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A Work on New Method to Find Initial Basic Feasible Solution of Transportation Problem in Contrast to Existing Methods

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Abstract

In the present paper, we focused on finding an initial basic feasible solution for transportation problems. The new method is compared to four different proposed mean methods namely, Proposed Arithmetic Mean (PAM), Proposed Harmonic Mean (PHM), Proposed Geometric Mean (PGM) and Proposed Quadratic Mean (PQM) methods along with existing traditional methods like North West Corner Method (NWCM), Least Cost Method (LCM) and Vogel Approximation Method (VAM). Also, the deviation percentage of the outcomes has been figured out in contrast to distinct methods. The approach thus developed, involves elementary arithmetic and statistics, which could be of keen interest even for beginners and is beneficial to decision makers dealing with the problem of supply chain as well as logistics. Suitable examples are given for examining optimality and to validate the results obtained.

2010 Mathematics Subject Classification: 90C47, 90C08.

Keywords: Transportation Problem, IBFS, NWCM, VAM, LCM, MODI Method, PAM, PHM, PGM and PQM.

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1. Introduction

A transportation problem is distinctive type of linear programming problem, where the intent is to cut down the cost of spreading products from number of sources (like- factories) to number of destinations (like-warehouse) while balancing both the supply limits and demand requirements. A transportation problem is named such, as many of its applications sought to determine the way for transporting goods optimally. It is logistical problem for organizations like manufacturing and transport companies. Whenever there is physical movement of goods from the producer or manufacturer to final consumer through various channels of distribution, there is need to cut down the operative cost so as to scale up profit on sales. Due to these reasons transportation problem has applicability in many disciplines such as minimizing shipping cost, determining low cost allocation, finding minimum cost production plan, in military distribution system and many more. Hence, it has very great significance in sphere of economy and mathematics. The Transportation problem was formalized by Gaspard Monge in 1781. A N Tolstoi in 1920, was the first to study Transportation problem mathematically. Major advancement was made in transportation problem theory by soviet mathematician Leonid Kantorovich during World War 2. Many other mathematicians like Hitchcock, Koopman, Charnes and Cooper have worked on such kind of problems and made significant contribution to this field. In the recent times, when industries and factories are growing at fast pace and as they are expanding every second, it is necessary to find a special method of solution which eases situation for policy makers.

2. Algorithm of the New Method

Step 1- Check whether the given transportation problem is balanced or not, if not balance it.

Step 2- Now look in columns D_1, D_2 and select the least cost element from them. Subtract it in column D_1. Similarly, apply the same process for columns (D_2, D_3), (D_3, D_4) and (D_4, D_1).

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Step 3- Calculate the arithmetic mean for each row i.e. for S_1, S_2 and S_3 and write calculated arithmetic mean along the right side of the table against the corresponding row and similarly calculate the arithmetic mean for each column i.e. for D_1, D_2, D_3 and D_4 then write the calculated arithmetic mean just below the corresponding columns of the table.

Step 4- Select the highest calculated arithmetic mean and then choose the corresponding row or column in which it lies. Look for the least cost element from the chosen row or column and then allocate supply or demand whichever is minimum in the least cost cell.

Step 5- Repeat the step3 and step 4 till all rows and columns are exhausted.

Step 6- Now reallocate the allocations thus obtained to the matrix of original transportation problem.

Step 7- Finally, compute the corresponding transportation cost of the transportation problem.

EXAMPLE -1: Consider the transportation problem formulated in following table;

	D_1	D_2	D_3	D_4	SUPPLY
S_1	4	6	8	8	40
S_2	6	8	6	7	60
S_3	5	7	6	8	50
DEMAND	20	30	50	50	150

Step 1- Check whether the given transportation problem is balanced or not, if not balance it.

