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On The Construction of Triangular Fuzzy Neutrosophic Metric Space

[1]G Veeramalai, [2]M. Angayarkanni, [3]P.Ramesh, [4]M.Prabakaran

[1]Department of Mathematics, M.Kumarasamy College of Engineering (Autonomous), Karur,

[2]Department of Mathematics, Kandaswami Kandar's College, P. Velur, Namakkal.

[3]Department of Mathematics, SVST Kongunad Arts and Science College, Namakkal.

[4]Department of Mathematics, Vivekanandha college of Education for women, Tiruchengode, Namakkal

Corresponding Author: veeramusiri@gmail.com

ABSTRACT

In this Paper, New approaches to find the distance of two triangular fuzzy neutrosophic numbers on the neutrosophic set and discuss about a basic properties of triangular fuzzy neutrosophic metric and also establish the triangular fuzzy neutrosophic metric space. To end with numerical example has been provided to validate the planned method for triangular fuzzy neutrosophic metrics.

AMS 2010 Subject Classification: 08A72, 41A65

Keywords: Neutrosophic triangular fuzzy distance, triangular fuzzy neutrosophic set, Triangular fuzzy -metric space, etc.

Introduction:

"In his pioneering work, Zadeh proposed fuzzy set in 1965 [1], which is a great theory to cope with ambiguity. According to him, a fuzzy set provides a membership grade to each component of a specified crisp universal set from a set of membership values. This concept set the groundwork for a large number of mathematical applications, as well as a wide range of real-life problems.

Later, Atanassov 1986[17] concentrated on intuitionistic fuzzy, which is defined by a membership value and nonmembership in each of the universe sets, and smarandache (19982005)[6]developed a new concept termed neutrosophic sets by adding an intermediates membership.

In 1992 [18], Felbin assigned a fuzzy numerical value to each member of subspace and presented the notion of a fuzzy averaging on a linear space and shown that finite dimensional domains are limited dimensional.

A topological space is just a nonempty set coupled with a two-variable function that allows us to calculate the distance between two locations [16], [18]. We need to calculate the distance not just between integers and vectors in advanced mathematics, but also between more intricate objects like sequences, sets, and functions [2425]. Numerous techniques exist in this sector in order to develop a suitable idea of a metric space. Many famous mathematicians have considered a variety of generalizations of a metric space."

The construction of this paper is as focus: In section 2 for a short time review of neutrosophic set, Neutrosophic single valued functions and the basic operations. In section 3, the planned method for gives some basic definition and properties

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of absolute distance and deals the triangular fuzzy neutrosophic metric space. In section 4, Numerical example given in the proposed method and Conclusion are discussed in Section5.

2. Preliminaries.

A review of important concept and representations of neutrosophic set is discussed in this part [2], $[3],[4],[6],[7],[9],[11-18]$ &[20].

2.1. Definition [3]

A neutrosophic set with a single value N through X taking from $N=\{x,\langle p_N(x),\ \delta_N(x),\ \eta_N(x)\rangle; x\in X\}$, where X be a universe of discourse, $p_N(x): X\to I, \delta_N(x): X\to I$ and $\eta_N(x): X\to I$, where $1=[0,1]$ with $0\le p_N(x)+\delta_N(x)+\eta_N(x)\le 3$ for $x\in X$. $p_N(x),\ \delta_N(x)\ and\ \eta_N(x)$ respectively represent the truth membership function (TMF), the indeterminacy membership function (IMF), and the falsity membership function are three types of membership functions (FMF)of x *to* N.

2.2. Definition [3]

If N be a single valued neutrosophic set, then the complement of N is calculated as follows $p'_N(x)=\eta_N(x)$, $\delta'_N(x)=1-\delta_N(x)$, $\eta'_N(x)=p_N(x)$, $\forall x\in X$.

2.3 Definition [4]

The union (Intersection) of two single valued neutrosophic set N and M is a singled valued neutrosophic set R.

$$p_R(x)=\max(p_N(x),p_M(x))$$
$$\delta_R(x)=\max(\delta_N(x),\delta_M(x))$$
$$\eta_R(x)=\min(\eta_N(x),\eta_M(x)),\forall x\in X$$

The intersection between 2 different single valued neutrosophic with single values N and M is a neutrosophic set with a single value S.

$$p_S(x)=\min(p_N(x),p_M(x))$$
$$\delta_s(x)=\min(\delta_N(x),\delta_M(x))$$
$$\eta_S(x)=\max(\eta_N(x),\eta_M(x)),\forall x\in X$$

2.4 Definition [3]

A single valued TFN number is denoted by $N=[(n^l,n^m,n^u),p_N,\delta_N,\eta_N]$, and its three membership functions are given as follows

