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On Pseudo Regular Picture Fuzzy Soft Graph

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ABSTRACT

In this paper, we define the notion of picture fuzzy soft graph and introduce the concepts of pseudo degree picture fuzzy soft graph, pseudo regular picture fuzzy soft graph, totally pseudo regular picture fuzzy soft graph. And also we studied about their properties.

1. INTRODUCTION

The concept of fuzzy set theory was introduced by Zadeh [8] to solve difficulties in dealing with uncertainties. Then the theory of fuzzy sets and fuzzy logic has been examined by many researchers to solve many real life problems involving ambiguous and uncertain environment. Santhi Maheswari. N. R and Sekar. C [7] introduced the Pseudo regular Fuzzy Graphs. Cuong and Kreinovich [3] proposed the concept of picture fuzzy set which is a modified version of fuzzy set and Intuitionistic fuzzy set. Durgadevi. S and Akilandeswari. B [4] introduced the idea of pseudo regular interval valued fuzzy soft graph. Cen Zuo [2] introduced picture fuzzy graph. Muhammad Akram, Sairam Nawaz and Maji [1,5,6] described soft graph, fuzzy soft graph and fuzzy soft set. In this paper, we define the notion of picture fuzzy soft graph and introduce the concepts of pseudo degree picture fuzzy soft graph, pseudo regular picture fuzzy soft graph, totally pseudo regular picture fuzzy soft graph. And also we studied about their properties.

2. PRELIMINARIES

Definition: 2.1

If Z is a collection of object (or element) bestowed by z. Then **fuzzy set [8]** A' in Z is expressed as a set of ordered pair.

$$A' = \{(z, \lambda_{A'}(z)) : z \in Z\}$$

where, $\lambda_{A'}(z)$ is called the membership function (or characteristic function) which maps Z to the closed interval [0,1].

Definition: 2.2

Let D be initial universal set, Q be a set of parameters, $\wp(D)$ be the power set of D and $K \subseteq Q$. A pair (J, K) is called **soft set [5]** over D if and only if J is a mapping of K into the set of all subsets of the set D.

Definition: 2.3

A pair (J, K) is called **fuzzy soft set[1,6]** over D, where J is a mapping given by $J : K \to I^D$, I^D denote the collection of all fuzzy subset of $D, K \subseteq Q$

Definition: 2.4

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Let A' be a **picture fuzzy set[2,3].** A' in Z defined by
$A' = \{(z, \lambda_{A'}(z), \delta_{A'}(z), \varphi_{A'}(z)) : z \in Z\}$
where, $\lambda_{A'}(z) \in [0,1]$, $\delta_{A'}(z) \in [0,1]$ and $\varphi_{A'}(z) \in [0,1]$ follow the condition $0 \le \lambda_{A'}(z) + \delta_{A'}(z) + \varphi_{A'}(z) \le 1$. The $\lambda_{A'}(z)$ is used to represent the positive membership degree, $\delta_{A'}(z)$ is used to represent the neutral membership degree and $\varphi_{A'}(z)$ is used to represent the negative membership degree of the element z in the set A'. For each picture fuzzy set A' in Z, the refusal membership degree is described as $\pi_{A'}(z) = 1 - (\lambda_{A'}(z) + \delta_{A'}(z) + \varphi_{A'}(z))$.

3. PICTURE FUZZY SOFT GRAPH

Definition: 3.1

A ordered pair (J, K) is called **picture fuzzy soft set** over D, where J is a mapping given by $J : K \to IP^D$, where IP^D denote the collection of all picture fuzzy subset of D, $K \subseteq Q$.

Definition: 3.2

Let $G'^* = (W, Y)$ be a graph, $W = \{w_1, w_2, \ldots w_n\}$ be a non-empty set, $Y \subseteq W \times W$, Q be parameter set and $K \subseteq Q$. Also let,

i. a) λ_A is a positive membership function defined on W by

$\lambda_A : K \to IP^D(W)$ ($IP^D(W)$ denote collection of all picture fuzzy subset in W)

$k \to \lambda_A(k) = \lambda_{Ak}$ (say) , $k \in K$ and

$\lambda_{Ak} : W \to [0,1]$, $w_i \to \lambda_{Ak}(w_i)$

(K, λ_A) picture fuzzy soft vertex of positive membership function.

