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Optimization Of Octagonal Fuzzy Number in Eoq Model Using Lagrangian Method with Beta Distribution

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ABSTRACT

An Economic Order Quantity is used in this research to apply the inventory model to an uncertain environment. To handle an inventory problem with octagonal fuzzy numbers in EOQ, various writers have proposed various solutions. Here, we made the assumption that the ratio of demand to returned goods is consistent over cycles. and shortages are permitted. Demand, Order Quantity, Ordering Cost, Holding Cost, and Carrying Cost are all taken into consideration as octagonal fuzzy numbers. The Lagrangian approach with beta distribution is used for defuzzification to identify the optimal order quantity in order to reduce the average total cost.

Keywords: Octagonal fuzzy number, Beta distribution, Lagrangian method

I. INTRODUCTION

Operational research is the study of a system's behaviour through logical analysis and analytical techniques. OR includes a variety of problem-solving approaches and methodologies used to improve decision-making and efficiency and Its applications are used in science to deal with complex numbers and large systems, as well as to analyse problems and solutions in terms of economics.

By 1915, inventories had been implemented. It consists of products in more stock components for potential production and sales. An inventory system keeps track of inventory levels and sets order timelines and quantities.

Fuzzy set theory is primarily concerned with how to quantitatively deal with imprecision and uncertainty and it is most beneficial for real world problems that involve the computation of hidden knowledge.In the previous period, probability theory was used to manage the uncertainties in inventory models, which were viewed as random events.

Since L.A. Zadeh first proposed the fuzzy set theory, it has been used in inventory control systems to better accurately simulate behaviour. Fuzzy numbers can be calculated using Chen's function principle, and optimization is done using the Lagrangian approach. For defuzzification of the annual integrated total cost for EOQ, graded mean integration is employed. Harris and Wilson researched stock management and ideal costs as they created the strategy. Zimmerman examined the fuzzy set theory is used in a wide variety of contexts. The octagonal fuzzy number model for transportation issues was created by S. U. Malini and Felbin C. Kennedy. In their new approach, S. Gajalakshmi and P. Parvathi used octagonal fuzzy numbers to resolve a fuzzy inventory problem.

In this paper to deals with octagonal fuzzy numbers using a Lagrangian method with beta distribution and we were able to reduce the total cost and increase the optimum time period.

II. OCTAGONAL FUZZY NUMBERS

A fuzzy number A is said to be a generalized octagonal fuzzy number denoted by $\widetilde{A_T} = (s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8)$ where s_1 , $s_2, s_3, s_4, s_5, s_6, s_7, s_8$ are real numbers and its membership function χ_A is given by,

$$\chi_A(x) = \begin{cases} 0, & for\ t < s_1 \\ m\left(\dfrac{t - s_1}{s_2 - s_1}\right), & for\ s_1 \le t \le s_2 \\ m, & for\ s_2 \le t \le s_3 \\ m + (l - m)\left(\dfrac{t - s_3}{s_4 - s_2}\right), & for\ s_3 \le t \le s_4 \\ m, & for\ s_4 \le t \le s_5 \\ m + (l - m)\left(\dfrac{s_6 - t}{s_6 - s_5}\right), & for\ s_5 \le t \le s_6 \\ m, & for\ s_6 \le t \le s_7 \\ m + (l - m)\left(\dfrac{s_8 - t}{s_8 - s_7}\right), & for\ s_7 \le t \le s_8 \\ 0, & for\ t \ge s_8 \end{cases}$$

Graphical representation in octagonal number,

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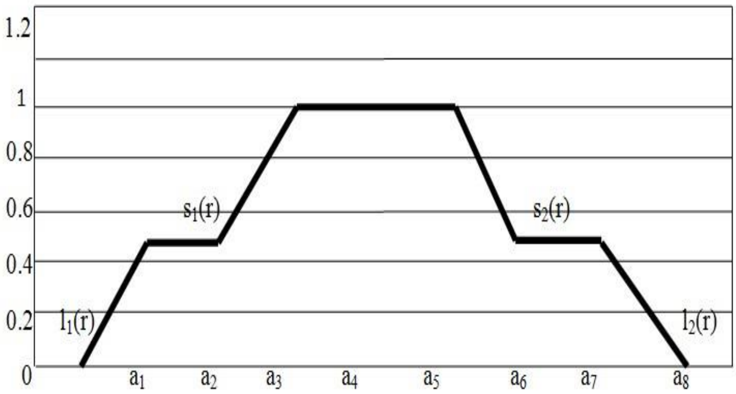

α $cuts$: To find $\propto$ cut of a normal octagonal fuzzy number
$\bar{S} = (s_1, s_2, s_3, s_4, s_5, s_6, s_{7,}s_8)$

$$\begin{cases} \left[\left(s_1 + \frac{s}{k}\right)(s_2 - s_1)\left(s_8 - \frac{s}{k}\right)(s_8 - s_7)\right] & for \propto \in [0, k] \\ [s_3 + \left(\frac{s-k}{1-k}\right)(s_4 - s_3)(s_6 - \frac{s-k}{1-k})(s_6 - s_3)] & for \propto \in [k, 1] \end{cases}$$

Arithmetical Operations on Octagonal Fuzzy Numbers

Let (A^a, A^b) $\&(B^c B^d)$ $are\ the\ two\ terms$ $(A^{ab1}, A^{ab2}, A^{ab3}, A^{ab4}, A^{ab5}, A^{ab6}, A^{ab7}, A^{ab8})$

and $(B^{cd1}, B^{cd2}, B^{cd3}, B^{cd4}, B^{cd5}, B^{cd6}, B^{b7}, B^{b8})$

Addition :

$(A^a, A^b) \oplus (B^c, B^d) = (A^{ab1}, A^{ab2}, A^{ab3}, A^{ab4}, A^{ab5}, A^{ab6}, A^{ab7}, A^{ab8}) +$
$(B^{cd1}, B^{cd2}, B^{cd3}, B^{cd4}, B^{cd5}, B^{cd6}, B^{cd7}, B^{cd8})$ are any real numbers.