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$$T_A(x) = \begin{cases} \frac{(x-a)}{(b-a)}p, & a \le x < b \\ p, & x = b \\ \frac{(c-x)}{(c-b)}p, & b \le x < c \\ 0, & otherwise \end{cases}$$

$$I_A(x) = \begin{cases} \frac{(b-x)}{(b-a)}\delta, & a \le x < b \\ \delta, & x = b \\ \frac{(x-c)}{(c-b)}\delta, & b \le x < c \\ 1, & otherwise \end{cases}$$

$$F_A(x) = \begin{cases} \frac{(b-x)}{(b-a)}\eta, & a \le x < b \\ \eta, & x = b \\ \frac{(x-c)}{(c-b)}\eta, & b \le x < c \\ 1, & otherwise \end{cases}$$

2. 5. Definitions [12]

Let $N = [p_N(x), \delta_N(x), \eta_N(x), x \in X]$, $M = [p_M(x), \delta_M(x), \eta_M(x), x \in X]$ and

$X = \{x_1, x_2, x_3, ... x_n\}$,

Then $d_{NS}(N, M) = \sum_{i=1}^{k} (p_N(x_i) - p_M(x_i)| + |\delta_N(x_i) - \delta_M(x_i)| + |\eta_N(x_i) - \eta_M(x_i)|)$

2.6. Definition [12]

Assume that X be the non-empty set and $F[O,1]$ be the set of all TFN on $[0,1]$. A Triangular fuzzy number neutrosophic set (TFNNS) $\overline{N}$ in X represented by

$\overline{N} = [\bar{p}(x), \bar{\delta}(x), \bar{\eta}(x), x \in X]$

Where $\bar{p}(x): X \to F[O,1]$, $\bar{\delta}(x): X \to F[O,1]$ and $\bar{\eta}(x): X \to F[O,1]$

The TFN $\bar{p}(x) = (p_N^1(x), \ p_N^2(x), \ p_N^3(x))$, $\bar{\delta}(x) = (\delta_N^1(x), \ \delta_N^2(x), \ \delta_N^3(x))$ and $\bar{\eta}(x) = (\eta_N^1(x), \ \eta_N^2(x), \ \eta_N^3(x))$ respectively denote the TMF, IMF and FMF of x in $\overline{N}$ for every $x \in X$, and also $0 \le p_N^3(x) + \delta_N^3(x) + \eta_N^3(x) \le 3$.

3. Triangular Fuzzy Neutrosophic Approaches on distance

In this area construct the metric in the TFNN

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3.1 A Metric on triangular fuzzy neutrosophic set

Assume that X be the non-empty finite set of all TFN on I, here $1 = [0,1]$. Let S be a space of objects in X and $\overline{A}$ be a TFNS on S is characterised by TMF (x), IMF $\delta(x)$ and FMF $\eta(x)$. Where $\rho(x)$, $\delta(x)$ and $\eta(x)$ are real standard and non-standard subset $of I.$

Thus the TFN set $\overline{A}$ over S is defined as $\overline{A} = [\overline{p}(x), \overline{\delta}(x), \overline{\eta}(x), x \in S] on$ the same of $\overline{p}(x), \overline{\delta}(x)$ and $\overline{\eta}(x)$ there is no restriction and also $0 \leq p_A^3(x) + \delta_A^3(x) + \eta_A^3(x) \leq 3.$

Clearly $\bar{p}_A(x) = (p_A^1(x),\ p_A^2(x),\ p_A^3(x)), \overline{\delta}(x) = (\delta_A^1(x),\ \delta_A^2(x),\ \delta_A^3(x))\&\ \overline{\eta}(x) = (\eta_A^1(x),\ \eta_A^2(x),\ \eta_A^3(x))$

Let $\overline{A} = [\overline{p}(x), \overline{\delta}(x), \overline{\eta}(x), x \in S]$ and $\overline{B} = [\overline{p}(x), \overline{\delta}(x), \overline{\eta}(x), x \in S]\ in\ S = \{x_1, x_2, x_3, x_n\}$, $S \subseteq X$. Then the TFN distance of two points defined as

$$d_{TFNS}(\overline{A}, \overline{B}) = \sum_{i=1}^{n} t_{\overline{p}}\ (x_i) - \overline{p}(x_i)| + |\overline{\delta}(x_i) - \overline{\delta}(x_i)| + |\overline{\eta}(x_i) - \overline{\eta}(x_i$$

$$d_{TFNS}(\overline{A}, \overline{B}) = (|p_A^1(x) - p_B^1(x)| + |\delta_A^1(x) - \delta_B^1(x)| + |\eta_A^1(x) - \eta_B^1(x)|)$$
$$+(|p_A^2(x) - p_B^2(x)| + |\delta_A^2(x) - \delta_B^2(x)| + |\eta_A^2(x) - \eta_B^2(x)|)$$
$$+(|p_A^3(x) - p_B^3(x)| + |\delta_A^3(x) - \delta_B^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|)$$

