non-empty closed set.

Theorem 3.4 (i) Every βg -closed set is β -closed set, and thus gsp -closed set and pre-semiclosed set. (ii) Every closed (resp. α -closed, regular closed) set is βg -closed set.

Proof. Follows from the definitions
The following example shows that these implications are not reversible.

Example 3.5 (i). Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$ be topology on X . Then $\{a, c\}$ is a β -closed set but not βg -closed set in X .

(ii). Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a\}, \{a, c\}, X\}$. Then $\{b\}$ is a gsp-closed set but not βg -closed set in X .

(iii). Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$. Then the set $\{a, b, d\}$ is a pre-semi-closed set but not βg -closed set in X .

(iv). Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}\}$ be a topology on X . Then $\{b\}$ is a βg -closed set but not a closed set in X .

(v). Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a, b\}\}$ be a topology on X . Then the set $\{a, c\}$ is βg -closed set but not α -closed set in X .

(vi). Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a, b\}, \{a, b, c\}\}$ be a topology on X . Then $\{b, c\}$ is a βg -closed set but not regular-closed set in X .

(vii). Every gp^* -closed set is βg -closed set and thus every αg^* -closed set is βg -closed set.

The converse of the above theorem is not true as shown in the following examples.

(viii). Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{a, b\}, X\}$. Then $\{b\}$ is a βg -closed set but not gp^* -closed set in X .

(ix). Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, \{a, b\}, X\}$. Now $\alpha g^*c(X) = \{X, \emptyset, \{c\}\}$ and $\beta gc(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. Then $\{a\}$ is a βg -closed set but not αg^* -closed set in X .

Remark 3.6 The concept of βg -closedness is independent of semi-closedness and rwg -closedness as seen from the following examples.

Example 3.7 (i). Let $X = \{a, b, c, d\}$ be given the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then the set $\{b, c\}$ is semi-closed set but not βg -closed set

in X .

(ii). Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a, b\}\}$ be a topology on X . Then the set $\{b, d\}$ is βg -closed set but not semi-closed set in X .

(iii). Let $X = \{a, b, c, d\}$ be given the topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. The set $\{a, b, d\}$ is rwg -closed set but not βg -closed set. Also, the sets $\{a\}$ and $\{b\}$ are βg -closed but not rwg -closed set.

Theorem 3.8 Every pre-closed set is βwg -closed and thus every $g^\#$ -closed set, $(gsp)^*$ -closed set and $g^\#s$ -closed set is βg -closed.

Proof. Follows from the definitions.

The converse of the above theorem is need not be true as seen from the following examples.

Example 3.9 (i) Let $X = \{a, b, c, d\}$ be given the topology, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then the set $\{a, c\}$ is βg -closed set but not pre closed set in X .

(ii). Let $X = \{a, b, c, d\}$ be given the topology, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $\{a\}$ is a βg -closed set but not $g^\#$ -closed set in X .

(iii). Let $X = \{a, b, c\}$ with topology, $\tau = \{X, \emptyset, \{a\}, \{a, c\}\}$. Then $\{c\}$ is a βg -closed set but not $(gsp)^*$ -closed set in X .

(iv). Let $X = \{a, b, c, d\}$ be given the topology, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $\{a, b, d\}$ is a βg -closed set but not $g^\#s$ -closed set in X .

Remark 3.10 The following examples show that βg -closed sets are independent of g -closed sets, g^* -closed sets, sg -closed sets, gs -closed sets, αg -closed sets.

Example 3.11 (i). Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a, b\}\}$ be a topology on X .

Then the sets $\{a\}$ and $\{b\}$ are βg -closed set but not g -closed set, g^* -closed set, sg -closed set, gs -closed set in X .

(ii). Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. The set $\{a, b, d\}$ is g -closed set, g^* -closed set, sg -closed set, gs -closed set, αg -closed set but not βg -closed set in X .

Theorem 3.12 Let A be a βg -closed set in (X, τ) . Then A is β -closed set iff $\beta cl(A) - A$ is g -closed set.

