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A Fuzzy Differential Approach to An Overage Management with Customer Acquisition Inventory Problem Via Neutrosophic Fuzzy Geometric Programming Technique.

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ABSTRACT

In this paper, a total inventory cost for overage management with customer acquisition has been analyzed in Neutrosophic fuzzy environment. The overage to maximum stock level depends on the demand and due to impreciseness of demand , the model is formulated using fuzzy differential equation. This equation is solved by fuzzy geometric programming technique and this approach has been illustrated with numerical example.

Keywords: Maximum stock level to overage, Special inventory, Neutrosophic trapezoidal fuzzy number, Fuzzy differential equation, Geometric programming technique.

1.INTRODUCTION

Inventory includes a usable however idle assets along with men, substances, machines and money. Inventory management is intently affiliated in industries has been performed sizeable role because the usage of the assets via way of means of guy. As stock get up in lots of situations, the methods and strategies of coping with stock relevant in a single situation, which isn't appropriate for another. This manner is the system of various stock fashions with the goal of figuring out the most suitable amount with minimal fees to satisfy the requirements. The lifestyles of the current guy has made greater cushy via way of means of the manufacturing sectors, which produces merchandise to satisfy his needs. But the actual truth is that, the manufacturing sectors face outstanding problems in engaging in every task, concerned in making out the favored product from the to be had uncooked material. Unfortunately, if there arises any deficiency in the amount of the specified substances for the manufacturing of the product, the survival of the person receives significantly affected.

Uncertainty theory plays an important role in modelling science and engineering problems. In reality, both demand and supply are uncertain due to change of orders, random capacity of suppliers and unpredictable events. The uncertainties are treated with the fuzzy logic theory. Fuzzy logic is widely used in solving problems of artificial intelligence as in building expert systems, and in combination with artificial neural networks.

Presence of fuzzy demand leads to a fuzzy differential equation for instantaneous state of inventory level. Till now, fuzzy differential equation is used to formulate and solve fuzzy inventory models. But, the topics on fuzzy differential equations have been rapidly growing in the recent years. The first rule on solving fuzzy differential equation was made by Kandel et al. . An extended version of their work was published after two years . After that different approaches have been presented by several authors for solving fuzzy differential equations . Recently, Chalco-Cano and Roman-Flores compare some approaches to solve fuzzy differential equations. Though, a demand of an item depends on several factors, coefficients of its functional form should be estimated from the past data to make inventory control decisions. But, due to liberisation of world economy and introduction of multinationals, there is a stiff competition among different multinational companies of all countries. To survive in the market, they have very frequently changed their product specifications with new name and features. As a result, small size of past data of their products is available to estimate the demand coefficients. In reality, these coefficients are uncertain and estimated as random parameters with a probability distribution if sufficient past data is available. These uncertain quantities can also
be estimated as fuzzy-parameters if past data is insufficient.

Nowadays, Neutrosophic Sets is used in different fields of research work. **Roy and Das** has solved multi objective production planning problem by neutrosophic linear programming approach. **Banerjee** et al. has discussed single objective linear goal programming problem in neutrosophic number environment. In recent era**, Pramanik and Banerjee** formulated three new neutrosophic goal programming model to solve multi objective programming problems with neutrosophic number coefficients**. Basset et al. S. Pramanik** analyzed neutrosophic goal programming problem under neutrosophic sets environment. Again, a multi objective neutrosophic optimization technique is investigated and its application to structural design is developed by **Sarker** et al. Then, **Basset** et al. introduced and solved a neutrosophic linear programming model where the parameters are considered as trapezoidal neutrosophic numbers. Ye and Ye et al. has developed neutrosophic numberfor linear and non-linear programming methods respectively. In both cases, authors made applications under Neutrosophic environment. In spite of the above developments, there are several gaps in the literature of Neutrosophic Optimization. Till now, none has demonstrated that a non-linear Neutrosophic Optimization Problem can be reduced to a Geometric Programming Problem (GPP) with posynomial terms and solved by GP technique. Thus the motivation of the present investigation is to develop a procedure to reduce a non-linear Neutrosophic Problem to a corresponding GPP and then to solve it by the appropriate technique depending upon its degree of difficulty.