3.2. Properties of TFN metric.

Let X every TFN set that isn't empty. A function $d_{TFNS}: X \times X \to R$ is said to be metric on X If essential requirements have been met:

i). $d_{TFNS}(\overline{A}, \overline{B}) \geq 0,\ \forall \overline{A}, \overline{B} \in X$

ii). $d_{TFNS}(\overline{A}, \overline{B}) = 0 \Leftrightarrow \overline{A} = \overline{B}$

iii). $d_{TFNS}(\overline{A}, \overline{B}) = d_{TFNS}(\overline{B}, \overline{A})$

iv). $d_{TFNS}(\overline{A}, \overline{C}) = d_{TFNS}(\overline{A}, \overline{B}) + d_{TFNS}(\overline{B}, C),\ \forall \overline{A}, \overline{B}, \overline{C} \in X$

Proof:

Let X be the finite universe of discourse of all TFNN and $\{\overline{A}, \overline{B}, \overline{C}\} \subseteq X,$

here $\overline{A} = (\overline{p}_A, \overline{\delta}_A, \overline{\eta}_A)$, $\overline{B} = (\overline{p}_B, \overline{\delta}_B, \overline{\eta}_B)$ and $\overline{C} = (\overline{p}_C, \overline{\delta}_C, \overline{\eta}_C)$.

Clearly $\overline{A} = \{(p_A^1, \delta_A^1, \eta_A^1), (p_A^2, \delta_A^2, \eta_A^2),\ (p_A^3, \delta_A^3, \eta_A^3)$

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$$\overline{B} = \{(p_B^1, \delta_B^1, \eta_B^1), (p_B^2, \delta_B^2, \eta_B^2), (p_B^3, \delta_B^3, \eta_B^3)\}$$

$$\overline{C} = \{(p_C^1, \delta_c^1, \eta_C^1), (p_C^2, \delta_c^2, \eta_C^2), (p_C^3, \delta_c^3, \eta_C^3)\}$$

and $0 \leq p_A^3 + \delta_A^3 + \eta_A^3 \leq 3, \quad 0 \leq p_B^3 + \delta_B^3 + \eta_B^3 \leq 3 \quad \& \quad 0 \leq p_C^3 + \delta_c^3 + \eta_C^3 \leq 3$

i). $d_{TFNS}(\overline{A}, \overline{B}) \geq 0, \forall \overline{A}, \overline{B} \in X$

$$d_{TFNS}(\overline{A}, \overline{B}) = \sum_{i=1}^{n} \left(\left|\widetilde{\rho}_A(x_i) - \widetilde{\rho}_B(x_i)\right| + \left|\widetilde{\delta}_A(x_i) - \widetilde{\delta}_B(x_i)\right| + \left|\widetilde{\eta}_A(x_i) - \widetilde{\eta}_B(x_i)\right| \right)$$

$$d_{TFNS}(\overline{A}, \overline{B}) = (|p_A^1(x) - p_B^1(x)| + |\delta_A^1(x) - \delta_B^1(x)| + |\eta_A^1(x) - \eta_B^1(x)|)$$
$$+(|p_A^2(x) - p_B^2(x)| + |\delta_A^2(x) - \delta_B^2(x)| + |\eta_A^2(x) - \eta_B^2(x)|)$$
$$+(|p_A^3(x) - p_B^3(x)| + |\delta_A^3(x) - \delta_B^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|)$$

Where $\overline{p}_A \leq \overline{p}_B, \overline{\delta}_A \leq \overline{\delta}_B$ and $\overline{\eta}_A \leq \overline{\eta}_B \Rightarrow |\overline{p}_A - \overline{p}_B| \neq 0 \Rightarrow |\overline{p}_A - \overline{p}_B| > 0$

Similarly $\left|\overline{\delta}_A - \overline{\delta}_B\right| \neq 0 \Rightarrow \left|\overline{\delta}_A - \overline{\delta}_B\right| > 0$

$\&| \overline{\eta}_A - \overline{\eta}_B \neq 0 \Rightarrow | \overline{\eta}_A - \overline{\eta}_B| > 0,$

Then $(|p_A^1(x) - p_B^1(x)| + |\delta_A^1(x) - \delta_B^1(x)| + |\eta_A^1(x) - \eta_B^1(x)|) \neq 0$

$(|\rho_A^2(x) - \rho_B^2(x)| + |\delta_A^2(x) - \delta_B^2(x)| + |\eta_A^2(x) - \eta_B^2(x)|) \neq 0$

