

Generalized Pre-Semi Closed Mappings in Intuitionistic Fuzzy Topological Spaces

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ABSTRACT

The notions of intuitionistic fuzzy generalised pre-semi closed mappings, intuitionistic fuzzy generalised pre-semi open mappings, and intuitionistic fuzzy I-generalized pre-semi closed mappings are discussed and introduced in this chapter.

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology, Intuitionistic fuzzy topological spaces, Generalized pre-semi closed mappings

Introduction:

Gurcay et al [1] proposed and researched intuitionistic fuzzy closed mappings in 1997. The features of open and closed mappings in intuitionistic fuzzy topological spaces were explored by Lee & Lee [8] in 2000. In intuitionistic fuzzy topological spaces, Jeon et al [2] introduced and researched semiopen mappings, α -open mappings, and pre-open mappings. Several writers have recently contributed to the generalisation of closed and open mappings in intuitionistic fuzzy topological spaces [3,4,5,6,7,9,10].

Intuitionistic Fuzzy Generalized Pre-Semi Closed mapping:

Definition 1:

A function $f : (X, \square) \square (Y, \square)$ is said to be the intuitionistic fuzzy generalized pre-semi closed mapping (*IFGPSCM*) if $f(A)$ is *IFGPSCS* in Y for each *IFCS* A in X .

Example 2:

Let $X = \{a, b\}$, $Y = \{u, v\}$ and $H_1 = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$, $H_2 = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle$. Then $\square = \{0\sim, H_1, 1\sim\}$ and $\sigma = \{0\sim, H_2, 1\sim\}$ are *IFTS* on X and Y respectively.

The function is defined as $f : (X, \square) \square (Y, \square)$ by $f(a) = u$ and $f(b) = v$. Here f is an *IFGPSCM*.

Theorem 3:

Each *IFCM* is an *IFGPSCM* but not contrariwise.

Proof:

If $f : (X, \square) \square (Y, \square)$ be an *IFCM*. If A is an *IFCS* in X .

Here $f(A)$ is an *IFCS* in Y .

Then $f(A)$ is an *IFGPSCS* in Y .

Hereafter f is an *IFGPSCM*.

Theorem 4:

Each *IF α CM* is an *IFGPSCM* but not contrariwise.

Proof:

If $f : (X, \square) \square (Y, \square)$ is an *IF α CM*.

If A is an *IFCS* in X .

Here $f(A)$ is an $IF\alpha CS$ in Y .

Then $f(A)$ is an $IFGPSCS$ in Y .

Hereafter f is an $IFGPSCM$.

Theorem 5:

Each $IFWCM$ is an $IFGPSCM$ but not contrariwise.

Proof:

If $f : (X, \square) \square (Y, \square)$ be an $IFWCM$.

If A be an $IFCS$ in X .

Here $f(A)$ be an $IFWCS$ in Y .

Then $f(A)$ be an $IFGPSCS$ in Y .

Hereafter f be an $IFGPSCM$.

Theorem 6:

Each $IFPCM$ is an $IFGPSCM$ but not contrariwise.

Proof:

If $f : (X, \square) \square (Y, \square)$ be an $IFPCM$.

If A be an $IFCS$ in X .

Here $f(A)$ be an $IFPCS$ in Y .

Then $f(A)$ be an $IFGPSCS$ in Y .

Hereafter f is an $IFGPSCM$.

Example 7:

If $X = \{a, b\}, Y = \{u, v\}$ and $H_1 = \langle x, (0.1, 0.4), (0.9, 0.6) \rangle,$

$H_2 = \langle y, (0.2, 0.1), (0.8, 0.9) \rangle$ and $H_3 = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle.$

Then $\square = \{0 \sim, H_1, 1 \sim\}$ and $\square = \{0 \sim, H_1, H_2, 1 \sim\}$ are $IFTs$ on X and Y respectively.

The function is defined as $f : (X, \square) \square (Y, \square)$ by $f(a) = u$ and $f(b) = v$. Here f is an $IFGPSCM$ but not an $IFPCM$.

Theorem 8:

Each $IFGPSCM$ is an $IFGSPCM$ but not contrariwise.

Proof:

If $f : (X, \square) \square (Y, \square)$ be an $IFGPSCM$. If A is an $IFCS$ in X .

Here $f(A)$ be an $IFGPSCS$ in Y .

Then $f(A)$ be an $IFGSPCS$ in Y .

Hereafter f be an $IFGSPCM$.

Example 9:

Let $X = \{a, b\}, Y = \{u, v\}$ and $H_1 = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle,$

$H_2 = \langle y, (0.3, 0.2), (0.7, 0.8) \rangle.$ Then $\square = \{0 \sim, H_1, 1 \sim\}$ and $\square = \{0 \sim, H_2, 1 \sim\}$ are

$IFTs$ on X and Y respectively.