In this paper, a total inventory cost for overage management with customer acquisition has been analyzed in neutrosophic fuzzy environment. The overage to maximum stock level depends on the demand and due to impreciseness of demand , the model is formulated using fuzzy differential equation. This equation is solved by fuzzy geometric programming technique.

2. PRELIMINARIES

2.1. Definition: Fuzzy Set

A fuzzy set $\tilde{A}$ = $\{(x, \mu_{\tilde{A}}(x)); x \in X, \mu_{\tilde{A}} \in [0,1]\}$

In the pair $(x, \mu_{\tilde{A}}(x))$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$ belong to the interval [0,1], called membership function or grade membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

2.2. Definition : α − cut .

An α-cut of a fuzzy set $\tilde{A}$ is a crisp set A_α , that contains all the elements of universal set X having a membership grade in A greater than or equal to the specific value of α.

ie., A_α = $\{x \in X \setminus \mu_{\tilde{A}}(x) \geq \alpha\}$

If R is the real line, then a fuzzy number is a fuzzy set A with membership function $\mu A : X \to [0,1]$, having the following properties:

(i) A is normal, i.e., there exists $x \epsilon R$ such that (x) =1

(ii) A is piece-wise continuous

(iii) supp(A) = cl{ $x \in R$: μA (x) > 0}, where cl represents the closure of a set

(iv) A is a convex fuzzy set.

2.3. Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x)$ = 0, -∞< x ≤a1,

(iii) $\mu_{\tilde{A}}(x)$ = L(x) is strictly increasing on [a1, a2],

(iv) $\mu_{\tilde{A}}(x)$ = 1, a2≤x ≤a3,

(v) $\mu_{\tilde{A}}(x)$ = R(x) is strictly decreasing on [a3, a4],

(vi) $\mu_{\tilde{A}}(x)$ = 0, a4 ≤x < ∞, where a1, a2, a3 and a4 are real numbers.

2.4. Trapezoidal Fuzzy Number:

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The fuzzy number $\widetilde{A}$ = (a_1,a_2, a_3,a_4), where $a_1 < a_2 < a_3 < a_4$ and defined on R is called the trapezoidal fuzzy number, if the membership function is given by

$$\mu_{\bar{A}}(x) = \begin{cases} 0 : x < a_1 or x > a_4 \\ \dfrac{(x-a_1)}{(a_2-a_1)} : a_1 \le x \le a_2 \\ 1 : a_2 \le x \le a_3 \\ \dfrac{(x-a_4)}{(a_3-a_4)} : a_3 \le x \le a_4 \end{cases}$$

2.5 Neutrosophic set:

Let X be a universe set. A Neutrosophic set A on X is defined as A = {TA(x), IA(x), FA(x) : x ∈ X}, where TA(x), IA(x), FA(x) : X →−]0, 1[+ represents the degree of membership, degree of indeterministic, and degree of non-membership respectively of the element x ∈ X, such that 0 ≤ TA(x) + IA(x) + FA(x) ≤ 3for all x ∈ X.

2.6 Neutrosophic Number:

A neutrosophic set A defined on the universal set of real numbers R is said to be neutrosophic number if it has the following properties.

i) A is normal if there exists $x_0 \in R$, such that $T_A(x_0) = I$ ($I_A(x_0) = F_A(x_0) = 0$)
ii) A is convex set for the truth function $T_A(x)$, $T_A(\mu\, x_1 + (1-\mu)x_2\ ) \ge \min(T_A(x_1),T_A(x_2))$ for all $x_1, x_2 \in R$, μ[0,1]
iii) A is concave set for the indeterministic function and false function $I_A(x)$, $F_A(x)$
iv) $I_A(\mu\, x_1 + (1-\mu)x_2\ ) \ge \max(T_A(x_1),T_A(x_2))$ for all $x_1, x_2 \in R$, μ[0,1] and
$T_A(\mu\, x_1 + (1-\mu)x_2\ ) \ge \max(T_A(x_1),T_A(x_2))$ for all $x_1, x_2 \in R$, μ[0,1]