$(|p_A^3(x) - p_B^3(x)| + |\delta_A^3(x) - \delta_B^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|) \neq 0$

$$= d_{TFNS}(\overline{A}, \overline{B}) = \sum_{i=1}^{n} \left(\left|\overline{\rho}_A(x_i) - \overline{\rho}_B(x_i)\right| + \left|\overline{\delta}_A(x_i) - \overline{\delta}_B(x_i)\right| + \left|\overline{\eta}_A(x_i) - \overline{\eta}_B(x_i)\right| \right) \geq 0$$

Hence $d_{TFNS}(\overline{A}, \overline{B}) \geq 0$

ii). Let by $d_{TFNS}(\overline{A}, \overline{B}) = 0$

$$d_{TFNS}(\overline{A}, \overline{B}) = \sum_{i=1}^{n} \left(\left|\overline{\rho}_A(x_i) - \overline{\rho}_B(x_i)\right| + \left|\overline{\delta}_A(x_i) - \overline{\delta}_B(x_i)\right| + \left|\overline{\eta}_A(x_i) - \overline{\eta}_B(x_i)\right| \right)$$

$$d_{TFNS}(\overline{A}, \overline{B}) = (|p_A^1(x) - p_B^1(x)| + |\delta_A^1(x) - \delta_B^1(x)| + |\eta_A^1(x) - \eta_B^1(x)|)$$
$$+(|p_A^2(x) - p_B^2(x)| + |\delta_A^2(x) - \delta_B^2(x)| + |\eta_A^2(x) - \eta_B^2(x)|)$$
$$+(|p_A^3(x) - p_B^3(x)| + |\delta_A^3(x) - \delta_B^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|)$$

Where $\overline{p}_A \leq \overline{p}_B, \quad \overline{\delta}_A \leq \overline{\delta}_B$ and $\overline{\eta}_A \leq \overline{\eta}_B, If \left|\overline{p}_A - \overline{p}_A\right| = 0 \Rightarrow \overline{p}_A - \overline{p}_A = 0 \Rightarrow \overline{p}_A = \overline{p}_A$

Similarly $\left| \overline{\delta}_A \leq \overline{\delta}_B\right| = 0 \Rightarrow \overline{\delta}_A - \overline{\delta}_B = 0 \Rightarrow \overline{\delta}_A = \overline{\delta}_B$

$\&|\overline{\eta}_A - \overline{\eta}_B| = 0 \Rightarrow \overline{\eta}_A - \overline{\eta}_B = 0 \Rightarrow \overline{\eta}_A = \overline{\eta}_B$

Then $(|p_A^1(x) - p_B^1(x)| + |\delta_A^1(x) - \delta_B^1(x)| + |\eta_A^1(x) - \eta_B^1(x)| = 0$

$(|p_A^2(x) - p_B^2(x)| + |\delta_A^2(x) - \delta_B^2(x)| + |\eta_A^2(x) - \eta_B^2(x)|) = 0$

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$$(|\rho_A^3(x)-\rho_B^3(x)|+|\delta_A^3(x)-\delta_B^3(x)|+|\eta_A^3(x)-\eta_B^3(x)|)=0$$

$$\Rightarrow d_{TFNS}(\overline{A},\overline{B})=\sum_{i=1}^{n}\left(\left|\overline{\rho}_A(x_i)-\overline{\rho}_B(x_i)\right|+\left|\overline{\delta}_A(x_i)-\overline{\delta}_B(x_i)\right|+\left|\overline{\eta}_A(x_i)-\overline{\eta}_B(x_i)\right|\right)=0$$

Hence$d_{TFNS}(\overline{A},\overline{B})=0$

Again $\overline{A}=\overline{B}$

$$\Rightarrow (\overline{p}_A,\overline{\delta}_A,\overline{\eta}_A)=(\overline{p}_B,\overline{\delta}_B,\overline{\eta}_B)$$

$$\Rightarrow \overline{p}_A=\overline{p}_B \Rightarrow \rho_A^1=\rho_B^1,\quad \rho_A^2=\rho_B^2 \ \ \& \ \rho_A^3=\rho_B^3 \quad (i.e)|\overline{p}_A-\overline{p}_B|=0$$

Similarly $\overline{\delta}_A=\overline{\delta}_B \Rightarrow \delta_A^1=\delta_B^1$ & $\delta_A^2=\delta_B^2$ & $\delta_A^3=\delta_B^3(i.e)|\overline{\delta}_A-\overline{\delta}_B|=0$

Similarly $\overline{\eta}_A=\overline{\eta}_B \Rightarrow \eta_A^1=\eta_B^1$ & $\eta_A^2=\eta_B^2$ & $\eta_A^3=\eta_B^3$ $(i.e)|\overline{\eta}_A-\overline{\eta}_B|=0$