Subtraction :

$(A^a, A^b) \ominus (B^c, B^d) = (A^{ab1}, A^{ab2}, A^{ab3}, A^{ab4}, A^{ab5}, A^{ab6}, A^{ab7}, A^{ab8}) -$
$(B^{cd1}, B^{cd2}, B^{cd3}, B^{cd4}, B^{cd5}, B^{cd6}, B^{cd7}, B^{cd8})$ are any real numbers.

Multiplication:

$(A^a, A^b) \otimes (B^c, B^d) = (A^{ab1}, A^{ab2}, A^{ab3}, A^{ab4}, A^{ab5}, A^{ab6}, A^{ab7}, A^{ab8}) \times$
$(B^{cd1}, B^{cd2}, B^{cd3}, B^{cd4}, B^{cd5}, B^{cd6}, B^{cd7}, B^{cd8})$ are any zero positive real numbers.

Division:

$(A^a, A^b) \oslash (B^c, B^d) =(A^{ab1}, A^{ab2}, A^{ab3}, A^{ab4}, A^{ab5}, A^{ab6}, A^{ab7}, A^{ab8})/$
$(B^{cd1}, B^{cd2}, B^{cd3}, B^{cd4}, B^{cd5}, B^{cd6}, B^{cd7}, B^{cd8})$ are all non-zero positive real numbers.

Defuzzification methods for octagonal fuzzy numbers

GRADED MEAN INTEGRATION METHOD :

Let $\tilde{A}$ be a fuzzy set defined on G. The graded mean integration method for Defuzzifying octagonal fuzzy number $\tilde{A} = (A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8)$ is defined as
Let $\tilde{A}$ be a generalized octagonal fuzzy number.

$$\mathrm{R(A)} = \frac{\frac{1}{4}\int_0^m xV_1^{-1}(x)\,dx}{\int_0^m x\,dx} + \frac{\frac{1}{4}\int_m^l xV_1^{-1}(x)\,dx}{\int_m^l x\,dx} + \frac{\frac{1}{4}\int_l^m xH_1^{-1}(x)\,dx}{\int_l^m x\,dx} + \frac{\frac{1}{4}\int_m^0 xH_1^{-1}(x)dx}{\int_m^0 x\,dx}$$

$$= \frac{1}{2}\left[\frac{A_1+A_3+A_6+A_8}{2} + \frac{A_2-A_1+A_7-A_8}{3} + \frac{A_4-A_3}{l-m}\left(\frac{l^2+ml+m^2}{3(l+m)} - \frac{m}{2}\right) + \frac{A_6-A_5}{l-m}\left(\frac{m}{2} - \frac{m^2+l^2+ml}{l+m}\right)\right]$$

If m $= \frac{1}{2}, l = 1$, then

$$D_X(\tilde{A}) = \frac{3A_1+6A_2+4A_3+5A_4+5A_5+4A_6+6A_7+3A_8}{36}$$

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LAGRANGIAN METHOD

In mathematical optimization, the method of Lagrange multipliers is a strategy for locating the neighbourhood maxima and minima of a feature issue to equality constraints (i.e., problem to the condition that one or greater equations must be glad exactly through the selected values of the variables).

Beta Distribution of octagonal fuzzy number

$$\beta(\tilde{E}) = \frac{3a_1 + 6a_{2+}4a_3 + 5a_4 + 5a_5 + 4a_6 + 6a_7 + 3a_8}{36}$$

NOTATIONS:

G = Order quantity for materials
H = Inventory objects for total demand
S = Holding cost per month
P = Rotation plan per cycle
R = Carrying cost per cycle
TC = Total cost
$\tilde{G}$ = Fuzzy Order quantity for materials
$\tilde{H}$ = Fuzzy Inventory objects for total demand
$\tilde{S}$ = Fuzzy Holding cost per month
$\tilde{P}$ = Fuzzy Rotation plan per cycle
$\tilde{R}$ = Fuzzy Carrying cost per cycle
$\widetilde{TC}$ = Fuzzy Total cost.

ASSUMPTION:

1) The proportion of the demand and returned product are constant per cycle.
2) Shortage is allowed.
3) Time is dependent.
4) Backorders will change according to the holding cost.