2.7 Trapezoidal Neutrosophic fuzzy number: A trapezoidal neutrosophic fuzzy number A (a,b,c,d,u_A, $\cancel{v_A, w_A}$) in R with the following truth function, indeterministic function and falsity function which is given by the following

T

$$T_{\bar{A}}(x) = \begin{cases} 0 : x < a_1 or x > a \\ \dfrac{(x-a)}{(b-a)} u_A : a \le x \le b \\ u_A : b \le x \le c \\ \dfrac{(d-x)}{(d-c)} u_A : c \le x \le d \\ 1, otherwise \end{cases} \qquad I_{\bar{A}}(x) = \begin{cases} 0 : x < a\ \ or x > a \\ \dfrac{(b-x)}{(b-a)} v_A : a \le x \le b \\ v_A : b \le x \le c \\ \dfrac{(d-x)}{(d-c)} v_A : c \le x \le d \\ 1, otherwise \end{cases}$$

$$F_{\bar{A}}(x) = \begin{cases} 0 : x < a\ \ or x > a \\ \dfrac{(b-x)}{(b-a)} w_A : a \le x \le b \\ w_A : b \le x \le c \\ \dfrac{(d-x)w_A}{(d-c)} : c \le x \le d \\ 1, otherwise \end{cases}$$

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2.8 Geometric programming problem:

Primal problem: Primal Geometric Programming(PGP) problem is

$$\text{Minimize } g_0(t) = \sum_{k=1}^{T_0} C_{0k} \prod_{j=1}^{m} t_j^{\alpha_{0kj}}$$

Subject to $$\sum_{k=1}^{T_0} C_{rk} \prod_{j=1}^{m} t_j^{\alpha_{rkj}} \leq 1 \ , (r= 1,2,\ldots..l) , j= (1,2,3\ldots..m) \ t_j>0$$

Where $C_{0k} > 0$ $(k=1,2,\ldots T_0)$ C_{rk} and α_{rk} are real numbers. It is constrained polynomial geometric problem. The number of term each polynomial constrained functions varies and it is denoted by T_r for each r= 0,1,2….. Let $T= T_0+T_1+T_2+\ldots\ldots +T_l$ be the total number of terms in the primal program.

The Degree of difficulty is (DD) = T – (m+1)

Dual Problem:

$$\text{Maximize } = \prod_{r=0}^{l} \prod_{k=1}^{T^r} \left(\frac{C_{rk}}{\delta_{rk}}\right)^{\delta rk} \left(\sum_{s=1+T_{r+1}}^{T} (\delta_{rs}\right)^{\delta rk}$$

Subject to $$\sum_{k=1}^{T_0} \delta_{0k} = 1 \quad (\text{ Normality condition})$$

$$\sum_{r=0}^{l} \sum_{k=1}^{T_r} \alpha_{rkj} \delta_{rk} = 0 \quad (\text{ Orthogonality conditions})$$

$\delta_{rk} > 0$, (Positive constant)

3 Fuzzy Differential equations:

General form of first order differential equation is dy/dt = f(x,y,k), y(0)=c ----------(1)

where y is dependent variable , defined on I which contains zero and x = $(x_1,x_2,x_3,\ldots..x_n)$ is vector of constants.

Let $\bar{x} = (\overline{x_1}, \overline{x_2}, \ldots\ldots \overline{x_n})$ be a vector of Trapezoidal fuzzy number and $\bar{c}$ be also trapezoidal fuzzy number, then equation (1) reduces to $\frac{d\bar{y}}{dt} = f(x,\bar{y},\bar{k}), \bar{y}(0) = \bar{c}$ --------------(2) which is fuzzy differential equation . Then the solution of equation (2) is $\bar{y}(t) = g(x,\bar{k},\bar{c})$ ------------(3)

And its α-cut $\bar{y}$ (t)[α] = $[y_L(t,\alpha), y_R(t,\alpha)]$ for each $t \in I$ and $\alpha \in (0,1)$.

