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Model Order Approximation of Linear Time Invariant System using Dragonfly Algorithm

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ABSTRACT

This paper deals with an integrated algorithm for the approximation of higher dimensional system (HDS). Framework of this research is initiated by considering different higher order transfer functions of LTI system. Numerator as well as denominator coefficients of the corresponding lower dimensional model (LDM) are computed using dragonfly algorithm (DA) and Routh approximation (RA) respectively. The proposed technique is verified by considering standard test cases. Further, the performance accuracies are evaluated by comparing the step and frequency responses of HDS and LDM.

KEYWORDS Model order approximation, dragonfly algorithm, Routh approximation, higher dimensional system, lower dimensional model.

1. INTRODUCTION

Modelling of physical system often results in higher-order transfer function. Model order approximation (MOA) techniques are used to reduce the order of HDS. MOA techniques are applied in different fields such as theory of control system [1-4] and large scale power system [5-7]. MOA techniques are essential to reduce the HDS into its corresponding LDM by retaining the fundamental characteristics of the system.
In earlier articles, several approximation techniques are mentioned for the order reduction of large scale systems in both continuous and discrete domains such as Padé approximation [8], RA technique [9, 10], stability equation [11], biased factor division method [12] and hybrid techniques in discrete system [13-21] etc. These techniques are based on traditional mathematical approaches. The limitations of traditional technique don't guarantee for the stability in LDM when HDS is stable.
Soft computing techniques [22-27] are one of the accurate MOA methodology as compared to existing mathematical approaches. However, limited performance is proved in many cases when these techniques are applied in system approximation. In spite of several computational techniques, there is a great significance for global optimization methods (GOM). Researchers are still searching for GOM that can be applied to all multifarious problems. In this research paper nature inspired technique called Dragonfly optimization algorithm [28] is proposed for order approximation.

Dragonfly algorithm (DA) [29] is an intelligent swarm based optimization algorithm mimic the dynamic and static characteristics of artificial dragonflies. This is a global optimization algorithm used to solve a large variety of complex engineering problems. It has proved its superiority compared to several well-known meta-heuristics algorithm available in the literature.

The rest of the paper is organized as follows, Section 2 gives an insights to the problem statement and proposed methodology of model approximation is described in Section 3 in two steps: (i) the denominator of the LDM is derived using RA technique (ii) DA is used to compute numerator of the LDM by minimizing integral square error (ISE) between HDS and LDM. A numerical test case solved in section 4 to verify efficacy of the proposed technique. Results obtained from simulation and its relatable discussions are illustrated in Section 5. Finally, section 6 concludes the entire article.

2. PROBLEM STATEMENT

The hth order single input single output system is represented by the transfer function

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$$T_h(s) = \frac{N_h(s)}{D_h(s)} = \frac{n_0 + n_1 s + n_2 s^2 + \cdots + n_{h-1} s^{h-1}}{d_0 + d_1 s + d_2 s^2 + \cdots + d_h s^h} \tag{1}$$

where $n_0, n_1, n_2, \cdots, n_{h-1}$ and $d_0, d_1, d_2, \cdots, d_h$ are coefficients of numerator and denominator respectively.

Similarly, the transfer function of the LDM $(l < h)$ becomes

$$\hat{T}_l(s) = \frac{\hat{N}_l(s)}{\hat{D}_l(s)} = \frac{\hat{n}_0 + \hat{n}_1 s + \hat{n}_2 s^2 + \cdots + \hat{n}_{l-1} s^{l-1}}{\hat{d}_0 + \hat{d}_1 s + \hat{d}_2 s^2 + \cdots + \hat{d}_l s^l} \tag{2}$$

where $\hat{n}_0, \hat{n}_1, \hat{n}_2, \cdots, \hat{n}_{l-1}$ and $\hat{d}_0, \hat{d}_1, \hat{d}_2, \cdots, \hat{d}_l$ are coefficients of LDM.