III. MATHEMATICAL MODEL FOR CRISP SENSE

TC = $\frac{GH}{S} + \frac{SPR}{2}$

Differentiate partially w.r.to. S then we get, $S^* = \sqrt{\frac{2GH}{PR}}$

Fuzzy total cost, $\widetilde{TC} = \frac{\tilde{G}\tilde{H}}{S} + \frac{S\tilde{P}\tilde{R}}{2}$

Defuzzifying for octagonal fuzzy number,

$$\beta(\tilde{E}) = \frac{3a_1+6a_{2+}4a_3+5a_4+5a_5+4a_6+6a_7+3a_8}{36}$$

$$S_1 = \sqrt{\frac{2[3(G_{a8}H_{C8})+6(G_{a7}H_{C7})+4\,(G_{a6}H_{c6})+5(G_{a5}H_{c5})+5(G_{a4}H_{c4})+4(G_{a3}H_{c3})+6(G_{a2}H_{c2})+3(G_{a1}H_{c1})]}{3(P_{b1}R_{d1})+6(P_{b2}R_{d2})+4(P_{b3}R_{d3})+5(P_{b4}R_{d4})+5(P_{b5}R_{d5})+4(P_{b6}R_{d6})+6(P_{b7}R_{d7})+3(P_{b8}R_{d8})}}$$

$$T\tilde{C} = \Big[\frac{Ga_1Hc_1}{S}\frac{SPb_1Rd_1}{2}, \frac{Ga_2Hc_2}{S}\frac{SPb_2Rd_2}{2}, \frac{Ga_3Hc_3}{S} + \frac{SPb_3Rd_3}{2}, \frac{Ga_4Hc_4}{S} + \frac{SPb_4Rd_4}{2},$$
$$\frac{Ga_5Hc_5}{S} + \frac{SPb_5Rd_5}{2}, \frac{Ga_6Hc_6}{S} + \frac{SPb_6Rd_6}{2}, \frac{Ga_7Hc_7}{S} + \frac{SPb_7Rd_7}{2}, \frac{Ga_8Hc_8}{S} + \frac{SPb_8Rd_8}{2}\Big]$$

STEP 1: $S_1 = \sqrt{\frac{2[3(G_{a8}H_{c8})]}{P_{b1}R_{d1}}}$

STEP 2 : $S_1 = S_2$, S_2= $\sqrt{\frac{2[3(G_{a8}H_{c8})+6(G_{a7}H_{C7})]}{3(P_{b1}R_{d1})+6(P_{b2}R_{d2})}}$

STEP 3: $S_1 = S_2 = S_3$, S_3=$\sqrt{\frac{2[3(G_{a8}H_{c7}\ )+6(G_{a7}H_{c7})+4(G_{a6}H_{c6})]}{3(P_{b1}R_{d1})+6(P_{b2}R_{d2})+4(P_{b3}R_{d3})}}$

By continuing this steps, then we get as

$$S_1 = S_2 = S_3 = S_4 = S_5 = S_5 = S_6 = S_7 = S_8 = S_9 = S_{10}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 187 - 195
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$$S_1 = \sqrt{\frac{2[3(G_{a8}H_{C8})+6(G_{a7}H_{C7})+4\,(G_{a6}H_{c6})+5(G_{a5}H_{c5})+\,5(G_{a4}H_{c4})+4(G_{a3}H_{c3})+6(G_{a2}H_{c2})+3(G_{a1}H_{c1})]}{3(P_{b1}R_{d1})+6(P_{b2}R_{d2})+4(P_{b3}R_{d3})+5(P_{b4}R_{d4})+5(P_{b5}R_{d5})+4(P_{b6}R_{d6})+6(P_{b7}R_{d7})+3(P_{b8}R_{d8})}}$$

We find the numerical solution by the fuzzy and crisp both are same or closer one another.

IV. OPTIMIZATION OF FUZZY INVENTORY MODEL USING BETA DISTRIBUTION

$$TC = \frac{GH}{S} + \frac{SPR}{2}$$

Differentiate partially w.r.to S equals to 0

$$\frac{\partial TC}{\partial S} = \frac{-GH}{S^2} + \frac{PR}{2}$$

$$S^* = \sqrt{\frac{2GH}{PR}}$$

$$\widetilde{TC} = \frac{\tilde{G}\tilde{H}}{S} + \frac{S\tilde{P}\tilde{R}}{2}$$

$\tilde{G} = (Ga_1, Ga_2, Ga_3, Ga_4, Ga_5, Ga_6, Ga_7, Ga_8)$,

$\tilde{P} = (Pb_1, Pb_2, Pb_3, Pb_4, Pb_5, Pb_6, Pb_7, Pb_8)$

$\tilde{H} = (Hc_1, Hc_2, Hc_3, Hc_4, Hc_5, Hc_6, Hc_7, Hc_8)$

$\tilde{R} = (Rd_1, Rd_2, Rd_3, Rd_4, Rd_5, Rd_6, Rd_7, Rd_8)$

Are non-negative octagonal fuzzy numbers

$$\widetilde{TC} = \frac{1}{S}[Ga_1Hc_1] + \frac{S}{2}[Pb_1Rd_1], \frac{1}{S}[Ga_2Hc_2] + \frac{S}{2}[Pb_2Rd_2],$$
$$\frac{1}{S}[Ga_3Hc_3] + \frac{S}{2}[Pb_3Rd_3], \frac{1}{S}[Ga_4Hc_4] + \frac{S}{2}[Pb_4Rd_4],$$
$$\frac{1}{S}[Ga_5Hc_5] + \frac{S}{2}[Pb_5Rd_5], \frac{1}{S}[Ga_6Hc_6] + \frac{S}{2}[Pb_6Rd_6],$$
$$\frac{1}{S}[Ga_7Hc_7] + \frac{S}{2}[Pb_7Rd_7], \frac{1}{S}[Ga_8Hc_8] + \frac{S}{2}[Pb_8Rd_8]$$