b) δ_A is a neutral membership function defined on W by

$\delta_A : K \to IP^D(W)$ ($IP^D(W)$ denote collection of all picture fuzzy subset in W)

$k \to \delta_A(k) = \delta_{Ak}$ (say) , $k \in K$ and

$\delta_{Ak} : W \to [0,1]$, $w_i \to \delta_{Ak}(w_i)$

(K, δ_A) picture fuzzy soft vertex of neutral membership function.

c) φ_A is a negative membership function defined on W by

$\varphi_A : K \to IP^D(W)$ ($IP^D(W)$ denote collection of all picture fuzzy

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subset in W)

$k \to \varphi_A(k) = \varphi_{Ak}$ (say) , $k \in K$ and

$\varphi_{Ak} : W \to [0,1]$, $w_i \to \varphi_{Ak}(w_i)$

(K, φ_A) picture fuzzy soft vertex of negative membership function.

such that $0 \le \lambda_{Ak}(w_i) + \delta_{Ak}(w_i) + \varphi_{Ak}(w_i) \le 1 \ \forall \ w_i \in W$, $k \in K$,

where A is a picture fuzzy soft set on W .

ii.a) λ_B is a positive membership function defined on Y by

$\lambda_B : K \to IP^D(W \times W)$ ($IP^D(W \times W)$ denote collection of all picture fuzzy subset in Y)

$k \to \lambda_B(k) = \lambda_{Bk}$ (say) , $k \in K$ and

$\lambda_{Bk} : W \times W \to [0,1]$, $(w_i, w_j) \to \lambda_{Bk}(w_i, w_j)$

(K, λ_B) picture fuzzy soft edge of positive membership function.

b) δ_B is a neutral membership function defined on Y by

$\delta_B : K \to IP^D(W \times W)$ ($IP^D(W \times W)$ denote collection of all picture fuzzy subset in Y)

$k \to \delta_B(k) = \delta_{Be}$ (say) , $k \in K$ and

$\delta_{Bk} : W \times W \to [0,1]$, $(w_i, w_j) \to \delta_{Bk}(w_i, w_j)$

(K, δ_B) picture fuzzy soft edge of neutral membership function.

c) φ_B is a negative membership function defined on Y by

$\varphi_B : K \to IP^D(W \times W)$ ($IP^D(W \times W)$ denote collection of all picture fuzzy subset in Y)

$k \to \varphi_B(k) = \varphi_{Bk}$ (say) , $k \in K$ and

$\varphi_{Bk} : W \times W \to [0,1]$, $(w_i, w_j) \to \varphi_{Bk}(w_i, w_j)$

(K, φ_B) picture fuzzy soft edge of negative membership function.

where, B is a picture fuzzy soft set on Y .

Also satisfying the following condition,

$$\lambda_{Bk}(w_i, w_j) \le \min(\lambda_{Ak}(w_i), \lambda_{Ak}(w_j)) , \delta_{Bk}(w_i, w_j) \le \min(\delta_{Ak}(w_i), \delta_{Ak}(w_j))$$

$\varphi_{Bk}(w_i, w_j) \ge \max(\varphi_{Ak}(w_i), \varphi_{Ak}(w_j))$ and

$0 \le \lambda_{Bk}(w_i, w_j) + \delta_{Bk}(w_i, w_j) + \varphi_{Bk}(w_i, w_j) \le 1 \ \forall \ (W_i, W_j) \in Y$, $i, j = 1,2,\ldots n$

and $k \in K$. Then

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$G'^{*} = (W, Y, (K, \lambda_A), (K, \delta_A), (K, \varphi_A), (K, \lambda_B), (K, \delta_B), (K, \varphi_B))$ is said to be **picture fuzzy soft graph** and this denoted by $G'^{*}_{K,W,Y}$.