The function is defined as $f : (X, \square) \square (Y, \square)$ by $f(a) = u$ and $f(b) = v$. Here f be an $IFGSPCM$ but not an $IFGPSCM$.

Theorem 10:

Each $IFGPSCM$ is an $IFGSPRCM$ but not contrariwise.

Proof:

If $f : (X, \square) \square (Y, \square)$ be an $IFGPSCM$.

If A is an $IFCS$ in X .

Here $f(A)$ be an $IFGPSCS$ in Y .

Then $f(A)$ be an $IFGSPRCS$ in Y .

Hereafter f be an $IFGSPRCM$.

Example 11:

Let $X = \{a, b\}, Y = \{u, v\}$ and $H_1 = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$,
 $H_2 = \langle y, (0.3, 0.2), (0.7, 0.8) \rangle$. Then $\square = \{0 \sim, H_1, 1 \sim\}$ and $\square = \{0 \sim, H_2, 1 \sim\}$ are IFTs on X and Y correspondingly. The function is defined as $f : (X, \square) \square (Y, \square)$ by $f(a) = u$ and $f(b) = v$. Here f is an IFGSPRCM but not an IFGPSCM.

Definition 12:

A mapping $f : (X, \square) \square (Y, \square)$ is supposed to be intuitionistic fuzzy generalized pre-semi open mapping (IFGPSOM) if $f(A)$ is an IFGPSOS in Y for all IFOS in X .

Definition 13:

A mapping $f : (X, \square) \square (Y, \square)$ is supposed to be intuitionistic fuzzy i -generalized pre-semi closed mapping (IF i GPSCM) if $f(A)$ is an IFGPSCS in Y for all IFGPSCS A in X .

Definition 14:

A mapping $f : (X, \square) \square (Y, \square)$ is supposed to be intuitionistic fuzzy i -generalized pre-semi open mapping (IF i GPSOM) if $f(A)$ is an IFGPSOS in Y for all IFGPSOS A in X .

Example 15:

Let $X = \{a, b\}, Y = \{u, v\}$ and $H_1 = \langle x, (0.3, 0.2), (0.7, 0.8) \rangle$,
 $H_2 = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle$. Then $\square = \{0 \sim, H_1, 1 \sim\}$ and $\square = \{0 \sim, H_2, 1 \sim\}$ are IFTs on X and Y correspondingly. The function is defined as $f : (X, \square) \square (Y, \square)$ by $f(a) = u$ and $f(b) = v$. Here f is an IF i GPSCM.

Theorem 16:

Each IF i GPSCM is an IFGPSCM but not contrariwise.

Proof:

If $f : (X, \square) \square (Y, \square)$ is an IF i GPSCM.

If A be an IFCS in X .

Here A is an IFGPSCS in Y .

By supposition $f(A)$ is an IFGPSCS in Y .

Hence f is an IFGPSCM.

Example 17:

Let $X = \{a, b\}, Y = \{u, v\}$ and $H_1 = \langle x, (0.8, 0.9), (0.2, 0.1) \rangle$,
 $H_2 = \langle y, (0.3, 0.2), (0.7, 0.8) \rangle$. Then $\square = \{0 \sim, H_1, 1 \sim\}$ and $\square = \{0 \sim, H_2, 1 \sim\}$ are IFTs on X and Y correspondingly. The function is defined as $f : (X, \square) \square (Y, \square)$ by $f(a) = u$ and $f(b) = v$. Here f is an IFGPSCM but not an IF i GPSCM.

Theorem 18:

If $f : (X, \square) \square (Y, \square)$ be the function. The subsequent declarations are comparable if Y is an IFPST_{1/2} space:

(i) f is an IFGPSCM,

(ii) $pcl(f(A)) \square f(cl(A))$ for all IFS A of X .

Proof:

(i) \Rightarrow (ii) If A is an IFS in X .

Here $cl(A)$ is an IFCS in Y .

(i) implies that $f(cl(A))$ is an IFGPSCS in Y .

Since Y is an IFPST_{1/2} space, $f(cl(A))$ is an IFPCS in Y .

Consequently $pcl(f(cl(A))) = f(cl(A))$.

Now $pcl(f(A)) \square l(f(cl(A))) = f(cl(A))$.

Hence $pcl(f(A)) \square f(cl(A))$ for each IFS A of X .

(ii) \Rightarrow (i) Let A be any IFCS in X .

Then $cl(A) = A$. (ii) implies that $pcl(f(A)) \sqsubseteq f(cl(A)) = f(A)$.

But $f(A) \sqsubseteq l(f(A))$.

Therefore $l(f(A)) = f(A)$.

This implies $f(A)$ be an *IFPCS* in Y .

