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PRIME LABELINGS OF SPLIT GRAPH OF SUNLET GRAPH S_n

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ABSTRACT

A graph G = (V, E) with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceed n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling for some classes of graph. In particular, we discuss on prime labeling of The Sunlet graph S_n. When n is even (or) odd.

Keywords: Graph labeling, Prime labeling , Cycle graph and Split graph of a graph G.

1. INTRODUCTION

In labeling of graphs, we consider only simple, finite , undirected, connected and non trivial graph G = (V,E) with the vertex set V and the edge set E. The number of elements of V, denoted as $|V|$ is called the order of the graph while the number of elements of E, denoted as $|E|$ is called the size of the graph G. M(G) denotes the middle graph of the cycle graph C_n.

The notion of prime labeling originated with Entringer and was introduced in a paper by Tout, Dabbouchy and Howalla [2]. Entringer conjectured that all trees have a prime labeling. Haxell, Pikhuriko and Taraz [8] proved that all large trees are prime graph. Many researchers have studied prime graphs, for example in Fu. H. C and Huany K.C [4] has proved that the path P_n on n vertices is a prime graph. In [6] Ganesan. V et. al proved that Middle graph of the path P_n admits prime labeling. In [7] S. Meena and Vaithelingam have proved that the prime labeling for some fan related graphs. For latest survey of graph labeling, we refer to [5] (Gallian J.A 2017) for various graph theoretic notations and terminology we follow Bondy J.A and U.S.R. Murty [1].

Now, we will give brief summary of definitions and other information which are useful for the present task.

2. PRELIMINARY

Definition 2.1

The graph labeling is an assignment of numbers to the vertices (or) edges (or) both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) or both then the labeling is called a vertex labeling (edge) labeling.

Definition 2.2

Let G = (V,E) be a graph with p vertices and q edges. A bijection f : V(G) $\rightarrow$ { 1,2,3,…$|V|$ } is called a prime labeling if for each edge e = uv, gcd (f(u), f(v)) = 1. A graph which admits prime labeling is called a prime graph.

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Definition 2.3

The Sunlet graph S_n is the graph obtained from a cycle C_n attaching a pendant edge at each vertex of a cycle C_n. A Sunlet graph S_n has 2n vertices and 2n edges.

Definition 2.4

For a graph G the split graph which is denoted as spl(G) is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

3. MAIN RESULTS

Algorithm

Prime labeling of split graph of Sunlet graph S_n , when n is odd

Step 1:

Let S_n be the Sunlet graph with the vertex set V(S_n) = { $u_1, u, \dots, u_n ; v_1, v_2, \dots, v_n$) and let $u'_1, u'_2, \dots, u'_n ; v'_1, v'_2, \dots, v'_n$ be the new vertices associated with each of $u_1, u, \dots, u_n$ and $v_1, v_2, \dots, v_n$ respectively.

Let G = Spl(S_n) be the split graph of Sunlet graph S_n

Step 2:

Obviously, $|V(G)|$ = 4n

Define a function f : V(G) → { 1,2,3, …,4n} as follows

f (u_i) = 4i – 3 for 1≤ i ≤ n

f (v_i) = 4i – 2 for 1≤ i ≤ n

f (u'_i) = 4i + 1 for 1≤ i ≤ n

f(v'_i) = 4i for 1≤ i ≤ n

Step 3 :

Checking the relative prime of adjacent vertices

There are six types of edges

i. $u_i u_{i+1}$ for 1≤ i ≤ n

ii. $u_i v_i$ for 1≤ i ≤ n

iii. $u_i v'_i$ for 1≤ i ≤ n

iv. $u'_i v_i$ for 1≤ i ≤ n

v. $u'_i u_{i+1}$ for 1≤ i ≤ n and $u_1 u'_n$

vi. $u'_i u_{i-1}$ for 1≤ i ≤ n and $u'_1 u_n$

vii. We need to check only the relative prime of edges of type $u_i u'_{i+1}$

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Step 4 :.

In this step, we check the relative prime of the pair of vertices $(v_i v'_{i+1})$ of the labeled graph G obtained in step 2

If gcd(f(v_i),f(v'_{i+1})) $= 1$ for i = 1,2,3,…,n-1 then the graph G admits prime labeling

If gcd(f(v_i),f(v'_{i+1})) $\neq 1$ for i = 1,2,3,…,n-1 then go to step 5.