	D_1	D_2	D_3	D_4	SUPPLY
S_1	4	6	8	8	40
S_2	6	8	6	7	60
S_3	5	7	6	8	50
DEMAND	20	30	50	50	150

Step 2- Now look in columns D_1, D_2 and select the least cost element from them. Subtract it in column D_1. Similarly, apply the same process for columns (D_2, D_3), (D_3, D_4) and (D_4, D_1).

	D_1	D_2	D_3	D_4	SUPPLY
S_1	0	0	2	4	40
S_2	2	2	0	3	60
S_3	1	1	0	4	50
DEMAND	20	30	50	50	

Step 3- Calculate the arithmetic mean for each row i.e. for S_1, S_2 and S_3 and write calculated arithmetic mean along the right side of the table against the corresponding row. Calculate the arithmetic mean for each column i.e. for D_1, D_2, D_3 and D_4 then write the calculated arithmetic mean just below the corresponding columns of the table.

	D_1	D_2	D_3	D_4	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	0	2	4	40	1.5
S_2	2	2	0	3	60	1.75
S_3	1	1	0	4	50	1.5
DEMAND	20	30	50	50		
$\frac{\sum_{i=1}^{n} x_n}{n}$	1	1	0.66	3.66		

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Step 4- Select the highest calculated arithmetic mean and then choose the corresponding row or column in which it lies. Look for the least cost element from the chosen row or column and then allocate supply or demand whichever is minimum in the least cost cell.

	D_1	D_2	D_3	~~D_4~~	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	0	2	~~4~~	40	1.5
S_2	2	2	0	~~3(50)~~	60/10	1.75
S_3	1	1	0	~~4~~	50	1.5
DEMAND	20	30	50	~~50/0~~		
$\frac{\sum_{i=1}^{n} x_n}{n}$	1	1	0.66	~~3.66~~		

Step 5- Repeat the step3 and step 4 till all rows and columns are exhausted.

	D_1	D_2	D_3	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	0	2	40	0.66
S_2	2	2	0	10	1.33
S_3	1	1	0	50	0.66
DEMAND	20	30	50		
$\frac{\sum_{i=1}^{n} x_n}{n}$	1	1	0.66		

	D_1	D_2	D_3	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	0	2	40	0.66
~~S_2~~	~~2~~	~~2~~	~~0(10)~~	~~10/0~~	~~1.33~~
S_3	1	1	0	50	0.66
DEMAND	20	30	50/40		
$\frac{\sum_{i=1}^{n} x_n}{n}$	1	1	0.66		

	D_1	D_2	D_3	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	0	2	40	0.66
S_3	1	1	0	50	0.66
DEMAND	20	30	40		
$\frac{\sum_{i=1}^{n} x_n}{n}$	0.5	0.5	1		

	D_1	D_2	~~D_3~~	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S1	0	0	~~2~~	40	0.66
S2	1	1	~~0(40)~~	50/10	0.66
DEMAND	20	30	~~50/40/0~~		
$\frac{\sum_{i=1}^{n} x_n}{n}$	0.5	0.5	~~1~~		

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	D_1	D_2	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S1	0	0	40	0
S2	1	1	10	1
DEMAND	20	30		
$\frac{\sum_{i=1}^{n} x_n}{n}$	0.5	0.5		

	D_1	D_2	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	0	40	0
~~S_2~~	~~1(10)~~	~~1~~	~~10/0~~	~~1~~
DEMAND	20/10	30		
$\frac{\sum_{i=1}^{n} x_n}{n}$	0.5	0.5		

	D_1	D_2	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	0	40	0
DEMAND	10	30		
$\frac{\sum_{i=1}^{n} x_n}{n}$	0	0		

	~~D_1~~	D_2	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	~~0(10)~~	0	40/30	0
DEMAND	~~10/0~~	30		
$\frac{\sum_{i=1}^{n} x_n}{n}$	~~0~~	0		

	D_2	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0	40/30	0
DEMAND	30		
$\frac{\sum_{i=1}^{n} x_n}{n}$	0		

	D_2	SUPPLY	$\frac{\sum_{i=1}^{n} x_n}{n}$
S_1	0(30)	40/30/0	0
DEMAND	30/0		
$\frac{\sum_{i=1}^{n} x_n}{n}$	0		

JOURNAL OF ALGEBRAIC STATISTICS
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Step 6- Now reallocate the allocations thus, obtained to the matrix of original transportation problem.