3. PROPOSED METHODOLOGY

In this paper, a hybrid methodology is proposed by combining the benefits of RA [30] and DA [31]. This technique is carried out in two phases. In first step, denominator coefficients are calculated using RA and in next step DA is carried out to compute coefficients of the numerator by minimizing the ISE between HDS and LDM.

3.1 OVERVIEW OF ROUTH APPROXIMATION

RA technique [30] is proposed by Hutton et al. for the reduction of HDS to its corresponding LDM in frequency domain. It is a simple technique which preserves all significant characteristics of HDS in LDM, if the HDS is stable. RA technique is applicable to compute denominator coefficients of the LDM based upon δ table as follows.

Step-1: Determine reciprocal transformation of the HDS

$$\bar{D}_h(s) = s^h D_h\left(\frac{1}{s}\right) \tag{3}$$

Step-2: From the coefficients of $\bar{D}(s)$, compute the values of $\delta_1, \delta_2, \cdots, \delta_m,$ by using delta table

Step-3: Calculate the lth-order denominator polynomial $\hat{D}_l$ from the delta coefficients of $\bar{D}_h(s)$

for second order polynomial , $\hat{D}_2(s) = \delta_1 \delta_2 s^2 + \delta_2 s + 1$

for third order polynomial , $\hat{D}_3(s) = \delta_1 \delta_2 \delta_3 s^3 + \delta_2 \delta_3 s^2 + (\delta_1 + \delta_3) s + 1$

General terms, the above equation becomes

$$\bar{D}_l = \delta_l s \bar{D}_{l-1}(s) + \bar{D}_{l-2}(s) \tag{4}$$

	$d_0^0 = d_0$	$d_2^0 = d_2$	$d_4^0 = d_4$	$d_6^6 = d_6$	$\cdots$
	$d_0^1 = d_1$	$d_2^1 = d_3$	$d_4^1 = d_5$	$\cdots$	
$\delta_1 = d_0^0 / d_0^1$	$d_0^2 = d_0^2 - \delta_1 d_2^1$	$d_2^2 = d_4^0 - \delta_1 d_4^1$	$d_4^2 = d_6^0 - \delta_1 d_6^1$	$\cdots$	
$\delta_2 = d_0^1 / d_0^2$	$d_0^3 = d_2^1 - \delta_2 d_2^2$	$d_2^3 = d_4^1 - \delta_2 d_4^2$	$\cdots$		
$\delta_3 = d_0^2 / d_0^3$	$d_0^4 = d_2^2 - \delta_3 d_2^3$	$d_2^4 = d_4^2 - \delta_3 d_4^3$	$\cdots$		
$\delta_4 = d_0^3 / d_0^4$	$d_0^5 = d_2^3 - \delta_4 d_2^4$	$\cdots$	$\cdots$		
$\delta_5 = d_0^4 / d_0^5$	$d_0^6 = d_2^4 - \delta_5 d_2^5$	$\cdots$			
$\delta_6 = d_0^5 / d_0^6$	$\cdots$	$\cdots$			

Table 1 Delta Table

3.2 DRAGON FLY ALGORITHM

Dragonfly algorithm (DA) [29, 32] is a nature inspired optimization algorithm, developed by Mirjalili. Dragonflies are small flying insects that hunt and eat a wide variety of small insects. The algorithm is formulated based on two phases exploitation and exploration. In the exploitation phase, a large number of dragonflies make the swarms migrate in one direction over long distances and distract from enemies. In the exploitation phase, a large number of swarm can

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fly in one direction to distract enemies. However, in exploration phase swarm makes small groups for searching food over a bounded area and attract flying preys.

In DA five basic principles are designed as follows. In the following equations, Z represents the position of the current agent, Z_j the position of the jth neighbouring agent, and K the number of neighbouring agent

- Separation is a strategy is used for collision avoidance with the other agents in the neighborhood. This procedure is represented by following mathematical equations:

$$Se_i = -\sum_{j=1}^{K} Z - Z_j \tag{5}$$

- Alignment represents the matching of velocity between agent and neighbourhood agent of the same group. The concept is shown as follows

$$Al_i = -\sum_{j=1}^{K} W_j \tag{6}$$

where velocity of jth agent is denoted by V_j

- Cohesion property draw attention towards the center of the swarm group. It is shown mathematically as

$$Co_i = \frac{\sum_{j-1}^{K} Z_j}{K} - Z \tag{7}$$

- In the attraction phase, food source is the center of attraction (Fa_i) for dragonflies and it is modeled as

$$Fa_i = F_Z - Z \tag{8}$$

where F_i and F_Z represents the position of ith agent and food source position respectively.