So, $\Rightarrow |\overline{p}_A-\overline{p}_B|=0, |\overline{\delta}_A-\overline{\delta}_B|=0$ & $|\overline{\eta}_A-\overline{\eta}_B|=0$

$$\Rightarrow |\overline{p}_A-\overline{p}_B|+|\overline{\delta}_A-\overline{\delta}_B|+|\overline{\eta}_A-\overline{\eta}_B|=0$$

$d_{TFNS}(\overline{A},\overline{B})=0$

Hence $d_{TFNS}(\overline{A},\overline{B})=0=\overline{A}=\overline{B}$

iii) Let$\overline{p}_A\le\overline{p}_B,\overline{\delta}_A\le\overline{\delta}_B$ & $\overline{\eta}_A\le\overline{\eta}_B$

$$\begin{aligned}
&=(|p_A^1(x)-p_B^1(x)|+|\delta_A^1(x)-\delta_B^1(x)|+|\eta_A^1(x)-\eta_B^1(x)|)\\
&+(|p_A^2(x)-p_B^2(x)|+|\delta_A^2(x)-\delta_B^2(x)|+|\eta_A^2(x)-\eta_B^2(x)|)\\
&+(|p_A^3(x)-p_B^3(x)|+|\delta_A^3(x)-\delta_B^3(x)|+|\eta_A^3(x)-\eta_B^3(x)|)\\
&=(|p_B^1(x)-p_A^1(x)|+|\delta_B^1(x)-\delta_A^1(x)|+|\eta_B^1(x)-\eta_A^1(x)|)\\
&+(|p_B^2(x)-p_A^2(x)|+|\delta_B^2(x)-\delta_A^2(x)|+|\eta_B^2(x)-\eta_A^2(x)|)\\
&+(|p_B^3(x)-p_A^3(x)|+|\delta_B^3(x)-\delta_A^3(x)|+|\eta_B^3(x)-\eta_A^3(x)|)
\end{aligned}$$

$$=\sum_{i=1}^{n}|\overline{p}_A(x_i)-\overline{p}_B(x_i)|+|\overline{\delta}_A(x_i)-\overline{\delta}_B(x_i)|+|\overline{\eta}_A(x_i)-\overline{\eta}_B(x_i)| \quad = d_{TFNS}(\overline{B},\overline{A})$$

iv). we have $\overline{A}=\{(p_A^1,\delta_A^1,\eta_A^1),(p_A^2,\delta_A^2,\eta_A^2),\ (p_A^3,\delta_A^3,\eta_A^3)$

$\overline{B}=\{(p_B^1,\delta_B^1,\eta_B^1),(p_B^2,\delta_B^2,\eta_B^2),\ (p_B^3,\delta_B^3,\eta_B^3)\}$ &

$\overline{C}=\{(p_C^1,\delta_c^1,\eta_C^1),(p_C^2,\delta_c^2,\eta_C^2),(p_C^3,\delta_c^3,\eta_C^3)\}$

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Such that $\overline{p}_A \leq \overline{p}_B \leq \overline{p}_C$, $\quad \overline{\delta}_A \leq \overline{\delta}_B \leq \overline{\delta}_C \quad$ & $\quad \overline{\eta}_A \leq \overline{\eta}_B \leq \overline{\eta}_C$

So, $d_{TFNS}(\overline{A},\overline{C}) = \sum_{i=1}^{n} |\overline{p}_A(x_i) - \overline{p}_C(x_i)| + |\overline{\delta}_A(x_i) - \overline{\delta}_C(x_i)| + |\overline{\eta}_A(x_i) - \overline{\eta}_C(x_i)|$

$$= (|p_A^1(x) - p_C^1(x)| + |\delta_A^1(x) - \delta_c^1(x)| + |\eta_A^1(x) - \eta_C^1(x)|)$$
$$(|p_A^2(x) - p_C^2(x)| + |\delta_A^2(x) - \delta_c^2(x)| + |\eta_A^2(x) - \eta_C^2(x)|)$$
$$+(|p_A^3(x) - p_C^3(x)| + |\delta_A^3(x) - \delta_c^3(x)| + |\eta_A^3(x) - \eta_C^3(x)|)$$