Defuzzifying of graded mean integration in octagonal fuzzy number

$$\beta(\tilde{E}) = \frac{3a_1 + 6a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 6a_7 + 3a_8}{36}$$

$$\beta(\widetilde{TC}) = \frac{1}{36S}[3[Ga_1Hc_1] + 6[Ga_2Hc_2] + 4[Ga_3Hc_3] + 5[Ga_4Hc_4] + 5[Ga_5Hc_5] + 4[Ga_6Hc_6] + 6[Ga_7Hc_7] + 3[Ga_8Hc_8] + \frac{S}{72}[3[Pb_1Rd_1] + 6[Pb_2Rd_2] + 4[Pb_3Rd_3] + 5[Pb_4Rd_4] + 5[Pb_5Rd_5] + 4[Pb_6Rd_6] + 6[Pb_7Rd_7] + 3[Pb_8Rd_8]]$$

Differentiate partially w.r.to S and equate to 0.

$$\frac{\partial\beta}{\partial S}(T\tilde{C}) = -\frac{1}{36S^2}[3(Ga_1Hc_1) + 6(Ga_2Hc_2) + 4(Ga_3Hc_3) + 5[Ga_4Hc_4] + 5[Ga_5Hc_5] + 4[Ga_6Hc_6] + 6[Ga_7Hc_7] + 3[Ga_8Hc_8] + \frac{S}{72}[3[Pb_1Rd_1] + 6[Pb_2Rd_2] + 4[Pb_3Rd_3] + 5[Pb_4Rd_4] + 5[Pb_5Rd_5] + 4[Pb_6Rd_6] + 6[Pb_7Rd_7] + 3[Pb_8Rd_8]]$$

$$S^2 = \frac{2[3(Ga_1Hc_1)+6(Ga_2Hc_2)+4(Ga_3Hc_3)+5(Ga_4Hc_4)+(Ga_5Hc_5)+4(Ga_6Hc_6)+6(Ga_7Hc_7)+3(Ga_8Hc_8)}{3(Pb_1Rd_1)+6(Pb_2Rd_2)+4(Pb_3Rd_3)+5(Pb_4Rd_4)+5(Pb_5Rd_5)+4(Pb_6Rd_6)+6(Pb_7Rd_7)+3(Pb_8Rd_8)}$$

$$S = \sqrt{\frac{2[3(Ga_1Hc_1) + 6(Ga_2Hc_2) + 4(Ga_3Hc_3) + 5(Ga_4Hc_4) + 5(Ga_5Hc_5) + 4(Ga_6Hc_6) + 6(Ga_7Hc_7) + 3(Ga_8Hc_8)}{3(Pb_1Rd_1) + 6(Pb_2Rd_2) + 4(Pb_3Rd_3) + 5(Pb_4Rd_4) + 5(Pb_5Rd_5) + 4(Pb_6Rd_6) + 6(Pb_7Rd_7) + 3(Pb_8Rd_8)}}$$

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 187 - 195
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LAGRAGEAN METHOD

Total cost $\widetilde{TC} = \frac{\widetilde{GH}}{S} + S\frac{\widetilde{PR}}{2}$ with $S = (S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8)$

$$0 < S_1 < S_2 < S_3 < S_4 < S_5 < S_6 < S_7 < S_8$$

$$\widetilde{TC} = [\frac{G_{a1}H_{C1}}{S_8} + \frac{S_1P_{b1}R_{d1}}{2}, \frac{G_{a2}H_{c2}}{S_7} + \frac{S_2P_{b1}R_{d2}}{2}, \frac{Ga_3Hc_3}{S_6} + \frac{S_3Pb_3Rd_3}{2}, \frac{Ga_4Hc_4}{S_5} + \frac{S_4Pb_4Rd_4}{2},$$
$$\frac{Ga_5Hc_5}{S_4} + \frac{S_5Pb_5Rd_5}{2}, \frac{Ga_6Hc_6}{S_3} + \frac{S_6Pb_6Rd_6}{2}, \frac{Ga_7Hc_7}{S_2} + \frac{S_7Pb_7Rd_7}{2}, \frac{Ga_8Hc_8}{S_1} + \frac{S_8Pb_8Rd_8}{2}]$$

Apply Beta distribution method,

$$\beta(\widetilde{TC}) = \frac{1}{36}\left[3\left(\frac{Ga_1Hc_1}{S_8} + \frac{S_1Pb_1Rd_1}{2}\right) + 6\left(\frac{Ga_2Hc_2}{S_7} + \frac{S_2Pb_2Rd_2}{2}\right) + 4\left(\frac{Ga_3Hc_3}{S_6} + \frac{S_3Pb_3Rd_3}{2}\right) + 5\left(\frac{Ga_4Hc_4}{S_5} + \frac{S_4Pb_4Rd_4}{2}\right) + 5\left(\frac{Ga_5Hc_5}{S_4} + \frac{S_5Pb_5Rd_5}{2}\right) + 4\left(\frac{Ga_6Hc_6}{S_3} + \frac{S_6Pb_6Rd_6}{2}\right) + 6\left(\frac{Ga_7Hc_7}{S_2} + \frac{S_7Pb_7Rd_7}{2}\right) + 3\left(\frac{Ga_8Hc_8}{S_1} + \frac{S_8Pb_8Rd_8}{2}\right)\right]$$

Step 1:

Differentiate partially W.r.to $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8$ and equal to 0

$$\frac{\partial\widetilde{TC}}{\partial S_1} = 0 \Rightarrow \frac{1}{36}\left[3\left(\frac{Ga_8Hc_8}{S_1} + \frac{S_1Pb_1Rd_1}{2}\right)\right] = 0 \quad \Rightarrow S_1 = \sqrt{\frac{2Ga_8Hc_8}{Pb_1Rd_1}}$$

$$\frac{\partial\widetilde{TC}}{\partial S_2} = 0 \Rightarrow \frac{1}{36}\left[6\left(\frac{Ga_7Hc_7}{S_7} + \frac{S_2Pb_2Rd_2}{2}\right)\right] = 0 \Rightarrow S_2 = \sqrt{\frac{2(Ga_7Hc_7)}{Pb_2Rd_2}}$$

$$\frac{\partial\widetilde{TC}}{\partial S_3} = 0 \Rightarrow \frac{1}{36}[4\left(\frac{S_3P_{b3}R_{d3}}{2}\right) + \frac{G_{a6}H_{c6}}{S_3}] = 0 \Rightarrow S_3 = \sqrt{\frac{2(Ga_6Hc_6)}{Pb_3Rd_3}}$$

$$\frac{\partial\widetilde{TC}}{\partial S_4} = 0 \Rightarrow \frac{1}{36}\left[5\left(\frac{S_4Pb_4Rd_4}{2}\right) + 5\left(\frac{Ga_54c_5}{S_4}\right)\right] = 0 \Rightarrow S_4 = \sqrt{\frac{2(Ga_5Hc_5)}{Pb_4Rd_4}}$$

$$\frac{\partial\widetilde{TC}}{\partial S_5} = 0 \Rightarrow \frac{1}{36}\left[5\left(\frac{S_5Pb_5Rd_5}{2}\right) + 5\left(\frac{Ga_44c_4}{S_5}\right)\right] = 0 \Rightarrow S_5 = \sqrt{\frac{2(Ga_4Hc_4)}{Pb_5Rd_5}}$$

$$\frac{\partial\widetilde{TC}}{\partial S_6} = 0 \Rightarrow \frac{1}{36}\left[4\left(\frac{S_6Pb_6Rd_6}{2}\right) + 4\left(\frac{Ga_34c_3}{S_6}\right)\right] = 0 \Rightarrow S_6 = \sqrt{\frac{2(Ga_3Hc_3)}{Pb_6Rd_6}}$$

$$\frac{\partial\widetilde{TC}}{\partial S_7} = 0 \Rightarrow \frac{1}{36}\left[6\left(\frac{S_7Pb_7Rd_7}{2}\right) + 6\left(\frac{Ga_24c_2}{S_7}\right)\right] = 0 \Rightarrow S_7 = \sqrt{\frac{2(Ga_2Hc_2)}{Pb_7Rd_7}}$$

$$\frac{\partial\widetilde{TC}}{\partial S_8} = 0 \Rightarrow \frac{1}{36}\left[6\left(\frac{S_8Pb_8Rd_8}{2}\right) + 3\left(\frac{Ga_1Hc_1}{S_8}\right)\right] = 0 \Rightarrow S_8 = \sqrt{\frac{2(Ga_1Hc_1)}{Pb_8Rd_8}}$$

Here $S_8 > S_7 > S_6 > S_5 > S_4 > S_3 > S_2 > S_1$ it does not satisfy the local optimum.

Step 2 :

Fix the constraint one $\lambda_1(S_2 - S_1)$and equate to 0

$$\beta(T\tilde{C}) = \frac{1}{36}\left[3\left(\frac{Ga_1Hc_1}{S_8} + \frac{S_1Pb_1Rd_1}{2}\right) + 6\left(\frac{Ga_2Hc_2}{S_7} + \frac{S_2Pb_2Rd_2}{2}\right) + 4\left(\frac{Ga_3Hc_3}{S_6} + \frac{S_3Pb_3Rd_3}{2}\right)\right.$$
$$+5\left(\frac{Ga_4Hc_4}{S_5} + \frac{S_4Pb_4Rd_4}{2}\right) + 5\left(\frac{Ga_5Hc_5}{S_4} + \frac{S_5Pb_5Rd_5}{2}\right) + 4\left(\frac{Ga_6Hc_6}{S_3} + \frac{S_6Pb_6Rd_6}{2}\right)$$
$$\left. + 6(\frac{G_{a7}H_{c7}}{S_2} + \frac{S_7P_{b7}R_{d7}}{2}) + 3(\frac{G_{a8}H_{c8}}{S_1} + \frac{S_8P_{b6}R_{d8}}{2}) + \lambda_2(S_2\text{-}S_1)\right]$$

Differentiate w.r to S_1, S_2 and λ_1, λ_2 and equate to 0.

$$\frac{\partial\widetilde{TC}}{\partial S_1} = 0 \Rightarrow \frac{1}{36}\left[3\left(\frac{Pb_1Rd_1}{2}\right) - 3\left(\frac{Ga_8Hc_8}{{S_1}^2}\right)\right] - \lambda = 0 \ldots\ldots (1)$$

$$\frac{\partial\widetilde{TC}}{\partial S_1} = 0 \Rightarrow \frac{1}{36}[6\left(\frac{Pb_2Rd_2}{2}\right) - 6\left(\frac{Ga_7Hc_7}{{S_2}^2}\right) + \lambda = 0 \ldots\ldots. (2)$$

Adding 1 and 2 we get

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 187 - 195
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$$=\frac{1}{36}\left[\tfrac{3}{2}\left(Pb_1Rd_1\right)-3\left(\frac{Ga_8Hc_8}{S_1^2}\right)+\tfrac{6}{2}\left(Pb_2Rd_2\right)-6\left(\frac{Ga_7Hc_7}{S_2^2}\right)=0\right.$$

$$\frac{\partial\widetilde{TC}}{\partial S_1}=0\Rightarrow S_2-S_1=0\Rightarrow S_2=S_1$$

$$S_1=\sqrt{\frac{2[3(4a_8Hc_8)+6(Ga_7Hc_7)]}{3(Pb_1Rd_1)+6(Pb_2Rd_2)}}$$

Step 3 :