Subsequently all *IFPCS* be an *IFGPSCS*,

$f(A)$ be an *IFGPSCS* in Y .

Here f be an *IFGPSCM*.

Theorem 19:

If $f : (X, \sqsubseteq) \rightarrow (Y, \sqsubseteq)$ be the function. The subsequent are correspondent if Y is an *IFPST*_{1/2} space:

(i) f is an *IFGPSOM*.

(ii) $f(int(A)) \sqsubseteq pint(f(A))$ for each *IFS* A of X .

(iii) $int(f^{-1}(B)) \sqsubseteq f^{-1}(pint(B))$ for all *IFS* B of Y .

Proof:

(i) \Rightarrow (ii) If f is an *IFGPSOM*.

Let A be any *IFS* in X .

Then $int(A)$ is an *IFOS* in X .

(i) implies that $f(int(A))$ is an *IFGPSOS* in Y .

Since Y is an *IFPST*_{1/2} space, $f(int(A))$ be an *IFPOS* in Y .

Consequently $nt(f(int(A))) = f(int(A))$.

Now $f(int(A)) = pint(f(int(A))) \sqsubseteq nt(f(A))$.

(ii) \Rightarrow (iii) If B is an *IFS* in Y .

Here $f^{-1}(B)$ be an *IFS* in X .

By (ii) $f(int(f^{-1}(B))) \sqsubseteq pint(f(f^{-1}(B))) \sqsubseteq pint(B)$.

Now $int(f^{-1}(B)) \sqsubseteq f^{-1}(f(int(f^{-1}(B)))) \sqsubseteq f^{-1}(pint(B))$.

(iii) \Rightarrow (i) If A is an *IFOS* in X .

Here $int(A) = A$ and $f(A)$ is an *IFS* in Y .

By (iii) $int(f^{-1}(f(A))) \sqsubseteq f^{-1}(pint(f(A)))$.

Now $A = int(A) \sqsubseteq int(f^{-1}(f(A))) \sqsubseteq f^{-1}(pint(f(A)))$.

Therefore $f(A) \sqsubseteq f(f^{-1}(pint(f(A)))) \sqsubseteq pint(f(A)) \sqsubseteq f(A)$.

This implies $pint(f(A)) = f(A)$ be an *IFPOS* in Y and hereafter an *IFGPSOS* in Y .

Accordingly f is an *IFGPSOM*.

Theorem 20:

A mapping $f : (X, \sqsubseteq) \rightarrow (Y, \sqsubseteq)$ is an *IFGPSCM* if $f(pint(A)) \sqsubseteq pint(f(A))$ for all $A \sqsubseteq X$.

Proof:

Let A be an *IFOS* in X .

Here $int(A) = A$. At present $f(A) = f(int(A)) \sqsubseteq f(pint(A)) \sqsubseteq nt(f(A))$, by hypothesis. But $pint(f(A)) \sqsubseteq (A)$.

Therefore $f(A)$ is an *IFPOS* in Y .

Then $f(A)$ be an *IFGPSOS* in Y .

Hereafter f is an *IFGPSCM*, by Theorem 19.

Theorem 21:

$f : (X, \sqsubseteq) \rightarrow (Y, \sqsubseteq)$ be the function here X and Y are *IFPST*_{1/2} space, where succeeding announcements are correspondent:

(i) f be an *IF i GPSCM*.

(ii) $f(A)$ is an *IFGPSOS* in Y for every *IFGPSOS* A in X .

(iii) $f(pint(B)) \sqsubseteq pint(f(B))$ for every *IFS* B in X .

(iv) $pcl(f(B)) \sqsubseteq f(pcl(B))$ for every *IFS* B in X .

Proof:

(i) \Rightarrow (ii) obvious.

(ii) \Rightarrow (iii) If B is any *IFS* in X .

Since $pint(B)$ be an *IFPOS*, it is an *IFGPSOS* in X .

Formerly by proposition, $f(pint(B))$ be an *IFGPSOS* in Y .

Subsequently Y be an *IFPST*_{1/2} space, $f(pint(B))$ be an *IFPOS* in Y .

Therefore $f(pint(B)) = pint(f(pint(B))) \sqsubseteq nt(f(B))$.

(iii) \Rightarrow (iv) is obvious by taking complement in (iii).

(iv) \Rightarrow (i) Let A be an *IFGPSCS* in X .

By Hypothesis, $pcl(f(A)) \sqsubseteq (pcl(A))$.

Since X is an *IFPST*_{1/2} space, A is an *IFPCS* in X .

Therefore $pcl(f(A)) \sqsubseteq f(pcl(A)) = f(A) \sqsubseteq (pcl(A))$.

Hereafter $f(A)$ be an *IFPCS* in Y and hereafter *IFGPSCS* in Y .

Here f be an *IF iGPSCM*.

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