Proof. Suppose A is βg -closed set in X . Then $\beta cl(A) = A$ and so $\beta cl(A) - A = \emptyset$ which is g -closed set in X . Conversely, Suppose $\beta cl(A) - A$ is g -closed set in X . Since A is βg -closed set, $\beta cl(A) - A$ does not contain any non-empty g -closed set in X . That is $\beta cl(A) - A = \emptyset$. Hence A is β -closed.

Theorem 3.13 If A is βg -closed in (X, τ) and if $A \subseteq B \subseteq \beta cl(A)$ then B is also βg -closed set in (X, τ) .

Proof. Let U be an g -open set of (X, τ) such that $B \subseteq U$. Since $A \subseteq U$ and A is βg -closed set, $\beta cl(A) \subseteq U$. Since $B \subseteq \beta cl(A)$, we have $\beta cl(B) \subseteq \beta cl(\beta cl(A)) = \beta cl(A)$. Thus $\beta cl(B) \subseteq U$. Hence B is a βg -closed set of (X, τ) .

Remark 3.14 Union of two βg -closed sets need not be βg -closed set.

Example 3.15 Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a, b\}\}$ be a topology on X . Now $\beta g(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, b\}, \{b, c, d\}\}$. Then the sets $\{a\}$ and $\{b\}$ are βg -closed set but their union $\{a, b\}$ is not a βg -closed set in X .

Theorem 3.16 The intersection of two βg -closed subsets of X is also βg -closed set.

Proof. Let A and B be any two βg -closed subsets of X then $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and $B \subseteq U$, U is an g -open. Let U be an g -open set in X such that $A \cap B \subseteq U$. Then we have $\beta cl(A \cap B) \subseteq \beta cl(A) \cap \beta cl(B) \subseteq U$, U is an g -open set in X . Hence $A \cap B$ is a βg -closed set of X .

Theorem 3.17 For an element $x \in X$, then the set $X - \{x\}$ is a βg -closed set (or) g -open set.

Proof. Let $x \in X$. Suppose that $X - \{x\}$ not g -open set. Then X is the only g -open set containing $X - \{x\}$. This implies $\beta cl(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is βg -closed set in X .

Theorem 3.18 If A is both g -open set and βg -closed set in X then A is β -closed set.

Proof. Suppose A is g -open set and βg -closed set in X . Since $A \subseteq A$, $\beta cl(A) \subseteq A$. But always $A \subseteq \beta cl(A)$. Therefore $A = \beta cl(A)$. Hence A is β -closed set.

Corollary 3.19 Let A be a g -open set and βg -closed set in X . Suppose that F is β -closed set in X . Then $A \subseteq F$ is βg -closed set in X .

Proof. By theorem 3.18, A is β -closed set. So $A \subseteq F$ is β -closed set and hence $A \subseteq F$ is a βg -closed set in X .

Theorem 3.20 If A is both open set and g -closed set in X , then A is βg -closed set in X .

Proof. Let $A \subseteq U$ and U be g -open set in X . Now $A \subseteq A$. By hypothesis $\alpha cl(A) \subseteq A$. Since every closed set is β -closed set, $\beta cl(A) \subseteq cl(A)$. Thus $\beta cl(A) \subseteq A \subseteq U$. Hence A is βg -closed set in X .

Definition 3.21 Let $B \subseteq A \subseteq X$. Then we say that B is βg -closed set relative to A if $\beta cl_A(B) \subseteq U$ where $B \subseteq U$ and U is g -open set in A .

Theorem 3.22 Let $B \subseteq A \subseteq X$ and suppose that B is βg -closed set in X . Then B is βg -closed set relative to A .

Proof. Given that $B \subseteq A \subseteq X$ and B is βg -closed set in X . Let us assume that $B \subseteq A \subseteq V$, where V is g -open set in X . Since B is βg -closed set, $B \subseteq V$ implies $\beta cl(B) \subseteq V$. It follows that $\beta cl_A(B) = \beta cl(B) \cap A \subseteq V \cap A$. Therefore B is βg -closed set relative to A .

Theorem 3.23 Let A and B be βg -closed sets such that $D(A) \subseteq D(A)_p(A)$ and $D(B) \subseteq D(A)_p(B)$ then $A \cup B$ is βg -closed set.