- Distraction from the enemies is represented mathematically as

$$Ed_i = Ed_Z + Z \tag{9}$$

where Ed_i indicates the position of ith agent and Ed_Z denotes the position of enemy.

Artificial dragon flies updated their positions inside the bounded search space by considering the step vector ΔZ and the position vector Z . The updated positions are indicated by following equations:

$$\Delta Z_i^{y+1} = \left(sSe_i + aAl_i + cCo_i + fFa_i + eEd_i \right) + \beta \Delta Z_i^{y} \tag{10}$$

where

s denotes the weight assigned for separation phase,
Se_i represents the separation phase of the ith agent,
a denotes the weight assigned in alignment phase,
Al_i indicates the alignment of the ith agent,
c is the weight assigned in cohesion phase,
Co_i is the cohesion of the ith agent,
f denotes the food factor,
Fa_i is the source of food for the ith agent,
e indicates the enemy factor,
Ed_i denotes the enemy position of the ith agent,
w is the inertia weight,
y indicated the iteration number.

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Then, the updated position of the ith dragonfly at $y+1$ as follows:

$$Z_i^y = Z_i^y + \Delta Z_i^{y+1} \tag{11}$$

3.3 TOOLS USED FOR VALIDATION OF OBJECTIVE FUNCTION

The DA algorithm is used to obtain the numerator coefficient of LDM by minimizing following fitness function for single input single output system (SISO) system.
Fitness function =Minimum (Integral Square Error)

$$ISE = \int_0^\infty \|Error\|^2 dt, \tag{12}$$

$$Error = \|hi(t) - lo(t)\|$$

where $hi(t)$ and $lo(t)$ are the time responses of HDS and LDM, respectively.