Definition:3.3

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. Then the degree of the vertex $w_i \in G'^{*}_{K,W,Y}$ is defined by $d_{G'^{*}_{K,W,Y}}(w_i) = (d_{\lambda_A}(w_i), d_{\delta_A}(w_i), d_{\varphi_A}(w_i))$ where, $d_{\lambda_A}(w_i) = \sum_{k\in K}\sum_{w_i \neq w_j} \lambda_{Bk}(w_i, w_j)$, $d_{\delta_A}(w_i) = \sum_{k\in K}\sum_{w_i \neq w_j} \delta_{Bk}(w_i, w_j)$, $d_{\varphi_A}(w_i) = \sum_{k\in K}\sum_{w_i \neq w_j} \varphi_{Bk}(w_i, w_j)$ for $w_i, w_j \in Y$ and $\lambda_{Bk}(w_i, w_j) = \delta_{Bk}(w_i, w_j) = \varphi_{Bk}(w_i, w_j) = 0$ for $w_i, w_j \notin Y$.

Definition:3.4

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. Then the total degree of a vertex $w_i \in G'^{*}_{K,W,Y}$ is defined by $td_{G'^{*}_{K,W,Y}}(w_i) = (td_{\lambda_A}(w_i), td_{\delta_A}(w_i), td_{\varphi_A}(w_i))$ where,

$$td_{\lambda_A}(w_i) = \sum_{k\in K}\left(\sum_{w_i \neq w_j} \lambda_{Bk}(w_i, w_j) + \lambda_{Ak}\right), \quad td_{\delta_A}(w_i) = \sum_{k\in K}\left(\sum_{w_i \neq w_j} \delta_{Bk}(w_i, w_j) + \delta_{Ak}\right),$$

$$td_{\varphi_A}(w_i) = \sum_{k\in K}\left(\sum_{w_i \neq w_j} \varphi_{Bk}(w_i, w_j) + \varphi_{Ak}\right)$$

4. PSEUDO DEGREE OF A VERTEX IN PICTURE FUZZY SOFT GRAPH

Definition:4.1

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. Then 2 degree of a vertex $w \in G'^{*}_{K,W,Y}$ is defined by $t_{G'^{*}_{K,W,Y}}(w) = (t_{\lambda_A}(w), t_{\delta_A}(w) t_{\varphi_A}(w),)$ where, $t_{\lambda_A}(w) = \sum_{w \neq x} d_{\lambda_{Ak}}(x)$, $t_{\delta_A}(w) = \sum_{w \neq x} d_{\delta_{Ak}}(x)$, $t_{\varphi_A}(w) = \sum_{w \neq x} d_{\varphi_{Ak}}(x)$, the vertex *x* is the adjacent to the vertex *w*.

Definition:4.2

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. Then pseudo (average) degree of a vertex *w* in picture fuzzy soft graph is defined by $d_{aG'^{*}_{K,W,Y}}(w) = (d_{a\lambda_A}(w), d_{a\delta_A}(w), d_{a\varphi_A}(w))$ where, $d_{a\lambda_A}(w) = \dfrac{\sum_{k\in K} t_{\lambda_{Ak}}(w)}{d'_{G'^{*}_{K,W,Y}}(w)}$, $d_{a\delta_A}(w) = \dfrac{\sum_{k\in K} t_{\delta_{Ak}}(w)}{d'_{G'^{*}_{K,W,Y}}(w)}$, $d_{a\varphi_A}(w) = \dfrac{\sum_{k\in K} t_{\varphi_{Ak}}(w)}{d'_{G'^{*}_{K,W,Y}}(w)}$ where, $d'_{G'^{*}_{K,W,Y}}(w)$ is the number of edges incident at *w*.

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Definition:4.3

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. Then total pseudo degree of a vertex w in picture fuzzy soft graph is defined by $td_{aG'^{*}_{K,W,Y}}(w)=\left(td_{a\lambda_A}(w),td_{a\delta_A}(w),td_{a\varphi_A}(w)\right)$ where, $td_{a\lambda_A}(w)=d_{a\lambda_A}(w)+\sum_{k\in K}\lambda_{Ak}(w)$, $td_{a\delta_A}(w)=d_{a\delta_A}(w)+\sum_{k\in K}\delta_{Ak}(w)$, $td_{a\varphi_A}(w)=d_{a\varphi_A}(w)+\sum_{k\in K}\varphi_{Ak}(w)$ for all $w\in G'^{*}_{K,W,Y}$

5. PSEUDO REGULAR AND TOTALLY PSEUDO REGULAR PICTURE FUZZY SOFT GRAPH

Definition:5.1

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. If $d_{aG'^{*}_{K,W,Y}}(w)=(M_1,M_2,M_3)$ for all w in W then $G'^{*}_{K,W,Y}$ is called (M_1,M_2,M_3) - pseudo regular picture fuzzy soft graph.