Fix the constraints as $\lambda_1(S_2-S_1)+\lambda_2(S_3-S_2)$and equate to 0.

$$\beta(\widetilde{TC})=\frac{1}{36}\left[3\left(\frac{Ga_1Hc_1}{S_8}+\frac{S_1Pb_1Rd_1}{2}\right)+6\left(\frac{Ga_2Hc_2}{S_7}+\frac{S_2Pb_2Rd_2}{2}\right)\right.$$
$$+4\left(\frac{Ga_3Hc_3}{S_6}+\frac{S_3Pb_3Rd_3}{2}\right)+5\left(\frac{Ga_4Hc_4}{S_5}+\frac{S_4Pb_4Rd_4}{2}\right)+5\left(\frac{Ga_5Hc_5}{S_4}+\frac{S_5Pb_5Rd_5}{2}\right)$$
$$+4\left(\frac{Ga_6Hc_6}{S_3}+\frac{S_6Pb_6Rd_6}{2}\right)+6\left(\frac{Ga_7Hc_7}{S_2}+\frac{S_7Pb_7Rd_7}{2}\right)+3\left(\frac{Ga_8Hc_8}{S_1}+\frac{S_8Pb_8Rd_8}{2}\right)$$
$$\left.+\lambda_1(S_2,S_1)+\lambda_2(S_3-S_2)\right]$$

Differentiate partially w.r to S_1, S_2, S_3 and $\lambda_1, \lambda_2, \lambda_3$ and equate to

$$\frac{\partial\widetilde{TC}}{\partial S_1}=0\Rightarrow\frac{1}{36}\left[3\left(\frac{Pb_1Rd_1}{2}\right)-3\left(\frac{Ga_8Hc_8}{S_1{}^2}\right)\right]-\lambda_1=0\ \ldots\ldots\ldots\ (3)$$

$$\frac{\partial\widetilde{TC}}{\partial S_2}=0\Rightarrow\frac{1}{36}[6\left(\frac{Pb_2Rd_2}{2}\right)-6\left(\frac{Ga_7Hc_7}{S_2{}^2}\right)+\lambda_1-\lambda_2=0\ldots\ldots\ (4)$$

$$\frac{\partial\widetilde{TC}}{\partial S_3}=0\Rightarrow\frac{1}{36}[4\left(\frac{Pb_3Rd_3}{2}\right)-4\left(\frac{Ga_6Hc_6}{S_3{}^2}\right)+\lambda_2=0\ldots\ldots\ldots..\ (5)$$

Adding 3 ,4 and 5 to get $S_2=S_1$

$$\frac{\partial\widetilde{TC}}{\partial\lambda_2}=0\Rightarrow S_3=S_2=0$$

$$S_1^2=\frac{2[3(Ga_8Hc_8)+6(Ga_7Hc_7)+4(Ga_6Hc_6)]}{3(Pb_1Rd_1)+6(Pb_2Rd_2)+4(Pb_3Rd_3)}\quad\Rightarrow\quad S_1=\sqrt{\frac{2[3(Ga_8Hc_8)+6(Ga_7Hc_7)+4(Ga_6Hc_6)]}{3(Pb_1Rd_1)+6(Pb_2Rd_2)+4(Pb_3Rd_3)}}$$

Step 4:

Fix the constraints three

$\lambda_1(S_2-S_1)+\lambda_2(S_3-S_2)+\lambda_3(S_4-S_3)$ and equate to 0.

Differentiate partially w.r to S_1, S_2, S_3, S_4 and $\lambda_1, \lambda_2, \lambda_3$ equate to 0.

$$\frac{\partial\widetilde{TC}}{\partial S_1}=0\Rightarrow\frac{1}{36}\left[3\left(\frac{Pb_1Rd_1}{2}\right)-3\left(\frac{Ga_8Hc_8}{S_1{}^2}\right)\right]-\lambda_1=0\ldots\ldots\ldots\ldots\ldots\ (6)$$

$$\frac{\partial\widetilde{TC}}{\partial S_2}=0\Rightarrow\frac{1}{36}[6\left(\frac{Pb_2Rd_2}{2}\right)-6\left(\frac{Ga_7Hc_7}{S_2{}^2}\right)+\lambda_1-\lambda_2=0\ldots\ldots\ldots\ (7)$$

$$\frac{\partial\widetilde{TC}}{\partial S_3}=0\Rightarrow\frac{1}{36}[4\left(\frac{Pb_3Rd_3}{2}\right)-4\left(\frac{Ga_6Hc_6}{S_3{}^2}\right)+\lambda_2-\lambda_3=0\ldots\ldots\ldots\ldots(8)$$

$$\frac{\partial\widetilde{TC}}{\partial S_4}=0\Rightarrow\frac{1}{36}\left[5\left(\frac{Pb_4Rd_4}{2}\right)-5\left(\frac{Ga_5Hc_5}{S_4{}^2}\right)\right]+\lambda_3=0\ldots\ldots\ldots\ldots\ (9)$$

$$\frac{\partial\widetilde{TC}}{\partial\lambda_2}=0\Rightarrow S_3-S_2=0\Rightarrow S_3=S_2$$

$$\frac{\partial\widetilde{TC}}{\partial\lambda_3}=0\Rightarrow S_4-S_3=0\Rightarrow S_4=S_3$$

$$S_1=S_2=S_3=S_4$$

Adding 6, 7, 8 and 9 to get

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Volume 13, No. 3, 2022, p. 187 - 195
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$$S_1 = \sqrt{\frac{2[3(Ga_8Hc_8)+6(Ga_7Hc_7)+4(Ga_6Hc_6)+5(Ga_5Hc_5)]}{3(Pb_1Rd_1)+6(Pb_2Rd_2)+4(Pb_3Rd_3)+5(Pb_4Rd_4)}}$$

Step 5: Fix the constrains four

$\lambda_1(S_2 - S_1) + \lambda_2(S_3 - S_2) + \lambda_3(S_4 - S_3) + \lambda_4(S_5 - S_4)$ and equal to 0.