Step 5 :

Suppose gcd(f(v_i),f(v'_{i+1})) $\neq 1$ for some i then select all those pairs of v_i and v'_{i+1} for which f(v_i) and f(v'_{i+1}) are not relatively prime and encircle each pairs within a circle. Now, interchange the label of v'_i and v'_{i+1} (where v'_{i+1} is the encircled vertex and v'_i is not). The procedure is repeated until all encircled vertex v'_i are exhausted. Now, the newly labeled graph admit prime labeling.

Hence M(W_n) is a prime graph.

Illustration: 1
Prime labeling of Split graph of Sunlet graph S_5

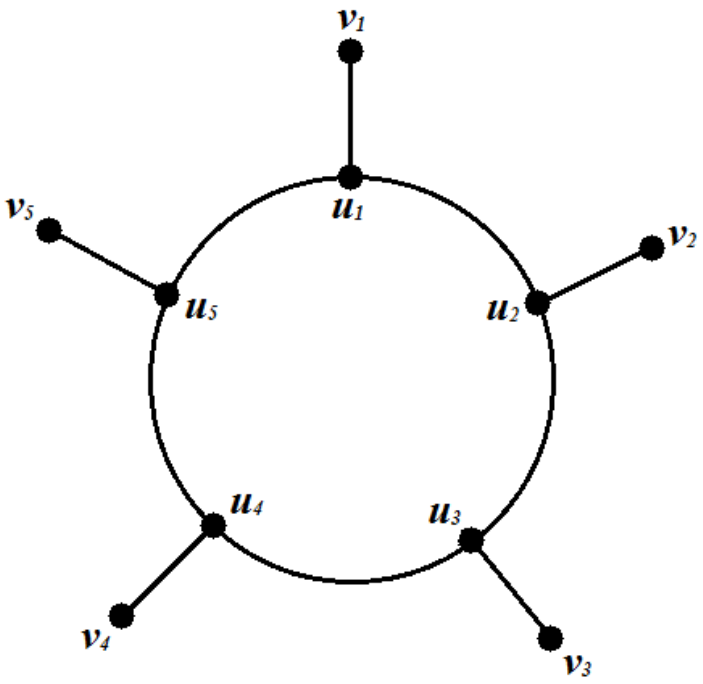

Figure 1. A Sunlet S_5

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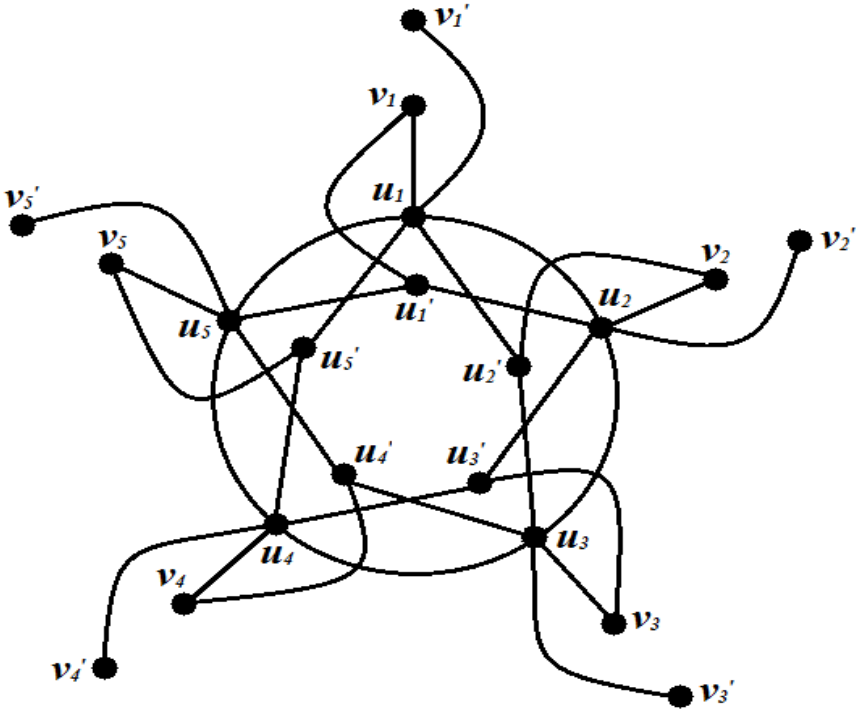

Figure 2. Split Graph of Sunlet S_5 , G = Spl(S_5)