Definition:5.2

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. If $td_{aG'^{*}_{K,W,Y}}(w)=(N_1,N_2,N_3)$ for all w in W then $G'^{*}_{K,W,Y}$ is called (N_1,N_2,N_3) - totally pseudo regular picture fuzzy soft graph.

Example:5.1

Consider, picture fuzzy soft graph $G'^{*}_{K,W,Y}=(W,Y,(K,\lambda_A),(K,\delta_A),(K,\varphi_A),(K,\lambda_B),(K,\delta_B),(K,\varphi_B))$, where $W=\{w_1,w_2,w_3\}$ and $Y=\{(w_1,w_2),(w_2,w_3),(w_1,w_3)\}$. Let $K=\{k_1,k_2,k_3\}$ be the parameter set.

Table:5.1 Picture fuzzy soft graph $G'^{*}_{K,W,Y}$

(a)

λ_{Ak}	w_1	w_2	w_3
k_1	0.2	0.2	0.2
k_2	0.3	0.2	0.3
k_3	0.2	0.3	0.2

(b)

δ_{Ak}	w_1	w_2	w_3
k_1	0.4	0.4	0.4
k_2	0.3	0.3	0.2
k_3	0.2	0.2	0.3

(c)

φ_{Ak}	w_1	w_2	w_3
k_1	0.4	0.2	0.4
k_2	0.2	0.4	0.4
k_3	0.4	0.4	0.2

(d)

λ_{Bk}	(w_1,w_2)	(w_2,w_3)	(w_1,w_3)
k_1	0.2	0.2	0.2
k_2	0.2	0.2	0.2
k_3	0.2	0.2	0.2

(e)

δ_{Bk}	(w_1,w_2)	(w_2,w_3)	(w_1,w_3)
k_1	0.2	0.2	0.2
k_2	0.2	0.2	0.2
k_3	0.2	0.2	0.2

(f)

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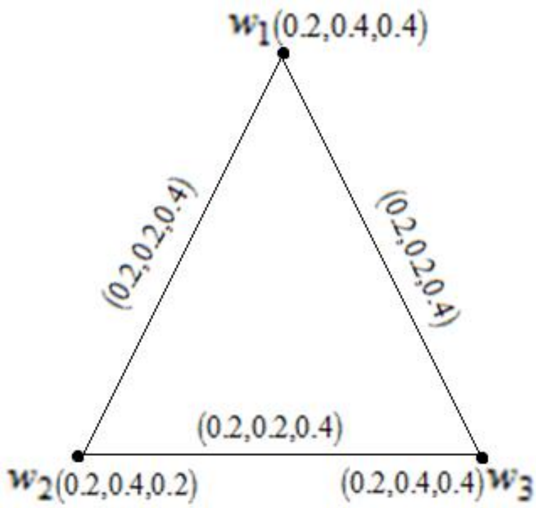
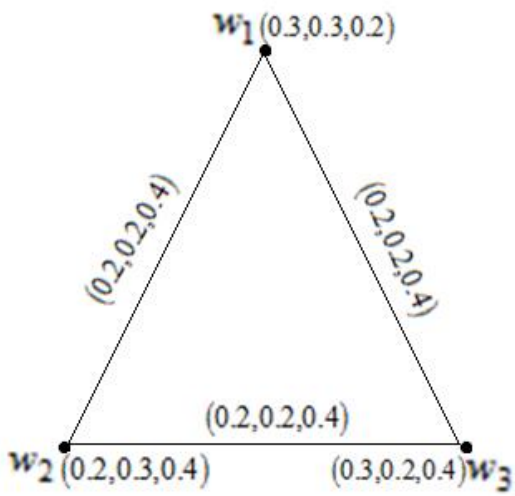
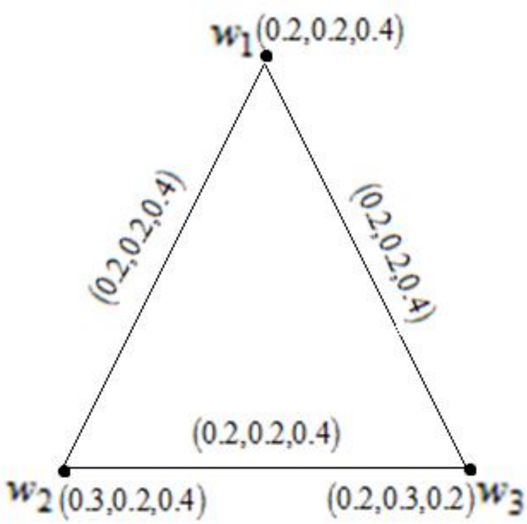
φ_{Bk}	(w_1, w_2)	(w_2, w_3)	(w_1, w_3)
k_1	0.4	0.4	0.4
k_2	0.4	0.4	0.4
k_3	0.4	0.4	0.4



