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PRIME LABELINGS OF MIDDLE GRAPH OF CYCLE C_n

[1]A. Dhandapani, [2]DR. K. Balamurugan

[1]Assistant Professor PG and Research Department of MathematicsThiru A. Govindasamy Government Arts College, Tindivanam,Tamilnadu,India. Email-madhandapani74@gmail.com
[2]Associate Professor, PG and Research Department of Mathematics Government Arts College, Thiruvannamalai, Tamilnadu, India. Email-dr.kb1963@gmail.com

ABSTRACT

A graph G = (V, E) with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceed n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling for some classes of graph. In particular, we discuss on prime labeling of middle graph of cycle C_n, when n is even (or) odd.

Keywords: Graph labeling, Prime labeling , Cycle graph and Middle graph of a graph G.

1. INTRODUCTION

In labeling of graphs, we consider only simple, finite , undirected, connected and non trivial graph G = (V,E) with the vertex set V and the edge set E. The number of elements of V, denoted as $|V|$ is called the order of the graph while the number of elements of E, denoted as $|E|$ is called the size of the graph G. M(G) denotes the middle graph of the graph G.

The notion of prime labeling originated with Entringer and was introduced in a paper by Tout, Dabbouchy and Howalla [2]. Entringer conjectured that all trees have a prime labeling. Haxell, Pikhuriko and Taraz [8] proved that all large trees are prime graph. Many researchers have studied prime graphs, for example in Fu. H. C and Huany K.C [4] has proved that the path P_n on n vertices is a prime graph. In [6] Ganesan. V et. al proved that Middle graph of the path P_n admits prime labeling. In [7] S. Meena and Vaithelingam have proved that the prime labeling for some fan related graphs. For latest survey of graph labeling, we refer to [5] for various graph theoretic notations and terminology we follow Bondy J.A and U.S.R. Murty [1].

Now, we will give brief summary of definitions and other information which are useful for the present task.

2. PRELIMINARY

Definition 2.1

The graph labeling is an assignment of numbers to the vertices (or) edges (or) both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) or both then the labeling is called a vertex labeling (edge) labeling.

Definition 2.2

Let G = (V,E) be a graph with p vertices and q edges. A bijection f : V(G) → { 1,2,3,…$|V|$ } is called a prime labeling if for each edge e = uv, gcd (f(u), f(v)) = 1. A graph which admits prime labeling is called a prime graph.

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Definition 2.3

For $n \geq 3$, an n – cycle (or simply cycle C_n) with n vertices is a connected graph consisting of all vertices with degree two. A cycle graph C_n with n vertices has n edges.

Definition 2.4

The middle graph M(G) of a graph G is the graph whose vertex set is V(G) ∪ E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G (or) one is a vertex of G and other is an edge incident on it.

3. MAIN RESULTS

Algorithm

Prime labeling of Middle graph of cycle C_n

Step 1:

Let C_n be the cycle with n vertices and n edges and $V(C_n) = \{v_1, v_2, \dots, v_n\}$ and $E(C_n) = \{e_1, e_2, \dots, e_n\}$

Let G = C_n and M(G) be the Middle graph of C_n

Clearly V(M(G)) = V(G) ∪ E(G). Obviously

$\therefore |V(M(G))| = |E(G) \cup V(G)|$ = n + n = 2n

Step 2:

Define a labeling f : V(M(G)) → $\{1,2,3, \dots, 2n\}$ as follows

$$f(e_i) = 2i - 1 \quad \text{for } i = 1,2,3,\dots,n$$

$$f(v_i) = 2i \quad \text{for } i = 1,2,3,\dots,n$$

Let G^* be the labeled graph of M(G) labeled as defined above.

Step 3 :

Check the relative prime of adjacent vertices of G^*

In M(G), there are three types of edges (i) $v_i e_i$ (ii) $e_i v_{i+1}$ (iii) $e_i e_{i+1}$ We need to check the relative prime of edges of the type $e_i v_{i+1}$ only.

Step 4 :

In this step, we check the relative prime of the pair of vertices (e_i, v_{i+1}) in M(G).

If gcd(f(e_i),f(v_{i+1})) $= 1$ for i = 1,2,3,…,n-1 then the graph G* admits prime labeling

If gcd(f(e_i),f(v_{i+1}))$\neq 1$ for i = 1,2,3,…,n-1 then we have to proceed the step5.