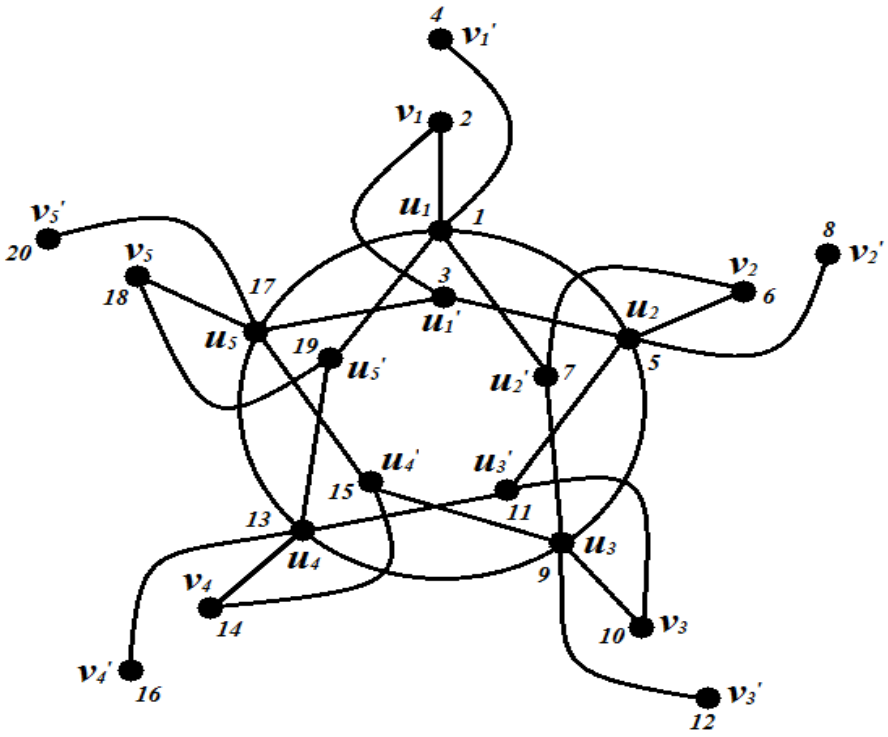

Figure 3. Labeled Split Graph of Sunlet graph S_5 , $G = \mathrm{Spl}(S_5)$

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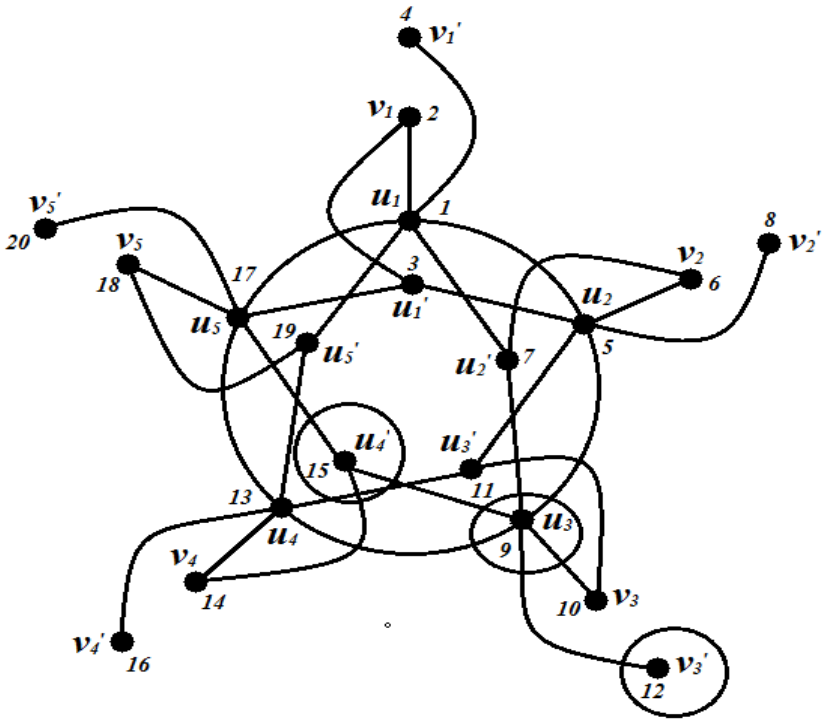