$$\leq \begin{Bmatrix} (|p_A^1(x) - p_B^1(x)| + |\delta_A^1(x) - \delta_B^1(x)| + \eta_A^1(x) - \eta_B^1(x)|) + \\ (|p_A^2(x) - p_B^2(x)| + |\delta_A^2(x) - \delta_B^2(x)| + \eta_A^2(x) - \eta_B^2(x)|) + \\ (|p_A^3(x) - p_B^3(x)| + |\delta_A^3(x) - \delta_B^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|) \end{Bmatrix} +$$
$$\begin{Bmatrix} (|p_B^1(x) - p_c^1(x)| + |\delta_B^1(x) - \delta_c^1(x)| + |\eta_B^1(x) - \eta_C^1(x)|) \\ (|p_B^2(x) - p_C^2(x)| + |\delta_B^2(x) - \delta_c^2(x)| + |\eta_B^2(x) - \eta_C^2(x)|) + \\ (|p_B^3(x) - p_C^3(x)| + |\delta_B^3(x) - \delta_c^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|) \end{Bmatrix}$$

$$\leq \sum_{i=1}^{n} \left(|\overline{p}_A(x_i) - \overline{p}_B(x_i)| + |\overline{\delta}_A(x_i) - \overline{\delta}_B(x_i)| + |\overline{\eta}_A(x_i) - \overline{\eta}_B(x_i)|\right)$$

$$+ \sum_{i=1}^{n} \left(|\overline{p}_B(x_i) - \overline{p}_C(x_i)| + |\overline{\delta}_B(x_i) - \overline{\delta}_C(x_i)| + |\overline{\eta}_B(x_i) - \overline{\eta}_C(x_i)|\right)$$

Hence $d_{TFNS}(\overline{A},\overline{C}) \leq d_{TFNS}(\overline{A},\overline{B}) + d_{TFNS}(\overline{B},\overline{C})$

3.2 Triangular fuzzy neutrosophic metric space (TFNMS)

The set X associated with a metric d_{TFNS} on X is called TFN metric space (X, d_{TFNS})

3.3 Theorem

The function $d_{TFNS} : X \times X \to R$, $d_{TFNS}(\overline{A},\overline{B}) = |\overline{A} - \overline{B}|$defined on a TFN metric on $\Re$.

Proof:

Let X be the non-empty set of all TFNN and$\{\overline{A}, \overline{B}, \overline{C}\} \subseteq X$,

Here $\overline{A} = (\overline{p}_A, \overline{\delta}_A, \overline{\eta}_A)$, $\overline{B} = (\overline{p}_B, \overline{\delta}_B, \overline{\eta}_B)$ and $\overline{C} = (\overline{p}_C, \overline{\delta}_C, \overline{\eta}_C)$

Clearly $\overline{A} = \{(p_A^1, \delta_A^1, \eta_A^1), (p_A^2, \delta_A^2, \eta_A^2),\ (p_A^3, \delta_A^3, \eta_A^3)$

$$\overline{B} = \{(p_B^1, \delta_B^1, \eta_B^1), (p_B^2, \delta_B^2, \eta_B^2), (p_B^3, \delta_B^3, \eta_B^3)\}$$
$$\overline{C} = \{(p_C^1, \delta_c^1, \eta_C^1), (p_C^2, \delta_c^2, \eta_C^2), (p_C^3, \delta_c^3, \eta_C^3)\}$$

and $0 \leq p_A^3 + \delta_A^3 + \eta_A^3 \leq 3$, $\quad 0 \leq p_B^3 + \delta_B^3 + \eta_B^3 \leq 3 \quad$ & $\quad 0 \leq p_C^3 + \delta_c^3 + \eta_C^3 \leq 3$

Let $\overline{A}, \overline{B}$ and $\overline{C}$ be a TFN numbers in $\Re$

Here compute $d_{TFNS}(\overline{A}, \overline{B}) \geq 0$ is valid by the definition.

Moreover $d_{TFNS}(\overline{A}, \overline{A}) = 0 \Leftrightarrow |\overline{A} - \overline{A}| = 0$ holds and $d_{TFNS}(\overline{A}, \overline{B}) = 0 \Leftrightarrow |\overline{A} - \overline{B}| = 0 \Leftrightarrow \overline{A} = \overline{B}$ also

$d_{TFNS}(\overline{A}, \overline{B}) = d_{TFNS}(\overline{B}, \overline{A})$ are satisfied obviously.