Step 5 :

Suppose gcd(f(e_i),f(v_{i+1}))$\neq 1$ for some i then select all those pairs of e_i and v_{i+1} for which f(e_i) and f(v_{i+1}) are not relatively prime and encircle each pairs within a circle. Now, interchange the label of v_{i+1} and v_{i+2} (where v_{i+1} is the encircled vertex and v_{i+2} is not). The

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procedure is repeated until all encircled vertex v_{i+1} are exhausted. Now, the newly labeled graph admits prime labeling. Hence, M(G) is a prime graph.

Illustration: 1
Prime labeling of Middle graph of Cycle graphC_5

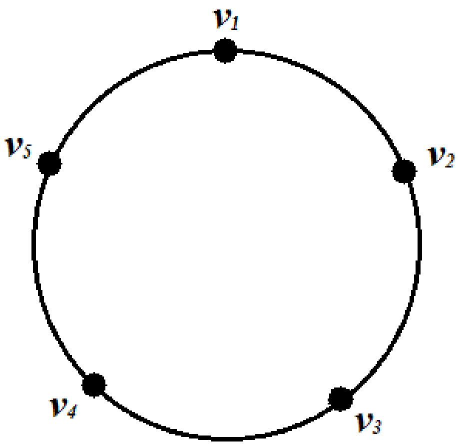

Figure 3.1 A Cycle c_5

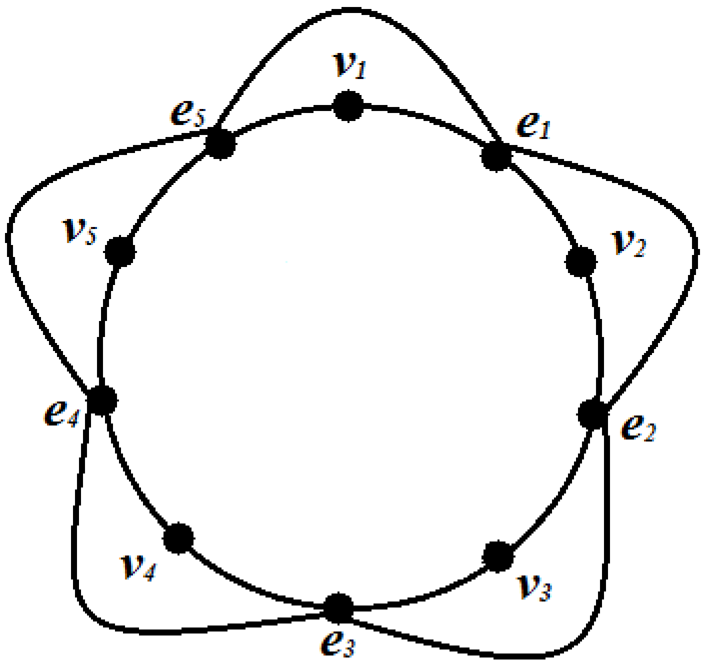

Figure 3.2 Middle graph of Cycle c_5

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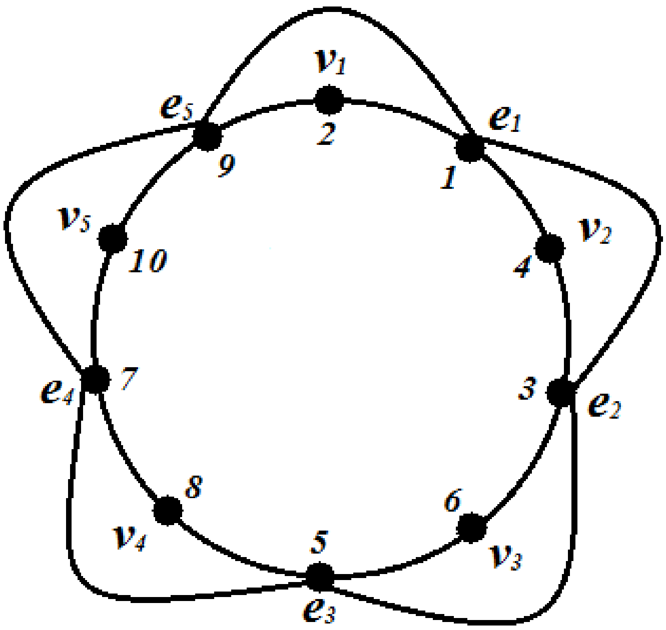