	D_1	D_2	D_3	D_4	SUPPLY
S_1	4(10)	6(30)	8	8	40/30/0
S_2	6	8	6(10)	7(50)	60/10/0
S_3	5(10)	7	6(40)	8	50/10/0
DEMAND	20/10/0	30/0	50/40/10/0	50/0	150

Step 7- Finally, compute the corresponding transportation cost of the transportation problem.

So, Minimum TC $z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$

$= 4 \times 10 + 6 \times 30 + 6 \times 10 + 7 \times 50 + 5 \times 10 + 6 \times 40$

$= 40 + 180 + 60 + 350 + 50 + 240 = 920$

EXAMPLE -2: Consider the transportation problem formulated in following table;

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	12	8	11	18	11	6
S_2	14	22	8	12	14	2
S_3	14	14	16	14	15	4
S_4	19	11	14	17	15	10
S_5	13	9	17	20	11	9
DEMAND	2	8	7	10	4	

	D_1	D_2	D_3	D_4	D_5	SUPPLY
S_1	12(2)	8	11(4)	18	11	6/4/0
S_2	14	22	8(2)	12	14	2/0
S_3	14	14	16	14(4)	15	4/0
S_4	19	11(3)	14(1)	17(6)	15	10/9/3/0
S_5	13	9(5)	17	20	11(4)	9/4/0
DEMAND	2/0	8/5/0	7/5/1/0	10/6/0	4/0	

So, Minimum TC $z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$

$= 12 \times 2 + 11 \times 4 + 8 \times 2 + 14 \times 4 + 11 \times 3 + 14 \times 1 + 17 \times 6 + 9 \times 5$

$= 24 + 44 + 16 + 56 + 33 + 14 + 102 + 45 + 44$

$= 378$

3. Comparison Table

Table no.	Problem size	NWCM	LCM	VAM	HARMONIC MEAN	GEOMETRIC MEAN	QUADRATIC MEAN	ARITHMETIC MEAN	NEW method	MODI method
1.	3×4	980	960	960	920	920	920	920	**920**	**920**
2.	5×5	459	383	383	385	385	385	385	**378**	**378**