Figure 4. Checking the relative prime of $(v_i v_{i+1}')$ in the labeled graph G

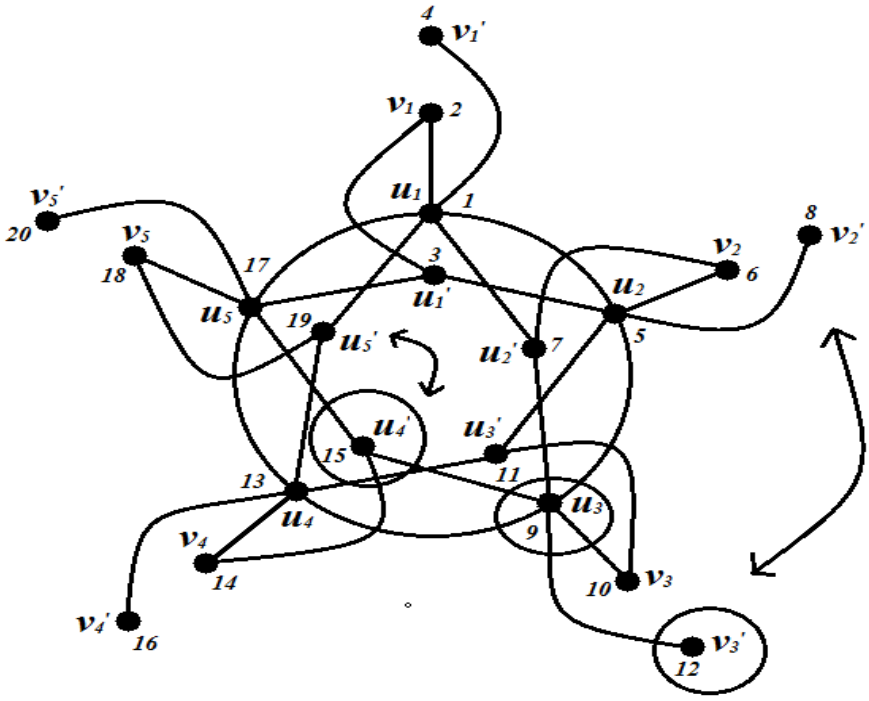

Figure 5. Prime labeling of Sunlet graph S_5

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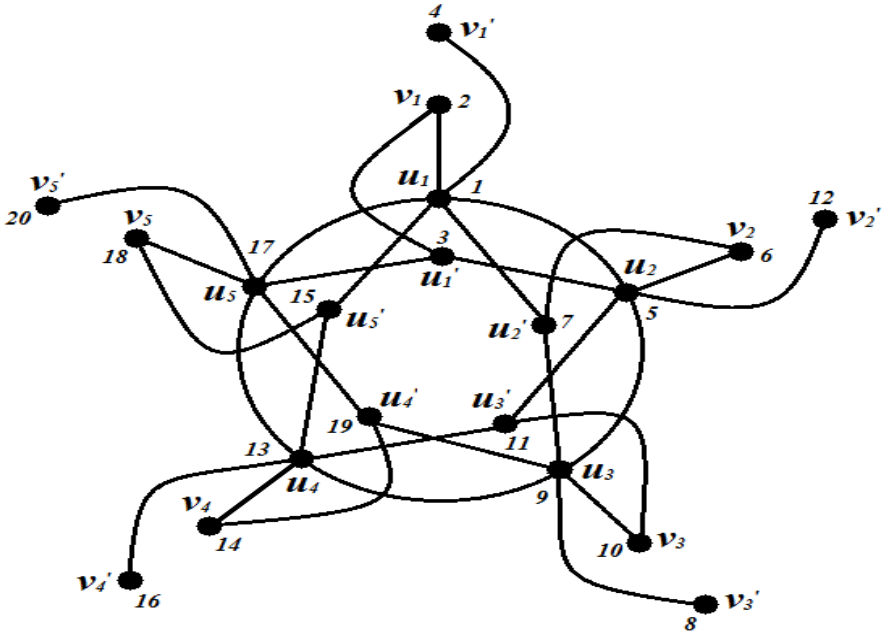