Figure 3.3 $G^* = \mathrm{M}(c_5)$, **labeled graph of** $\mathrm{M}(c_5)$

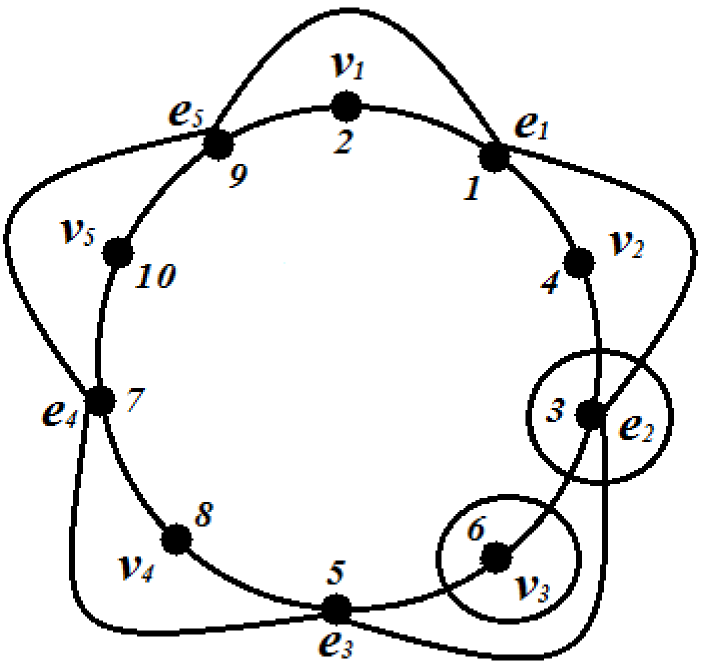

Figure 3.4 Checking the relative prime of (e_i, v_{i+1}) **in the labeled graph G**

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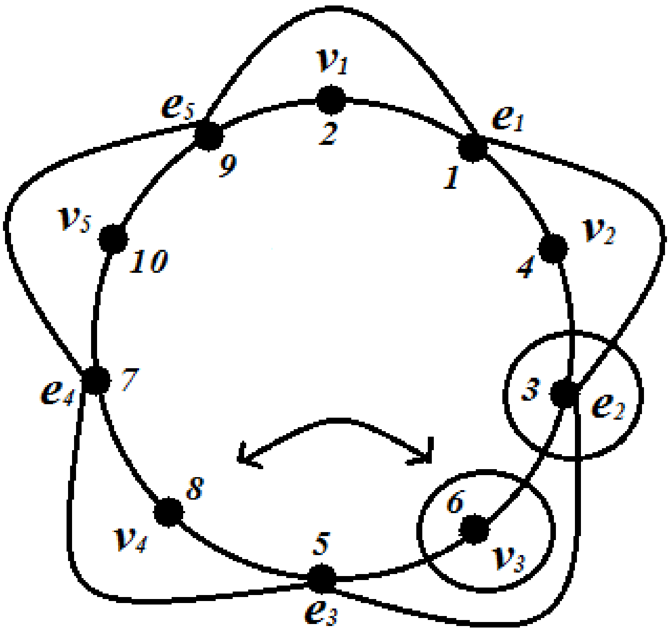