If the following conditions are satisfied then the equation (3) is the solution of the equation (2).

- $y'_L(t,\alpha)$ and $y'_R(t,\alpha)$ are continuous on Ix[0,1].
- $y'_L(t,\alpha)$ is an increasing function of α for each $t \in I$.
- $y'_R(t,\alpha)$) is decreasing function of α for each $t \in I$.

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- $y'_L(t,1) \leq y'_R(t,1)$ for each $t \in I$.

4.Defuzzification of Differential Neutrosophic Trapezoidal fuzzy number:

Let A= $[T^L, T^U]$, $[I^L, I^U]$, $[F^L, F^U]$ be an interval neutrosophic number, then the score function of A can be defined by S(A) = $\frac{T^L + T^U}{2} + 1 - \frac{I^L + I^U}{2} + \frac{F^L + F^U}{2}$

Signed distance method :

Let $\tilde{A}$ = (a,b,c,d) be a trapezoidal fuzzy number, then the signed distance method of $\tilde{A}$ is defined as d ($\tilde{A}$, 0) = ½ $\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$ where $\tilde{A}$ = $[A_L(\alpha), A_R(\alpha)]$ = [a+(b-a)α , d-(d-c)α] , α € [0,1] is a α-cut of fuzzy set $\tilde{A}$ which is a close interval.

d ($\tilde{A}$,0) = (a + b + c + d) / 4.

5. Assumptions and Notations:

5.1 Assumptions:

i) The maximum stock level reaches overage due to any one of the causes such as Improper transport services, Climatic barrier, Occurrences of calamity and Disputes and Strikes.

ii) The acquisition cost is the cost of acquiring new customer.

iii) The products are not of deteriorating type.

iv) Planning horizon is infinite.

5.2 Notations:

Q **o**rder size
D demand per unit time
p percentage of items in Q that adds to overage in the next cycle.
K fixed cost of placing an order
h holding cost per unit per unit of time in the maximum stock
h_0 holding cost per unit of time in the special inventory
r renovating cost per unit of an overage item
A acquisition cost per new customer
n number of new customers acquired
T cycle length
$\tilde{D}$ Fuzzy demand per unit time
$\tilde{h}$ Fuzzy holding cost per unit per unit of time in the maximum stock
$\tilde{h}_0$ Fuzzy holding cost per unit of time in the special inventory
$\tilde{A}$ Fuzzy acquisition cost per new customer
$\tilde{r}$ Fuzzy renovating cost per unit of an overage item

6.Mathematical Model:

6.1. Crisp Model

Maximum Stock level to overage:

Let us consider the items that add up to overage in n^{th} cycle is expressed as the percentage (p) of the inventory level in the $(n-1)^{th}$ cycle. Overage items are nothing but the percent of accumulated items in the $(n-1)^{th}$ cycle which increases the maximum stock level to overage in the n^{th} cycle.

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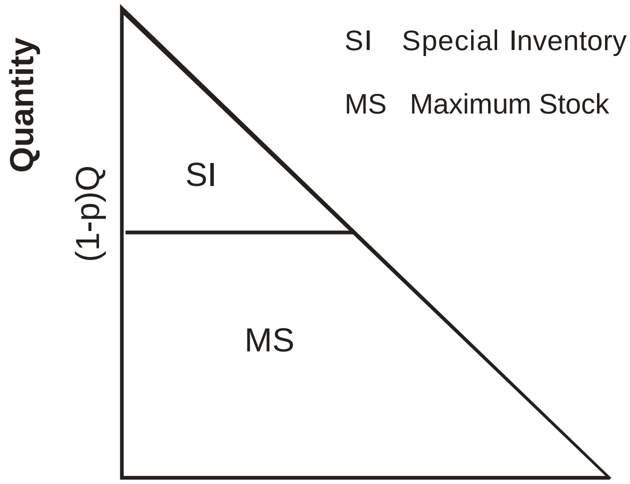
Special inventory:

As the accumulation of overage items in one cycle affects the next immediate cycle and also the consecutive cycles, the items that increase the maximum stock to overage in each cycle are segregated from the maximum stock and they are maintained as special inventory (fig.6.1). This special inventory is utilized in acquiring new customers who are worthier more than money.