Figure 6. Prime labeling of split graph of Sunlet graph S_5

Illustration: 2
Prime labeling of Split graph of Sunlet graph S_6

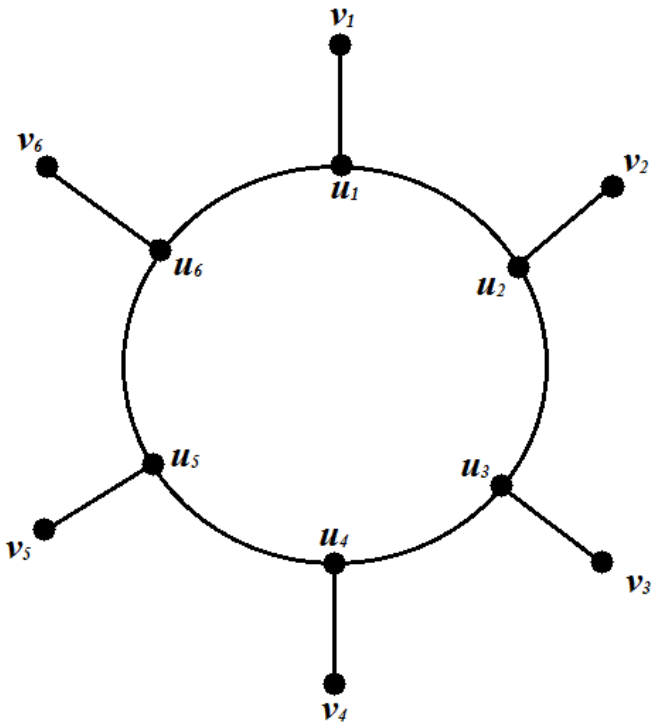

Figure 7. A Sunlet S_6

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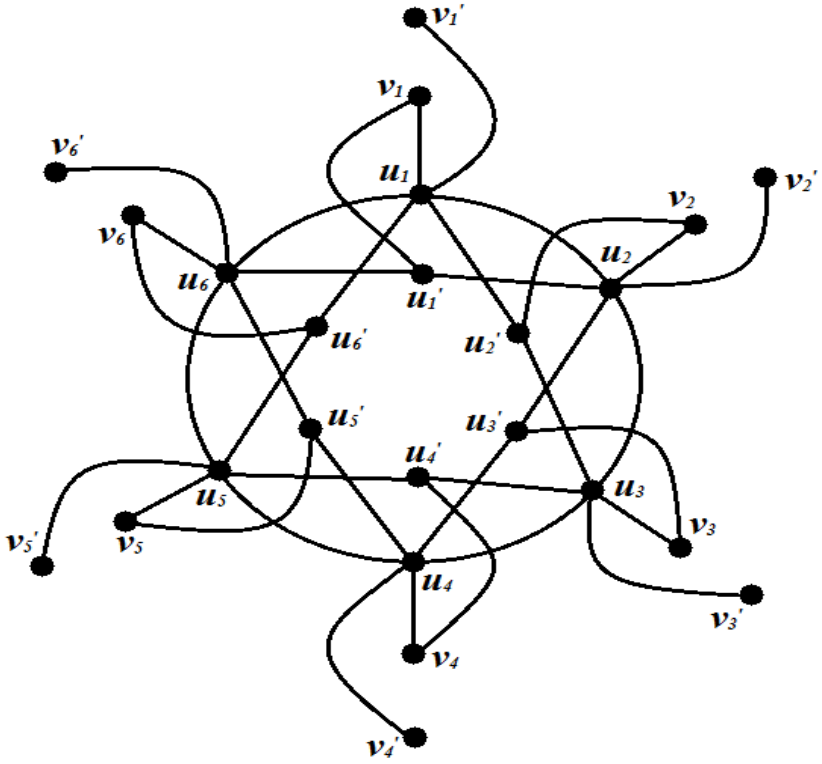

Figure 8. Split Graph of Sunlet S_6 , G = Spl(S_6)