Proof. Let U be an g -open set such that $A \cup B \subseteq U$. Then $\beta cl(A) \subseteq U$ and $\beta cl(B) \subseteq U$. However, for any set E , $D(A)_p(E) \subseteq D(E)$. Therefore $cl(A) = \beta cl(A)$ and $cl(B) = \beta cl(B)$ and this shows $cl(A \cup B) = cl(A) \cup cl(B) = \beta cl(A) \cup \beta cl(B)$. That is $\beta cl(A \cup B) \subseteq U$. Hence $A \cup B$ is βg -closed set.

Definition 3.24 A subset A of a topological space (X, τ) is called βg -open set iff $X - A$ is βg -closed set in X . We denote the family of all βg -open sets in X by $\beta gO(X)$.

Theorem 3.25 If $\beta int(A) \subseteq B \subseteq A$ and if A is βg -open in X then B is βg -open in X .

Proof. Suppose that $\beta int(A) \subseteq B \subseteq A$ and A is βg -open in X . Then $X - A \subseteq X - B \subseteq \beta cl(X - A)$. Since $X - A$ is βg -closed set in X , we have $X - B$ is βg -closed set in X .

Hence B is βg -open set in X .

Remark 3.26 The intersection of two βg -open sets in X is generally not βg -open set in X .

Example 3.27 Let $X = \{a, b, c, d\}$ with $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then the sets $\{a, d\}$ and $\{b, d\}$ are βg -open sets but their intersection $\{d\}$ is not βg -open set.

Theorem 3.28 A subset A is βg -open if and only if $F \subseteq \beta int(A)$ whenever F is g -closed set and $F \subseteq A$.

Proof. Necessity Let A be an open set and suppose $F \subseteq A$ where F is g -closed set. By definition, $X - A$ is contained in βg -closed set. Also, $X - A$ is contained in the g -open set $X - F$. This implies $\beta cl(X - A) \subseteq X - F$. Now $\beta cl(X - F) = X - \beta int(A)$. Hence $X - \beta int(A) \subseteq X - F$. that is $F \subseteq \beta int(A)$.

Sufficiency: If F is g -closed set with $F \subseteq \beta int(A)$ where $F \subseteq A$, it follows that $X - A \subseteq X - F$ and $\beta int(A) \subseteq X - F$. That is $\beta cl(X - A) \subseteq X - F$. Hence $X - A$ is βg -closed set and A becomes βg -open set.

Definition 3.29 Let x be a point in a topological spaces and let $x \in X$. A subset N of X is said to be a βg -neighbourhood (briefly, βg -nbd) of a point x iff there exists a βg -open set G such that $x \in G \subseteq N$.

Definition 3.30 A subset N of Space X is called a βg -nbd. of a set $A \subseteq X$ iff there exists an βg -open set G such that $A \subseteq G \subseteq N$.

Definition 3.31 Let A be a subset of a topological space (X, τ) . A point $x \in X$ is called a βg -interior point of A if there exists a βg -open set U such that $x \in U \subseteq A$. The set of all βg -interior points of A is called the βg -interior of A and is denoted by $\beta gInt(A)$.

Definition 3.32 Let A be a subset of a topological space X . A point $x \in X$ is said to be βg -limit point of A if every βg -open set containing x contains a point of A different from x .

The set of all βg -limit points of A is called the βg -derived set of A and is denoted by $\beta gD(A)$.

Remark 3.33 For a subset A of X , a point $x \in X$ is not a βg -limit point of A if and only if there exists a βg -open set H in X such that $x \in H$ and $H \cap (A - \{x\}) = \emptyset$ or, equivalently, $x \in H$ and $H \cap A = \emptyset$ or $H \cap A = \{x\}$.

Theorem 3.34 Every nbd N of $x \in X$ is a βg -nbd of X .

Proof. Let N be a nbd of point $x \in X$. To prove that N is a βg -nbd of x . By definition of nbd, there exists an open set G such that $x \in G \subseteq N$. Hence N is a βg -nbd of x .