Figure 3.5 Labeling of M(G), G^* = M(c_5), by using

f(e_i) = 2i – 1 for i = 1,2,3,…,n

f(v_i) = 2i for i = 1,2,3,…,n

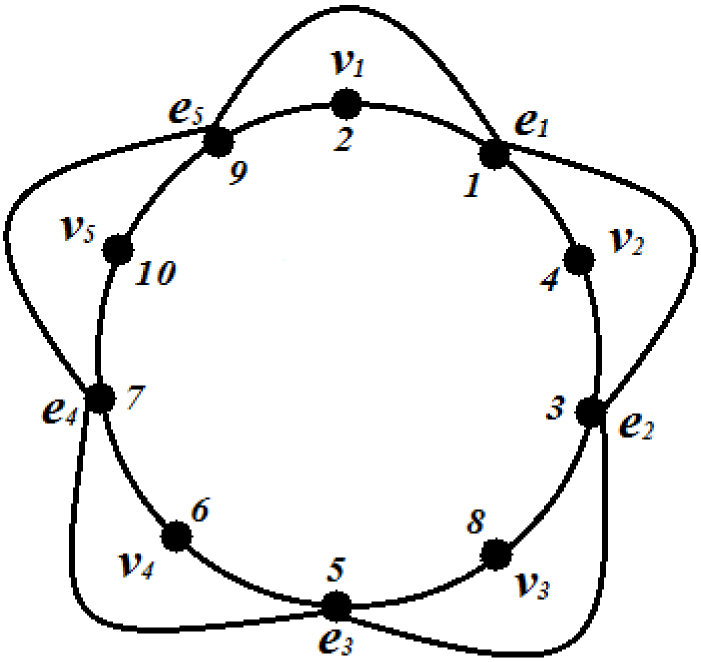

Figure 3.6 Interchanging the labels of v_{i+1} and v_{i+2}

Illustration: 2

Prime labeling of Middle graph of Cycle graphC_6

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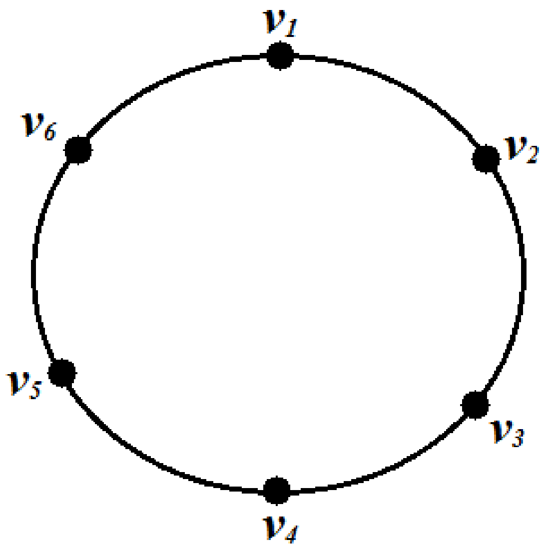

Fig 3.7 A Cycle c_6

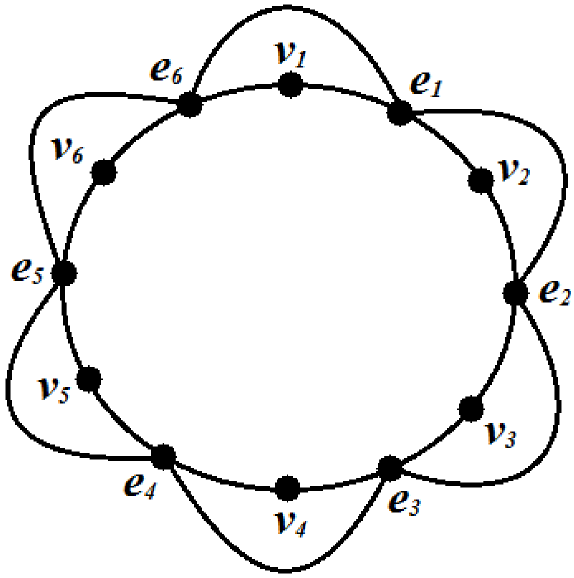

Fig 3.8 Middle graph of of Cycle c_6

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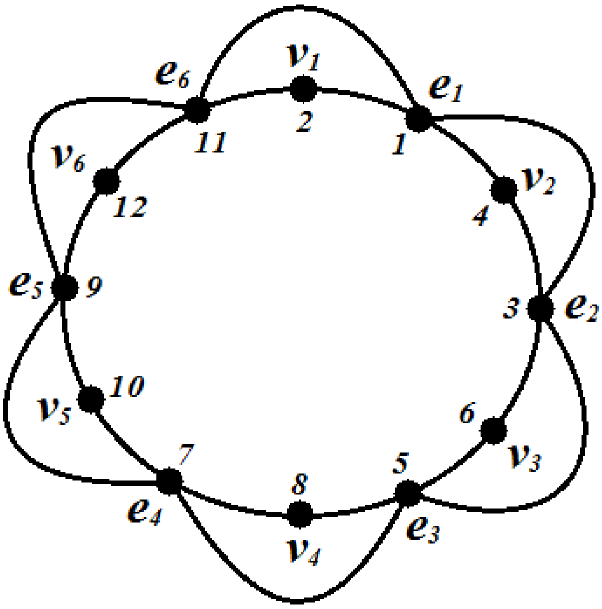