Time

Fig.6.1 Special Inventory

Costs involved in the inventory:

AC(Q) = n AD/Q is the Acquisition cost per unit time .

RC(Q) = r p D is the Renovating cost

Holding cost per unit of time is $\frac{hQ}{2}+\frac{Qp^2}{2}[h_0-h]$

Total inventory cost per unit time is TCU(Q) = $\frac{KD}{Q}+\frac{nAD}{Q}+rpD+\frac{hQ}{2}+\frac{Qp^2}{2}[h_0-h]$

The Optimal order quantity Q= $\sqrt{\frac{2D[K+nA]}{h+p^2[h_0-h]}}$

6.2 Fuzzy model:

Due to uncertainity in the environment, it is not easy to define all the parameters precisely. To account for situations closer to reality it may be assumed that some of these parameters namely, D , h,h_0 ,A and r may change within some limits. Let $\tilde{D},\tilde{h},\tilde{h}_0,\tilde{A},\tilde{r}$ be Neutrosophic fuzzy trapezoidal numbers.

6.2.1 Fuzzy Differential approach on Inventory model:

Inventory level q(t) increases owing to demand rate in the interval (0,t). Hence we apply the fuzzy differential equation which representing the inventory status is

$\frac{dq(t)}{dt}+\mu q(t)=\overline{D}$, where μ is a constant .--------------------------------(1)

with the initial conditions q(0)=Q and q(T) = Q_1

The solution of the differential equation (1) is q(t) = $\frac{\overline{D}}{\mu}+ce^{-\mu t}$

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When t=0, q(0) =Q, then Q= $\frac{\overline{D}}{\mu}+c$ and c = Q – $\frac{\overline{D}}{\mu}$

Q(t) = $\frac{\overline{D}}{\mu}+(Q-\frac{\overline{D}}{\mu})e^{-\mu t}$ - ----------------------------------(2)

When q(T) = Q_1, then Q_1 = $\frac{\overline{D}}{\mu}+(Q-\frac{\overline{D}}{\mu})e^{-\mu T}$

Q_1 = Q + ($\overline{D}-\mu Q)T$ ----------------------------------(3)

Inventory during 0 to T is $q_1(t)$ = $\int_0^T q(t)dt$

$$=\int_0^T \frac{\overline{D}}{\mu}+(Q-\frac{\overline{D}}{\mu})e^{-\mu t}dt$$

On simplification, we get $q_1(t)$ = $QT-\frac{1}{2}Q\mu T^2+\frac{\overline{D}}{2}T^2$ ------------------(4)

Now , we calculate the holding cost.

Holding cost in maximum inventory level is = $\frac{\overline{h}(1-p^2)}{2}q_1(t)$

$$=\frac{\overline{h}(1-p^2)}{2}(QT-\frac{1}{2}Q\mu T^2+\frac{\overline{D}}{2}T^2) \text{ -------(5)}$$

Holding cost in the special inventory level is = $\frac{\overline{h}_o p^2}{2}q_1(t)$

$$=\frac{\overline{h}_o p^2}{2}(QT-\frac{1}{2}Q\mu T^2+\frac{\overline{D}}{2}T^2) \text{ ------------(6)}$$

Acquisition cost for n intervals is n $\overline{A}$ and Ordering cost is K

The total cost function for the special inventory is

TCF = $\frac{1}{T}$ { (K+n $\overline{A}$) + $\frac{\overline{h}(1-p^2)}{2}(QT-\frac{1}{2}Q\mu T^2+\frac{\overline{D}}{2}T^2+\frac{\overline{h}_o p^2}{2}(QT-\frac{1}{2}Q\mu T^2+\frac{\overline{D}}{2}T^2)|$