In general, a βg -nbd of $x \in X$ need not be a nbd of x in X as seen from the following example.

Example 3.35 Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then $\beta gO(X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. The set $\{a, b, d\}$ is βg -nbd of the point d , since the βg -open sets $\{a, d\}$ is such that $d \in \{a, d\} \subseteq \{a, b, d\}$. However, the set $\{a, b, d\}$ is not a nbd of the point d , since no open set G exists such that $d \in G \subseteq \{a, b, d\}$.

Remark 3.36 The βg -nbd N of $x \in X$ need not be a βg -open set in X .

Example 3.37 Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{d\}, \{c, d\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}\}$ be a topology on X . Then $\beta gO(X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$.

The set

$\{a, d\}$ is βg -nbd of point a , since $a \in \{a\} \subseteq \{a, d\}$. However, the set $\{a, d\}$ is not a βg -open in X .

Theorem 3.38 If a subset N of a space X is βg -open, and then N is βg -nbd of each of its points.

Proof. Suppose N is βg -open. Let $x \in N$. We claim that N is βg -nbd of x . For N is a βg -open set such that $x \in N \subseteq N$. Since x is an arbitrary point of N , it follows that N is a βg -nbd of each of its points.

Theorem 3.39 Let X be a topological space. If F is βg -closed subset of X and $x \in F^c$. Then there exists a βg -nbd N of x such that $N \cap F = \emptyset$.

Proof. Let F be βg -closed subset of X and $x \in F^c$. Then F^c is a βg -open set in X .

By theorem 3.51, F^c contains a βg -nbd of each of its points. Hence there exists a βg -nbd N of x such that $N \subseteq F^c$. (i.e.) $N \cap F = \emptyset$.

Definition 3.40 For a subset A of X , the intersection of all βg -closed sets containing A is called the βg -closure of A and is denoted by $\beta gcl(A)$. That is, $\beta gcl(A) = \bigcap \{F : F \text{ is } \beta g\text{-closed in } X, A \subseteq F\}$.

Theorem 3.41 If A is a βg -closed subset of X then $\beta gcl(A) = A$.

Theorem 3.42 For a subset A of X and $x \in X$, $\beta gcl(A)$ contains x iff $V \cap A \neq \emptyset$ for every βg -open set V containing x .

Proof. Let $A \subseteq X$ and $x \in X$, where X is a topological space. Suppose that there exists a βg -open set V containing x such that $V \cap A = \emptyset$. Since $A \subseteq X - V$, $\beta gcl(A) \subseteq X - V$ and then $x \notin \beta gcl(A)$ which is a contradiction. Conversely, assume that $x \notin \beta gcl(A)$. Then there exists a βg -closed set F containing A such that $x \notin F$. Since $x \in X - F$ and $X - F$ is βg -open, $(X - F) \cap A = \emptyset$ which is a contradiction. Hence $x \in$

$\beta gcl(A)$ iff $\bigcap V \neq \emptyset$ for every βg - open set V containing x .

Theorem 3.43 Let A and B be subsets of a space X . Then

- (i). $\beta gcl(\emptyset) = \emptyset$ and $\beta gcl(X) = X$.
- (ii). $A \subseteq \beta gcl(A)$.
- (iii). $A \subseteq B \Rightarrow \beta gcl(A) \subseteq \beta gcl(B)$.
- (iv). $\beta gcl(\beta gcl(A)) = \beta gcl(A)$.
- (v). $\beta gcl(A \subseteq B) = \beta gcl(A) \cup \beta gcl(B)$.
- (vi). $\beta gcl(A \cap B) \subseteq \beta gcl(A) \cap \beta gcl(B)$.

Proof follows from definitions.

Theorem 3.44 Let A and B subsets of X . If A is βwg -closed, then $\beta wgCl(A \cap B) \subseteq A \cap \beta wgCl(B)$.

Proof: Let A be a βwg -closed set, then $\beta wgCl(A) = A$ and so $\beta wgCl(A \cap B) \subseteq \beta wgCl(A) \cap \beta wgCl(B) = A \cap \beta wgCl(B)$ which is the desired result. Now we define and study the followings

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