Differentiate partially w.r. to S_1, S_2, S_3, S_4, S_5 and $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, and equal to 0

$$\frac{\partial \widetilde{TC}}{\partial S_1} = 0 \Rightarrow \frac{1}{36}\left[3\left(\frac{Pb_1Rd_1}{2}\right) - 3\left(\frac{Ga_8Hc_8}{S_1^2}\right)\right] - \lambda_1 = 0 \ldots\ldots\ldots\ldots\ldots (10)$$

$$\frac{\partial \widetilde{TC}}{\partial S_2} = 0 \Rightarrow \frac{1}{36}\left[6\left(\frac{Pb_2Rd_2}{2}\right) - 6\left(\frac{Ga_7Hc_7}{S_2^2}\right)\right] + \lambda_1 - \lambda_2 = 0 \ldots\ldots\ldots (11)$$

$$\frac{\partial \widetilde{TC}}{\partial S_3} = 0 \Rightarrow \frac{1}{36}\left[6\left(\frac{Pb_2Rd_2}{2}\right) - 4\left(\frac{Ga_7Hc_7}{S_3^2}\right)\right] + \lambda_2 - \lambda_3 = 0 \ldots\ldots\ldots. (12)$$

$$\frac{\partial \widetilde{TC}}{\partial S_4} = 0 \Rightarrow \frac{1}{36}\left[\left(\frac{Pb_4Rd_4}{2}\right) - 5\left(\frac{Ga_5Hc_5}{S_4^2}\right)\right] + \lambda_3 - \lambda_4 = 0 \ldots\ldots\ldots \quad (13)$$

$$\frac{\partial \widetilde{TC}}{\partial S_5} = 0 \Rightarrow \frac{1}{36}\left[\left(\frac{Pb_5Rd_5}{2}\right) - 5\left(\frac{Ga_4Hc_4}{S_5^2}\right)\right] + \lambda_4 = 0 \ldots\ldots\ldots\ldots\ldots .(14)$$

$$\frac{\partial \widetilde{TC}}{\partial \lambda_1} = 0 \Rightarrow S_2 - S_1 = 0 \Rightarrow S_2 = S_1$$

$$\frac{\partial \widetilde{TC}}{\partial \lambda_2} = 0 \Rightarrow S_3 - S_2 = 0 \Rightarrow S_3 = S_2$$

$$\frac{\partial \widetilde{TC}}{\partial \lambda_3} = 0 \Rightarrow S_4 - S_3 = 0 \Rightarrow S_4 = S_3$$

$$\frac{\partial \widetilde{TC}}{\partial \lambda_4} = 0 \Rightarrow S_5 - S_4 = 0 \Rightarrow S_5 = S_4$$

$$S_1 = S_2 = S_3 = S_4 = S_5$$

Adding 10, 11, 12, 13 and 14 to get

$$S_1 = \sqrt{= \frac{2[(3(Ga_8Hc_8)+6(Ga_7Hc_7)+4(Ga_6HC_6)+5(Ga_5Hc_5)+5(Ga_6HC_6))}{3(pb_1Rd_1)+6(pb_2Rd_2)+4(pb_3Rd_3)+5(pb_4Rd_4)+5(pb_5Rd_5)}}$$

Step 6 : Fix the constraints five

$\lambda_1(S_2 - S_1) + \lambda_2(S_3 - S_2) + \lambda_3(S_4 - S_3) + \lambda_4(S_5 - S_4) + \lambda_5(S_6 - S_5)$ and equate 0

Differentiate partially w.r. to $\boldsymbol{S_1, S_2, S_3, S_4, S_5, S_6}$ and $\boldsymbol{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5}$ and equal to 0

$$S_1 = \sqrt{\frac{2\left[3\left(3(Ga_8Hc_8) + 6(Ga_7Hc_7) + 4(Ga_6HC_6) + 5(Ga_5Hc_5) + 5(Ga_4HC_4) + 4(Ga_3HC_3)\right)\right.}{3(pb_1Rd_1) + 6(pb_2Rd_2) + 4(pb_3Rd_3) + 5(pb_4Rd_4) + 5(pb_5Rd_5) + 4(pb_6Rd_6)}}$$

Step 7: **F**ix the constraints six

$\lambda_1(S_2 - S_1) + \lambda_2(S_3 - S_2) + \lambda_3(S_4 - S_3) + \lambda_4(S_5 - S_4) + \lambda_5(S_6 - S_5) + \lambda_6(S_7 - S_6)$ and equate to 0.

$$S_1^2 = \sqrt{\frac{2\left(3(Ga_8Hc_8) + 6(Ga_7Hc_7) + 4(Ga_6HC_6) + 5(Ga_5Hc_5) + 5(Ga_4HC_4) + 4(Ga_3HC_3) + 6(Ga_2HC_2)\right)}{3(pb_1Rd_1) + 6(pb_2Rd_2) + 4(pb_3Rd_3) + 5(pb_4Rd_4) + 5(pb_5Rd_5) + 4(pb_6Rd_6) + 6(Pb_7Rd_7)}}$$