$$d_{TFNS}(\overline{A}, \overline{C}) = \sum_{i=1}^{n} |\overline{p}_A(x_i) - \overline{p}_C(x_i)| + |\overline{\delta}_A(x_i) - \overline{\delta}_C(x_i)| + |\overline{\eta}_A(x_i) - \overline{\eta}_C(x_i)|$$

$$= (|p_A^1(x) - p_C^1(x)| + |\delta_A^1(x) - \delta_c^1(x)| + |\eta_A^1(x) - \eta_C^1(x)|)$$
$$(|p_A^2(x) - p_C^2(x)| + |\delta_A^2(x) - \delta_c^2(x)| + |\eta_A^2(x) - \eta_C^2(x)|)$$
$$+(|p_A^3(x) - p_C^3(x)| + |\delta_A^3(x) - \delta_c^3(x)| + |\eta_A^3(x) - \eta_C^3(x)|)$$
$$\leq \begin{Bmatrix} (|p_A^1(x) - p_B^1(x)| + |\delta_A^1(x) - \delta_B^1(x)| + \eta_A^1(x) - \eta_B^1(x)|) + \\ (|p_A^2(x) - p_B^2(x)| + |\delta_A^2(x) - \delta_B^2(x)| + \eta_A^2(x) - \eta_B^2(x)|) + \\ (|p_A^3(x) - p_B^3(x)| + |\delta_A^3(x) - \delta_B^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|) \end{Bmatrix} +$$
$$\begin{Bmatrix} (|p_B^1(x) - p_C^1(x)| + |\delta_B^1(x) - \delta_c^1(x)| + |\eta_B^1(x) - \eta_C^1(x)|) \\ (|p_B^2(x) - p_C^2(x)| + |\delta_B^2(x) - \delta_c^2(x)| + |\eta_B^2(x) - \eta_C^2(x)|) + \\ (|p_B^3(x) - p_C^3(x)| + |\delta_B^3(x) - \delta_c^3(x)| + |\eta_A^3(x) - \eta_B^3(x)|) \end{Bmatrix}$$

$$\leq \sum_{i=1}^{n} \left(|\overline{\rho}_A(x_i) - \overline{\rho}_B(x_i)| + |\overline{\delta}_A(x_i) - \overline{\delta}_B(x_i)| + |\overline{\eta}_A(x_i) - \overline{\eta}_B(x_i)|\right)$$

$$+ \sum_{i=1}^{n} \left(|\overline{\rho}_B(x_i) - \overline{\rho}_C(x_i)| + |\overline{\delta}_B(x_i) - \overline{\delta}_C(x_i)| + |\overline{\eta}_B(x_i) - \overline{\eta}_C(x_i)|\right)$$

Hence $d_{TFNS}(\overline{A}, \overline{C}) \leq d_{TFNS}(\overline{A}, \overline{B}) + d_{TFNS}(\overline{B}, \overline{C})$

$d_{TFNS}(\overline{A}, \overline{B}) \leq d_{TFNS}(\overline{A}, \overline{C}) + d_{TFNS}(\overline{C}, \overline{B})$

This is complete the proof of the metric properties.

3.4 Proposition

Let (X, d_{TFNS}) be a TFNM Space, for each $\overline{A}, \overline{B}, \overline{C} \in X$, then $d_{TFNS}(\overline{B}, \overline{C}) \geq |d_{TFNS}(\overline{A}, \overline{C}) - d_{TFNS}(\overline{A},$

$\overline{B})|$

3.5 Triangular fuzzy neutrosophic discrete metric space.

Let X be any non-empty TFN set and $\overline{A}, \overline{B} \in X$. The triangular filzzy neutrosophic discrete metric on X is a functions $d_{TFNS}: X \times X \to R$ defines as

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$$d_{TFNS}(\overline{A},\overline{B}) = \begin{cases} 0 & if\ \overline{A} \neq \overline{B} \\ 1 & if\ \overline{A} = \overline{B} \end{cases}$$

3.6 Proposition

Let X be any non-empty TFN set, then $(X, d^0_{TFNS}) is$ a TFN metric space.

Proof:

Let X be a TFN set

If $X = \phi$, then X has no element, here nothing to prove. So each properties of metric is vacuously satisfied.

Let us consider X is non empty

Since can take only the values of TMF is 0 and 1, we have $d^0_{TFNS}(\overline{A},\overline{B}) \geq 0$ for each choice $of \overline{A},\overline{B} \in X.$

By the definition of d^0_{TFNS} implies that $d^0_{TFNS}(\overline{A},\overline{B}) = 0\overline{A} = \overline{B}$

If $\overline{A} = \overline{B}$, then $d^0_{TFNS}(\overline{A},\overline{B}) = 0 = d^0_{TFNS}(,\overline{B},\overline{A})$

If $\overline{A} \neq \overline{B}$, then $d^0_{TFNS}(\overline{A},\overline{B}) = 1 = d^0_{TFNS}(,\overline{B},\overline{A}),\ \forall \overline{A},\overline{B} \in X$

Let $\overline{A},\overline{B} \in X$

Here try to prove triangular inequality, now choose two possible cases

$d^0_{TFNS}(\overline{A},\overline{C}) = 0$ & $d^0_{TFNS}(\overline{A},\overline{C}) = 1$