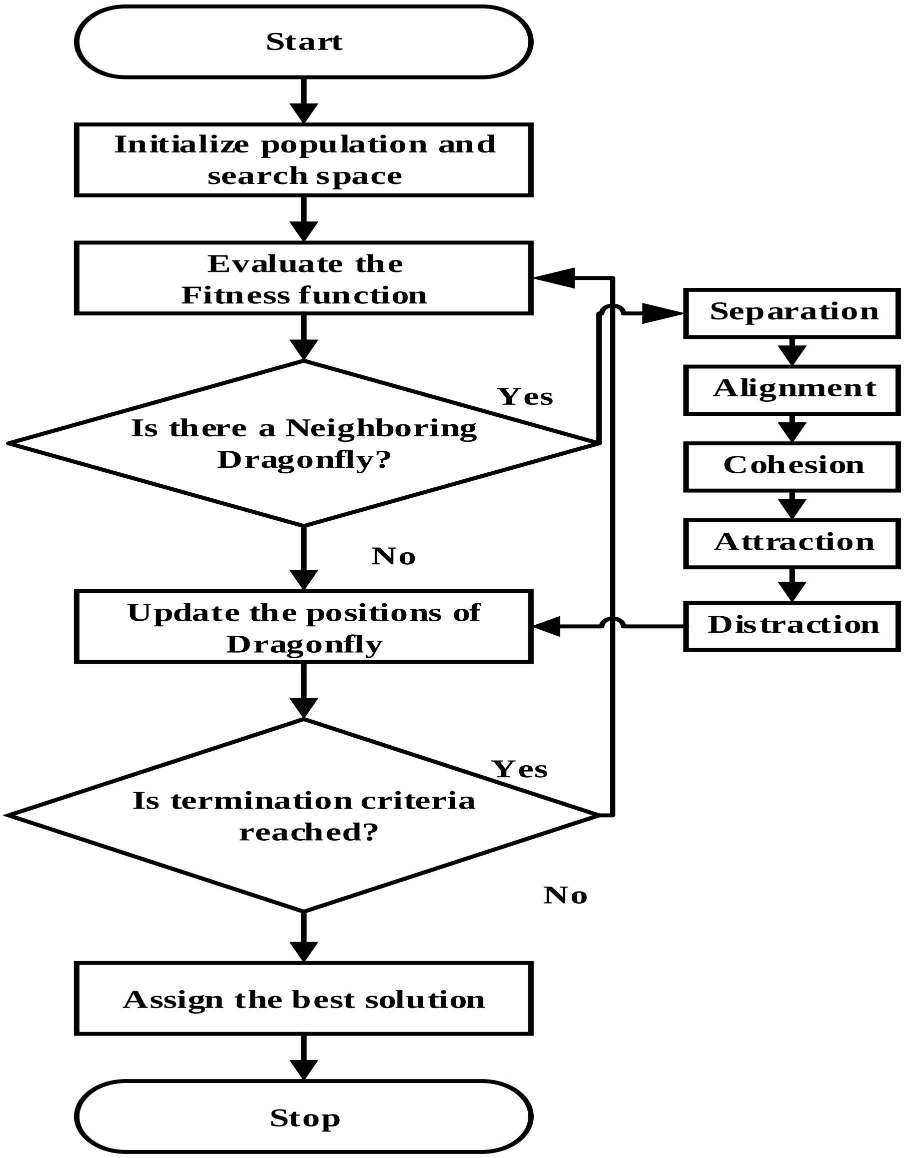

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Figure 2 Flow chart of dragonfly Algorithm

4. TEST CASE

Consider a fourth-order system [33] expressed in transfer function form as

$$T_h(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \quad (13)$$

Denominator polynomial of the model is computed using Equation (4) and Table 1 is given by

$$\hat{D}_l(s) = s^2 + 1.6501s + 0.7924 \quad (14)$$

After computation of denominator polynomial, numerator polynomial of the LDM is computed using DA, by minimizing the ISE between HDS and LDM. The lower order numerator polynomial becomes

$$\hat{N}_l(s) = 0.81479s + 0.79116 \quad (15)$$

Thus, the approximant i.e. LDM for the HDS is represented by following transfer function

$$\hat{T}_l(s) = \frac{0.8147s + 0.7911}{s^2 + 1.6501 + 0.7924} \quad (16)$$

The LDM obtained by [34] is

$$\hat{T}_{l_a}(s) = \frac{0.8147s + 0.7911}{s^2 + 1.65 + 0.7924} \quad (17)$$

The LDM computed using Routh stability criterion [35] becomes

$$\hat{T}_{l_b}(s) = \frac{20.571s + 24}{30s^2 + 42s + 24} \quad (18)$$

TABLE-2 PERFORMANCE INDEX ANALYSIS OF VARIOUS MODEL APPROXIMATION TECHNIQUE

Model approximation techniques	ISE	IAE	ITSE	MSE	RMSE
Higher order System	-	-	-	-	-
Proposed method	**1.4337×10^{-04}**	**0.0295**	**5.8384×10^{-04}**	**1.3814×10^{-05}**	**0.0037**
Stability-equation method and modified Cauer continued fraction method [34]	0.0346	0.4189	0.0770	0.0031	0.0561
Routh stability criterion [35]	0.0095	0.2170	0.0336	8.6202×10^{-04}	0.0294