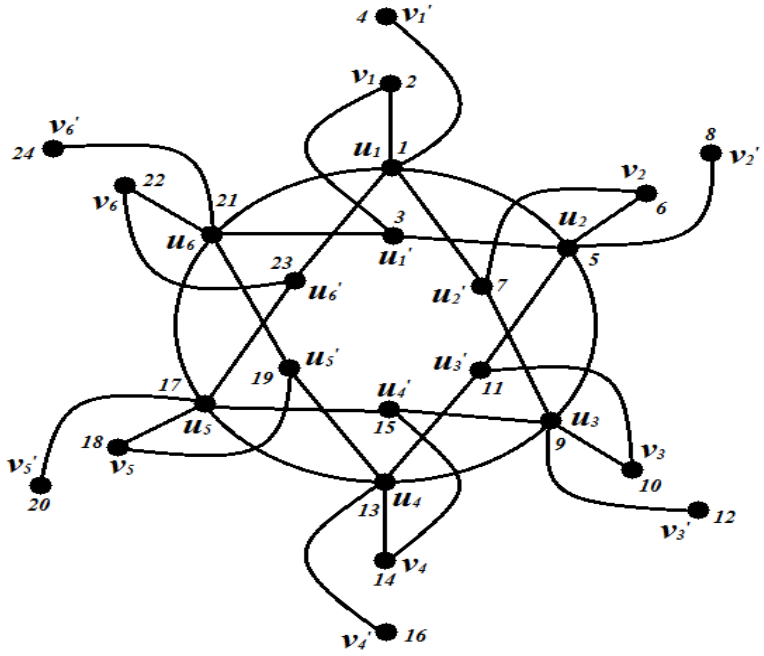

Figure 9. Labeled Split Graph of Sunlet graph S_6 , G = Spl(S_6)

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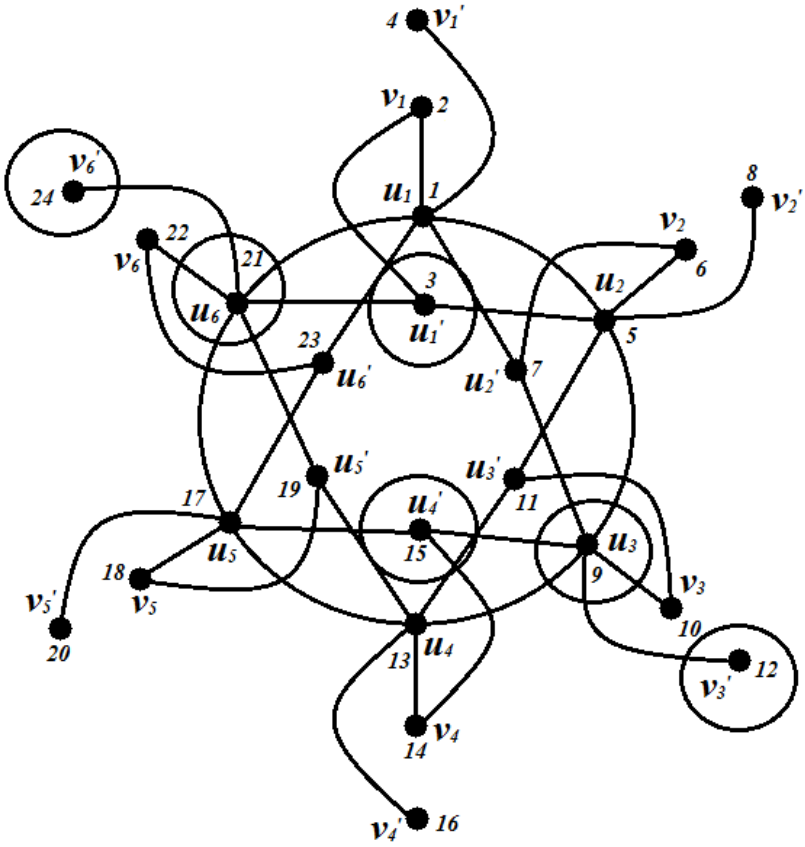

Figure 10. Checking the relative prime of $(v_i v_{i+1}')$ in the labeled graph G