Suppose $d^0_{TFNS}(\overline{A},\overline{C}) = 0$

Since $d^0_{TFNS}(\overline{A},\overline{B})$ and $d^0_{TFNS}(\overline{A},\overline{C})$ re non negative, it follows that

$d^0_{TFNS}(\overline{A},\overline{B}) + d^0_{TFNS}(\overline{B},\overline{C}) \geq 0$ and $d^0_{TFNS}(\overline{A},\overline{C}) \geq 0$

(i.e) $d^0_{TFNS}(\overline{A},\overline{B}) + d^0_{TFNS}(\overline{B},\overline{C}) \geq 0 = d^0_{TFNS}(\overline{A},\overline{C})$

Suppose $d^0_{TFNS}(\overline{A},\overline{C}) = 1$, then $\overline{A} \neq \overline{B}$ and so $\overline{B}$ not equal to both $\overline{A}$ and $\overline{B}$. Hence the property at least one of $d^0_{TFNS}(\overline{A},\overline{B})$ and $d^0_{TFNS}(\overline{B},\overline{C}) is$ non zero and must equal to 1. Thus

$d^0_{TFNS}(\overline{A},\overline{B}) + d^0_{TFNS}(\overline{B},\overline{C}) \geq 1 = d^0_{TFNS}(\overline{A},\overline{C})$

Hence the both case hold the triangular inequality.

Since d^0_{TFNS} satisfied the all properties of TFN metric on X and $(X, d^0_{TFNS}) is$ a metric space.

4. Numerical Example

This section contains an example that demonstrates the efficacy of our recommended approaches.

Example 1.

The set $X = \Re$ with $d(\overline{A},\overline{B}) = \sum_{i=1}^{n} t_{\overline{p}}(x_i) - \overline{p}(x_i)| + |\overline{\delta}(x_i) - \overline{\delta}(x_i)| + |\overline{\eta}(x_i) - \overline{\eta}(x_i)|$ the absolute value ofthe diff erence is $\overline{A} - \overline{B}$, for every $\overline{A},\overline{B} \in \Re.$

The metric conditions $d_{TFNS}(\overline{A},\overline{B}) \geq 0,$ $\forall \overline{A},\overline{B} \in \Re,$ $d_{TFNS}(\overline{A},\overline{B}) = 0 \Leftrightarrow \overline{A} = \overline{B}$ and

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$d_{TFNS}(\overline{A},\overline{B}) = d_{TFNS}(\overline{B},\overline{A})$ are satisfied obviously.

Then $d_{TFNS}(\overline{A},\overline{B}) = \sum_{i=1}^{n}\left|\overline{p}_A(x_i) - \overline{p}_B(x_i)\right| + \left|\overline{\delta}_A(x_i) - \overline{\delta}_B(x_i)\right| + \left|\overline{\eta}_A(x_i) - \overline{\eta}_B(x_i)\right|$

In order to consider two triangular fuzzy neutrosophic values.

$\overline{A} = ((0.70,0.75,0.80),(0.15,0.20,0.25),(0.10,0.15,0.20))$

$\overline{B} = ((0.40,0.45,0.50),(0.40,0.45,0.50),(0.35,0.40,0.45))$

$d_{TFNS}(\overline{A},\overline{B}) = ((0.30,0.30,0.30),(0.25,0.25,0.25),(0.25,0.25,0.25))$

$d_{TFNS}(\overline{A},\overline{B}) = 2.4$

$d_{TFNS}(\overline{A},\overline{A}) = 0$

$d_{TFNS}(\overline{B},\overline{A}) = ((0.30,0.30,0.30),(0.25,0.25,0.25),(0.25,0.25,0.25))$

$d_{TFNS}(\overline{B},\overline{A}) = 2.4$

Let $\overline{C} = ((0.50,0.55,0.60),(0.25,0.30,0.35),(0.20,0.25,0.30))$

$d_{TFNS}(\overline{A},\overline{C}) = ((0.20,0.20,0.20),(0.10,0.10,0.10),(0.10,0.10,0.10))$

$d_{TFNS}(\overline{A},\overline{C}) = 1.2$

$d_{TFNS}(\overline{A},\overline{B}) = 2.4$ and $d_{TFNS}(\overline{B},\overline{C}) = 1.2$

Hence $d_{TFNS}(\overline{A},\overline{C}) < d_{TFNS}(\overline{A},\overline{B}) + d_{TFNS}(\overline{B},\overline{C})$

These distance clearly satisfied the conditions of Triangular fuzzy neutrosophic sets.

5. Conclusions

The content of this research work is to construct a TFN metric space. These space and distance functions are used as cost function to minimize the optimization problem. The TFNN are powerful tool for dealing with Complete and accurate value of the environmental problem. Finally, the efficacy and application of the suggested technique were demonstrated, as well as their connected features.

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