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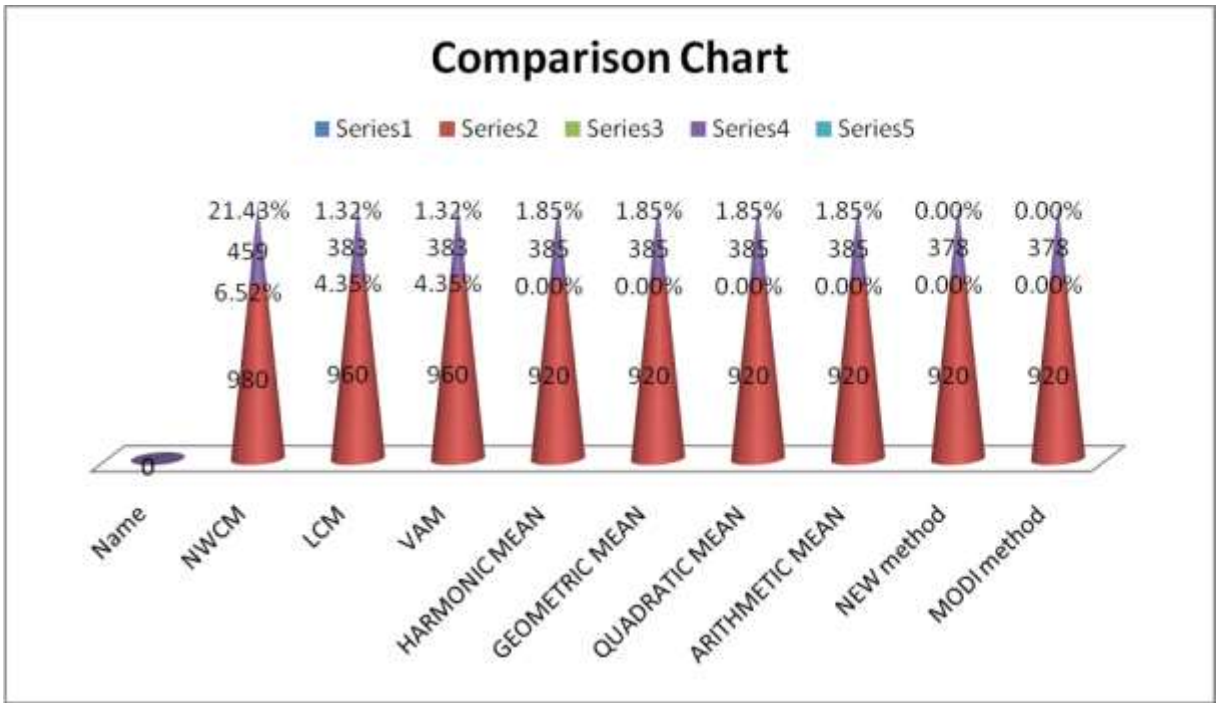
Solution and deviation values of the method

Name	NWCM	LCM	VAM	HARMONIC MEAN	GEOMETRIC MEAN	QUADRATIC MEAN	ARITHMETIC MEAN	NEW method	MODI method
Pro 1 Sol.(dev%)	980 6.52%	960 4.35%	960 4.35%	920 0.00%	920 0.00%	920 0.00%	920 0.00%	**920** **0.00%**	**920** **0.00%**
Pro 2 Sol.(dev%)	459 21.43%	383 1.32%	383 1.32%	385 1.85%	385 1.85%	385 1.85%	385 1.85%	**378** **0.00%**	**378** **0.00%**



4. Conclusion

The new algorithm thus established, reduces the iterative steps to find initial basic feasible solution. Further, we have compared the result obtained with different existing methods of transportation problems such as NWCM, LCM, VAM including different proposed means methods. Consequently, we found that the outcome is either very close or exactly same as that of MODI's method.

5. References

[1] F. L. Hitchcock: The distribution of a product from several sources to numerous localities, Journal of Mathematical Physics. 20 (1941) 224-230.

[2] T.C. Koopmans, Optimum Utilization of the Transportation system, proceeding of The International Statistical Conference, Washington D.C, (1947).

[3] G.B. Dantzig, Linear Programming and Extensions, America: Princeton University press, New Jersey, (1963).

[4] Hamdy A. Taha, Operations Research An Introduction, 9th Edition.

[5] M.Sathyavathy, M.Shalini: Solving Transportation Problem with Four Different Proposed Mean Method and Comparison with Existing Methods for Optimum Solution, International Conference on Physics and Photonics Processes in Nano Sciences. (2019) 1-9.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 1, 2022, p. 75-81
https://publishoa.com
ISSN: 1309-3452

[6] Reena G.Patel and P.H.Bhathawala: The advance method for the optimum solution of a transportation problem, International Journal of Science and Research. 6(14) (2015) 703-705.
[7] Reena G.Patel and P.H.Bhathawala: An innovative approach to optimum solution of a transportation problem, International Journal of Innovative Research in Science, Engineering Technology. 5(4) (2016) 5695-5700.
[8] Sushma Duraphe, Geeta Modi and Sarla Raigar: A new method for the optimal solution of a transportation problem, International Journal of Mathematics and Its Applications. 5 (2017) 309-312.