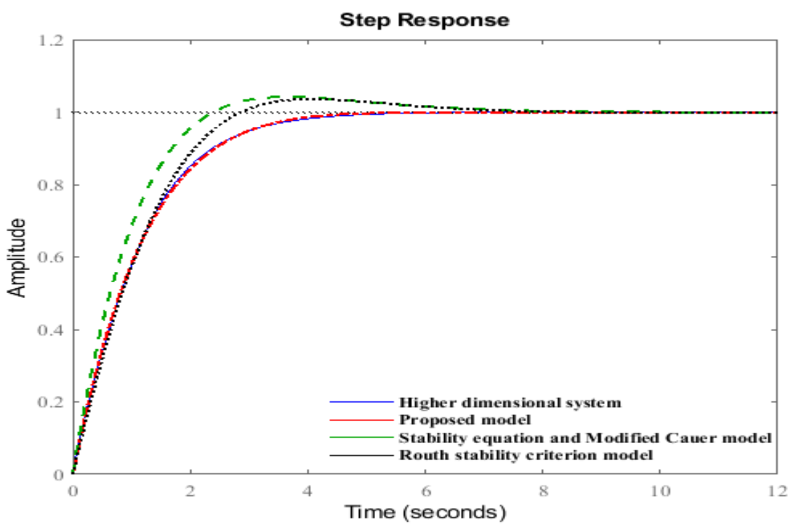

Figure 2: Comparison of step responses for the HDS and various LDM.

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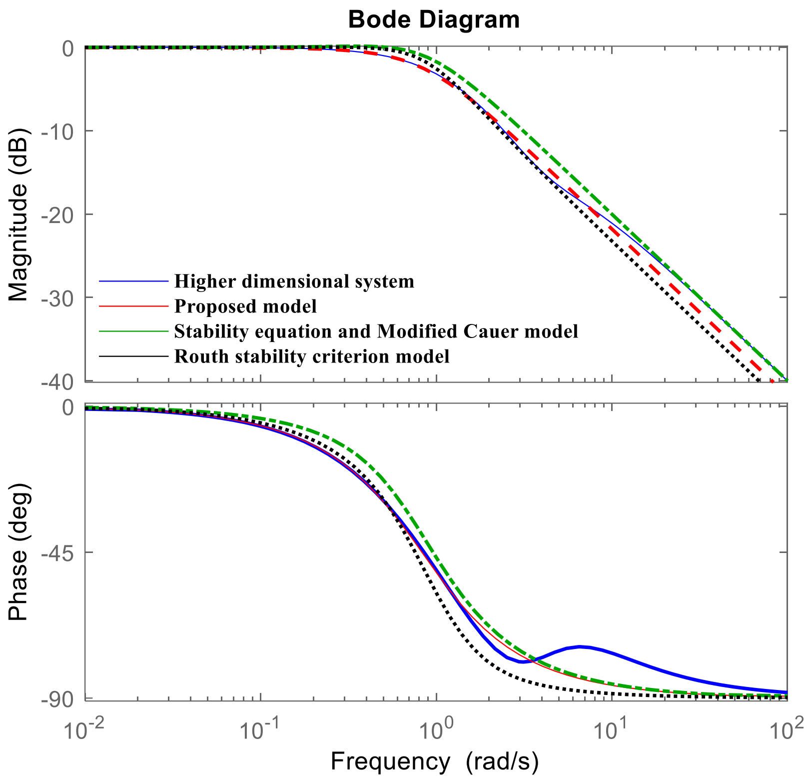

Figure 3: Comparison of frequency responses for the HDS and various LDM.

5. RESULTS AND DISCUSSIONS

The step and frequency responses of HDS and its LDM are computed by different model approximation techniques are shown in figure 2 and 3 respectively. The figure 2 and 3 clearly depict that the responses of the LDM computed by proposed technique is closed match with the transient and steady state behaviour of the HDS. Several performance indices of existing techniques and proposed method are tabulated in Table1. From the table, it is clearly observed that proposed technique has the minimum error as compared with other existing techniques.

6. CONCLUSION

In this paper, a mixed model approximation technique based on Routh approximation technique and dragonfly algorithm is proposed for the order approximation of higher order systems. In this proposed approach, the coefficients of the denominator are computed by using RA technique and numerator coefficients are determined by using DA. Main virtues of this technique is to produce stable LDM only for a stable HDS with minimizing the error bounds. The step and frequency responses depict the transient and steady-state behavior of HDS and LDM accordingly. Further, accuracy of the proposed scheme is compared with other MOA techniques incorporated with various performance indices such as ISE, IAE, ITSE, MSE and RMSE.

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