Step 8: Fix the constraints seven

$$\lambda_1(S_2 - S_1) + \lambda_2(S_3 - S_2) + \lambda_3(S_4 - S_3) + \lambda_4(S_5 - S_4) + \lambda_5(S_6 - S_5) + \lambda_6(S_7 - S_6) + \lambda(S_8 - S_7)$$

and equate to 0

$$S_1 = \sqrt{\frac{2[3(G_{a8}H_{C8)} + 6(G_{a7}H_{C7}) + 4(G_{a6}H_{c6}) + 5(G_{a5}H_{c5}) + 5(G_{a4}H_{c4}) + 4(G_{a3}H_{c3}) + 6(G_{a2}H_{c2}) + 3(G_{a1}H_{c1})]}{3(P_{b1}R_{d1}) + 6(P_{b2}R_{d2}) + 4(P_{b3}R_{d3}) + 5(P_{b4}R_{d4}) + 5(P_{b5}R_{d5}) + 4(P_{b6}R_{d6}) + 6(P_{b7}R_{d7}) + 3(P_{b8}R_{d8})}}$$

V. NUMERICAL EXAMPLE:

$S=\sqrt{\frac{2GH}{PR}}$ were G=1000 units , H=\$100 , P=\$200, R=\$10.

$S=\sqrt{\frac{1000\times100\times2}{200\times10}} = 10$

FUZZY SENSE

$$S_1 = \sqrt{\frac{2[3(G_{a8}H_{C8)} + 6(G_{a7}H_{C7}) + 4(G_{a6}H_{c6}) + 5(G_{a5}H_{c5}) + 5(G_{a4}H_{c4}) + 4(G_{a3}H_{c3}) + 6(G_{a2}H_{c2}) + 3(G_{a1}H_{c1})]}{3(P_{b1}R_{d1}) + 6(P_{b2}R_{d2}) + 4(P_{b3}R_{d3}) + 5(P_{b4}R_{d4}) + 5(P_{b5}R_{d5}) + 4(P_{b6}R_{d6}) + 6(P_{b7}R_{d7}) + 3(P_{b8}R_{d8})}}$$

$G_{a1} = 600$	$P_{b1} = 160$	$H_{c2} = 60$	$R_{d1} = 6$
$G_{a2} = 700$	$P_{b2} = 170$	$H_{c2} = 70$	$R_{d2} = 7$
$G_{a3} = 800$	$P_{b3} = 180$	$H_{c2} = 80$	$R_{d3} = 8$
$G_{a4} = 900$	$P_{b4} = 190$	$H_{c2} = 90$	$R_{d4} = 9$
$G_{a5} = 1100$	$P_{b5} = 210$	$H_{C5} = 100$	$R_{d5} = 11$
$G_{a6} = 1200$	$P_{b6} = 220$	$H_{c6} = 120$	$R_{d6} = 12$
$G_{a7} = 1300$	$P_{b7} = 230$	$H_{c7} = 130$	$R_{d7} = 13$
$G_{a8} = 1400$	$P_{b8} = 240$	$H_{c8} = 140$	$R_{d8} = 14$

$$S_1 = \sqrt{\frac{2[3(1400\times40)+6(1300\times130)+4(1200\times120)+5(1100\times110)+5(900\times90)+4(800\times80)+6(700\times70)+3(600\times60)]}{3(160\times6)+6(170\times7)+4(180\times8)+5(190\times9)+5(210\times11)+4(220\times12)+6(230\times13)+3(240\times14)}}$$

$= \sqrt{\frac{7692000}{74460}} = \sqrt{103}$

$S_1 = 10.14$

$T_C = \frac{GH}{S} + S\frac{PR}{2} = \frac{1000\times10}{10} + \frac{10\times200\times10}{2}$

$T_C = 20000$

CRISP SENSE

$\widetilde{T_C} = \frac{1}{36S}[3(G_{a1}H_{C1}) + 6(G_{a2}H_{c2}) + 4(G_{a3}H_{c3}) + 5(G_{a4}H_{C4}) + 5(G_{a5}H_{c5}) + 4(G_{a6}H_{c6}) + 6(G_{a7}H_{c7}) + 3(G_{a8}H_{C8})] + \frac{S}{72}[3(P_{b1}R_{d1}) + 6(P_{b2}R_{d2}) + 4(P_{b3}R_{d3}) + 5(P_{b4}R_{d4}) + 5(P_{b5}R_{d5}) + 4(P_{b6}R_{d6}) + 6(P_{b7}R_{d7}) + 3(P_{b1}R_{d1})]$

$= \frac{1}{36\times10}[3846000] + \frac{10}{72}[74460]$

$\widetilde{T_C} = 19908$

VI. CONCLUSION

In the optimization of a fuzzy inventory model, it deals with octagonal fuzzy numbers with beta distribution. Using a Lagrangian approach, we were able to minimize the total cost and increase the optimum time period. Fuzzification was achieved with the octagonal fuzzy number. After that, the beta distribution causes defuzzification of the octagonal fuzzy number Using graded mean integration. The fuzzy and crisp sense can be found using the strategy of reducing total cost in an optimal solution, and the solution's result is the same. Finally, the study decides how to maximise the overall cost of the fuzzy inventory model. The proposed model was evaluated using numerical examples.

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