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$$= \frac{(K+n\overline{A})}{T} + \left(\frac{\overline{h}(1-p^2)+\overline{h}_o p^2}{2}\right)\left(Q-\frac{1}{2}Q\mu T + \frac{\overline{D}}{2}T\right)$$

Since $\frac{\partial TCF}{\partial T} = 0 and \frac{\partial^2 TCF}{\partial T^2} > 0$

$$-\frac{(K+n\overline{A})}{T^2} + \left(\frac{\overline{h}(1-p^2)+\overline{h}_o p^2}{2}\right)\left(-\frac{1}{2}Q\mu + \frac{\overline{D}}{2}\right) = 0 \quad \text{----------------------------------(7)}$$

From (7), we get the optimal solution T is T= $\sqrt{\frac{2(K+n\overline{A})}{[\overline{h}(1-p^2)+\overline{h}_o p^2](\overline{D}-Q\mu)}}$ ----------(8)

6.2.2. Fuzzy differential Neutrosophic Geometric programming approach:

Let D'^N , h'^N ,$h_0'^N$, A'^N are Neutrosophic differential trapezoidal fuzzy number, then

$$TCF = \frac{1}{T}\{ (K+n\overline{A}'^N) + \frac{\overline{h}'^N(1-p^2)}{2}\left(QT-\frac{1}{2}Q\mu T^2+\frac{\overline{D}}{2}T^2\right) + \frac{\overline{h}'^N{}_o\, p^2}{2}\left(QT-\frac{1}{2}Q\mu T^2+\frac{\overline{D}}{2}T^2\right)\}$$

Using Geometric programming technique with degree of difficulty is zero,

$$\text{Max } G(w) = \prod_{r=1}^{n}\left(\frac{K+n\overline{A}'^N}{Tw_{1r}}\right)^{w^{1r}} * \left(\frac{\overline{h}'^N(1-p^2)+h_0'^N\, p^2)Q}{2w_{2r}}\right)^{w_{2r}} * \left(\frac{\overline{h}'^N(1-p^2)+h_0'^N\, p^2)(\overline{D}'^N-\mu Q)T}{2w_{3r}}\right)^{w_{3r}}$$

With normality and orthogonality conditions,

$w_{1r} + w_{2r} + w_{3r} = 1$ -----------------(i)

$-w_{1r} + w_{3r} = 0$, $w_{3r} = 0$ -----------------(ii)

From (i) , (ii) , we get , $w_{1r} = 1$, $w_{2r} = 0$, $w_{3r} = 0$

Using primal and dual relationship we get ,

$$\frac{K+n\overline{A}'^N}{T} = w_{1r}\ g(w_{1r}, w_{2r}, w_{3r}) \quad \text{---------------------------------(iii)}$$

$$\frac{\overline{h}'^N(1-p^2)+h_0'^N\, p^2)(\overline{D}'^N-\mu Q)T}{2} = w_2\ g(w_{1r}, w_{2r}, w_{3r}) \quad \text{----------(iv)}$$

From (iii) and (iv) , we get $T^2 = \dfrac{(K+n\overline{A}'^N)}{\dfrac{\overline{h}'^N(1-p^2)+h_0'^N\, p^2)(\overline{D}'^N-\mu Q)}{2}}$

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Hence, T = $\sqrt{\dfrac{2(K+n\overline{A'}^{N})}{\overline{h'}^{N}(1-p^{2})+h_{0}'^{N}\,p^{2})(\overline{D'}^{N}-\mu Q)}}$

NUMERICAL VALIDATION:

Consider an inventory system with following parametric values:

7.1 Crisp Model:
Demanad rate D= 50000 units/year
Ordering cost K= 100/cycle
Holding cost for maximum stock, h= 4 / unit/year
Holding cost for special inventory, h_0 = 5/ unit/year
Cost of acquiring new customers, A = 30/customer
Number of new customers acquired, say n=2
Renovation cost per item in overage, r = 5/item
Q = 10 units and p = 0.1,µ= 0.99
Economic order quantity Q = 2000 units

7.2 Fuzzy Differential approach on Inventory model:

Let $\tilde{D}, \tilde{h}, \tilde{h}_0, \tilde{A}, \tilde{r}$ be Trapezoidal fuzzy numbers

$$\tilde{D} = (48000, 49000, 52000, 53000)$$
$$K = 100$$
$$\tilde{h} = (2,3,5,6)$$
$$\tilde{h}_0 = (1,3,7,9)$$
$$, n = 2$$
$$\tilde{A} = (20,25,35,40)$$
$$\tilde{r} = (3,4,6,7)$$

Using Signed Distance Method ,

$$T = \sqrt{\frac{2(K+n\overline{A})}{[\overline{h}(1-p^{2})+\overline{h}_{o}\,p^{2}](\overline{D}-Q\mu)}}$$

T = 0.03162 units

Q_1 = Q + ($\overline{D}-\mu Q)T$ = 1590.85

Economic order Quantity Q_1 = 1590.85

7.3 Fuzzy differential Neutrosophic Geometric programming approach:

D= 50000 $\overline{D}$= (49600 , 49800 50200 , 50400)

D'^{N} = (49600,49800,50200,50400))(49300,49600,50300,50600) (49400,49800,50200,50600)

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$(T^L, T^U) = (49700,50300)$, $(I^L,I^U) = (49450,50450)$, $(F^L, F^U) = (49600,50400)$

By using Score function of fuzzy number, $S(\overline{D}'^N) = 50051$

ho = 5, $\overline{h} = (4.8, 4.9, 5.1, 5.2)$ $h_o'^N = (4.8, 4.9, 5.1, 5.2)(4.8, 4.9, 4.9, 5.1)(4.5, 4.9, 5.1, 5.5)$

$(T^L, T^U) = (4.85,5.15)$, $(I^L,I^U) = (5,5.85)$, $(F^L, F^U) = (4.7,5.3)$ and $S(\overline{h}_0'^N) = 5.575$

h = 4, $\overline{h} = (3.8, 3.9, 4.1, 4.2)$ $h'^N = (3.8, 3.9, 4.1, 4.2)(3.8, 3.9, 4.9, 4.1)(3.5, 3.9, 4.1, 4.5)$

$(T^L, T^U) = (3.85,4.15)$, $(I^L,I^U) = (4,4.85)$, $(F^L, F^U) = (3.7,4.3)$ and $S(\overline{h}'^N) = 4.575$

A=30, $\overline{A} = (28,29,31,32)(26,28,34,36)(25,29,31,35)$

$(T^L,T^U) = (28.5,31.5)$, $(I^L,I^U) = (27,35)$, $(F^L,F^U) = (27,33)$ AND $S(\overline{A}'^N)= 30$

$$T = \sqrt{\frac{2(K + n\overline{A}'^N)}{\overline{h}'^N(1-p^2) + h_0'^N\, p^2)(\overline{D}'^N - \mu Q)}}$$

T= 0.03162 units

Q_1 =1590.85

Economic order Quantity Q_1 = 1590.85 units

Conclusion:

In this paper, the first order differential equation involving neutrosophic fuzzy numbers have been solved. To solve this equation , the signed distance and Score function of the Neutrosophic numbers were used. The total cost is minimized as an optimal control problem. To avoid the overage item , it helpful for the business organization to decide how to minimize the total inventory cost for any company.. Geometric programming technique is used for solving non-linear equations for finding the optimal result for the objective function. The proposed model is the concept of customer acquisition in the special inventory level to the overage items with the additional cost parameters. The solution technique is based on a fuzzy differential equation which is formulated for the inventory control system. The eoq in crisp model and fuzzy model have been compared and fuzzy model gives the minimum inventory cost.

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