Fig 3.9 $G^* = \mathrm{M}(c_6)$ **labeled graph of** $\mathrm{M}(c_6)$

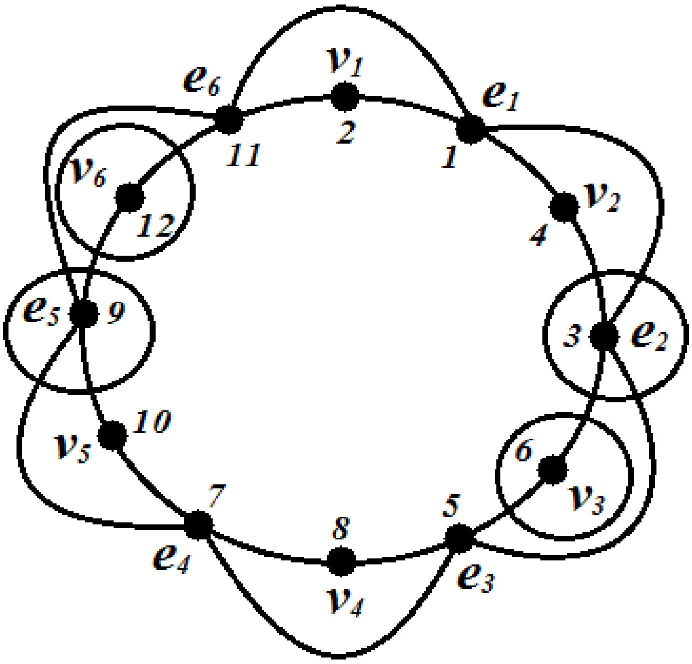

Fig 3.10 Checking the relative prime of (e_i, v_{i+1}) **in the labeled graph G**

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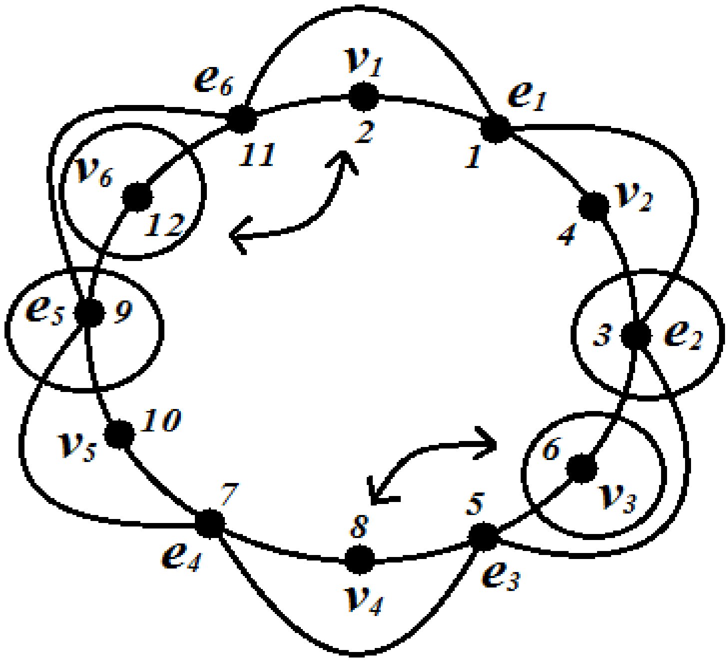

Fig 3.11 Labeling of M(G), G^* = M(c_6), by using
f(e_i) = 2i – 1 for i = 1,2,3,…,n
f(v_i) = 2i for i = 1,2,3,…,n

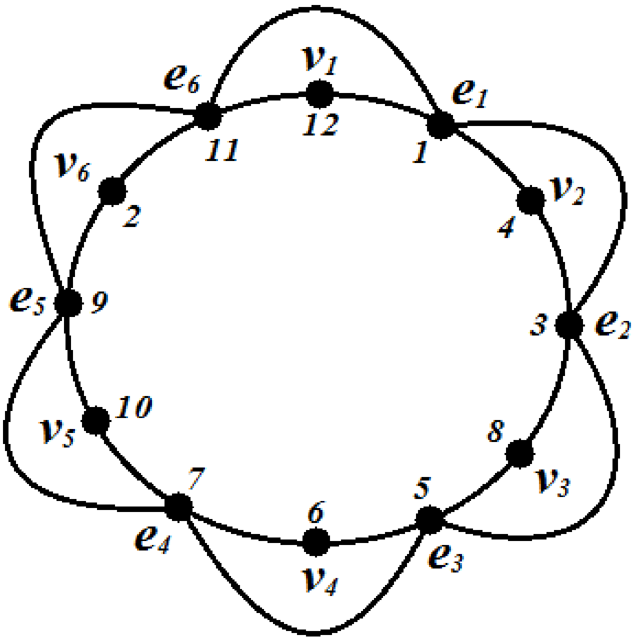

Fig 3.12 Interchanging the labels of v_{i+1} and v_{i+2}

CONCLUSION

We have presented an algorithm for prime labeling to some classes of graph such as middle graph of C_n and illustrate with two examples for the cases n is odd and even separately. Really, it will motivate the researcher to investigate the prime labeling of middle graph other families like star,sunlet and wheal etc., are prime graph.

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