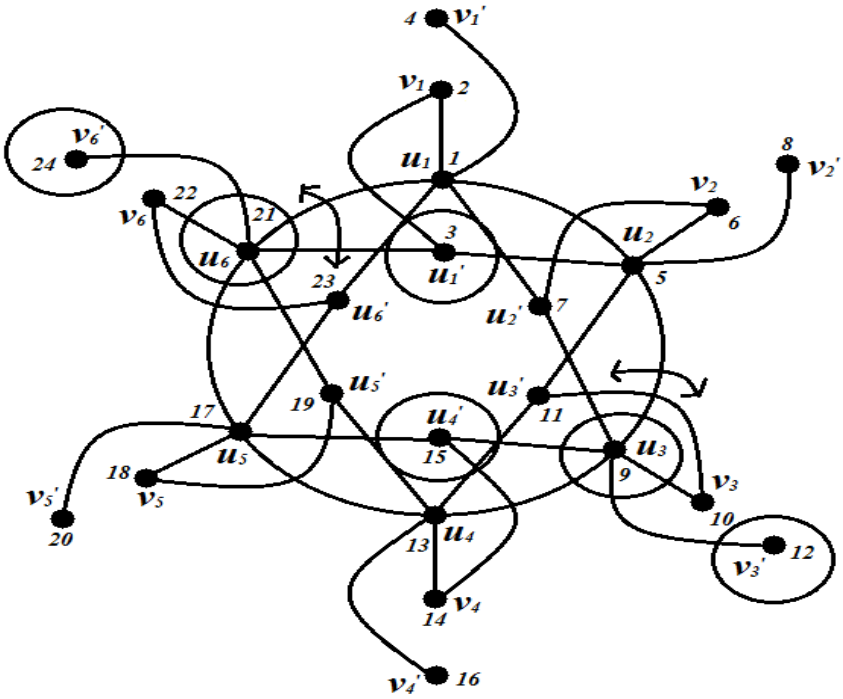

Figure 11. Prime labeling of Sunlet graph S_6

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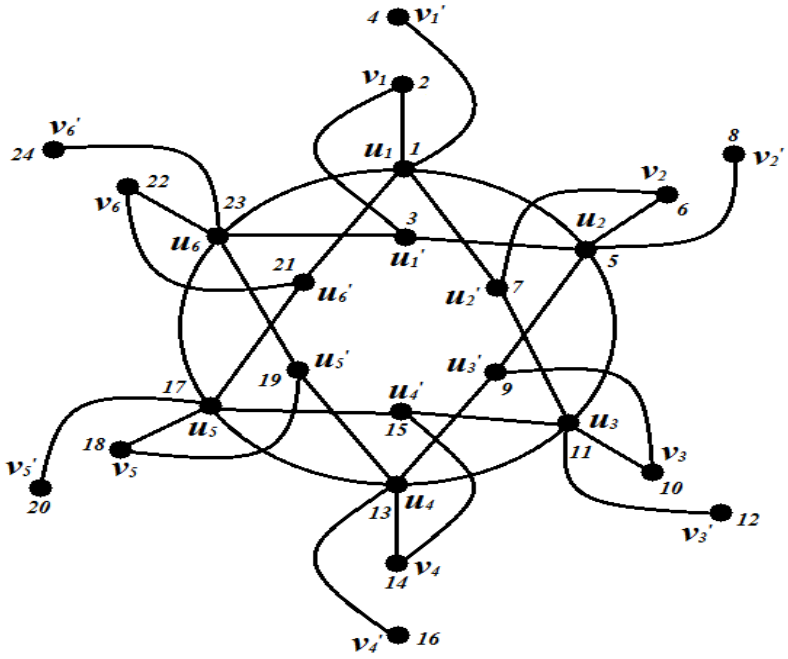

Figure 12. Prime labeling of split graph of Sunlet graph S_6

4. CONCLUSION

We have presented an algorithm for prime labeling to some classes of graph such as split graph of S_n and illustrate with two examples for the cases n is odd only. Really, it will motivate the researcher to investigate the prime labeling of splitb graph other families like star,sunlet and tree etc., are prime graph.

REFERENCES

1. J.A. Bondy and U.S.R. Murthy," Graph theory and applications" (North – Holland) , New York (1976)
2. A.N.Dabboucy , K. Howalla and Tout . "Prime labeling of graphs". Nat. Acad. Sci letters, 11 (1982) 365 – 368.
3. T.O.Dretsky et al. "On vertex prime labeling of graphs in graph theory". Combinatories and applications. Vol.I.J.Alari (Wilky.N.Y. 1991) 299 – 359.
4. H.C.Fu and K.C.Huany."On prime labeling discrete Math".127. (1994) 181 – 186.
5. J.A. Gallian."A dynamic survey of Graph Labeling". The Electronic Journal Combinatorics. Vol 18, 2011.
6. Dr. V. Ganesan et al " Prime Labeling of Split graph of path graph of cycle C_n " International journal of applied and advanced Scientific Research (IJAASR) volume 3, issue 2, 2018.
7. P. Haxel. O. Pikhurko and A. Taraz. " Primality of trees" J. Comb. 2(2011). 481 – 500.
8. Meena. S and Vaithlingam. K " Prime labeling for some form related graphs". International Journal of Engineering Research & Technology (IJERT)". Vol.1 issue 9, 2012.