Corresponding to the parameter (k_1)



Corresponding to the parameter (k_2)



Corresponding to the parameter (k_3)

Figure:5.1 Picture fuzzy soft graph $G'^{*}_{K,W,Y}$

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$d_{aG'^{*}_{K,W,Y}}(w_1)=(1.2,1.2,2.4)$, $d_{aG'^{*}_{K,W,Y}}(w_2)=(1.2,1.2,2.4)$, $d_{aG'^{*}_{K,W,Y}}(w_3)=(1.2,1.2,2.4)$

Here, all vertices have same pseudo degree. Hence, $G'^{*}_{K,W,Y}$ is (1.2,1.2,2.4) pseudo regular picture fuzzy soft graph.

$td_{aG'^{*}_{K,W,Y}}(w_1)=(1.9,2.1,3.4)$, $td_{aG'^{*}_{K,W,Y}}(w_2)=(1.9,2.1,3.4)$, $td_{aG'^{*}_{K,W,Y}}(w_3)=(1.9,2.1,3.4)$

Here, all vertices have same total pseudo degree. Hence, $G'^{*}_{K,W,Y}$ is (1.2,1.2,2.4) totally pseudo regular picture fuzzy soft graph.

Remark:5.1

A pseudo regular picture fuzzy soft graph need not be a totally pseudo regular picture fuzzy soft graph.

Remark:5.2

A totally pseudo regular picture fuzzy soft graph need not be a pseudo regular picture fuzzy soft graph.

Theorem:5.1

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph. Then $\alpha(w)=\sum_{k\in K}(\lambda_{Ak}(w),\delta_{Ak}(w),\varphi_{Ak}(w))$ for all $w\in W, k\in K$ is constant if and only if the following conditions are equivalent.

(i) $G'^{*}_{K,W,Y}$ is a pseudo regular picture fuzzy soft graph.

(ii) $G'^{*}_{K,W,Y}$ is a totally pseudo regular picture fuzzy soft graph.

Proof:

Let $G'^{*}_{K,W,Y}$ be the picture fuzzy soft graph.

Assume, $\alpha(w)$ is constant.

Let $\alpha(w)=\sum_{k\in K}(\lambda_{Ak}(w),\delta_{Ak}(w),\varphi_{Ak}(w))$

$=(c_1,c_2,c_3)$ for all $w\in W, k\in K$.

Suppose, $G'^{*}_{K,W,Y}$ is a pseudo regular picture fuzzy soft graph

then, $d_{aG'^{*}_{K,W,Y}}(w)=(M_1,M_2,M_3)$ for all $w\in W$

now, $td_{aG'^{*}_{K,W,Y}}(w)=d_{aG'^{*}_{K,W,Y}}(w)+\sum_{k\in K}(\lambda_{Ak}(w),\delta_{Ak}(w).\varphi_{Ak}(w))$

$=(M_1,M_2,M_3)+(c_1,c_2,c_3)$

$td_{aG'^{*}_{K,W,Y}}(w)=(M_1+c_1,M_2+c_2,M_3+c_3)$ for all $w\in W$.

Hence, $G'^{*}_{K,W,Y}$ is a totally pseudo regular picture fuzzy soft graph.

Suppose, $G'^{*}_{K,W,Y}$ is a totally pseudo regular picture fuzzy soft graph.

then, $td_{aG'^{*}_{K,W,Y}}(w)=(N_1,N_2,N_3)$ for all $w\in W$

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$$d_{aG'^{*}_{K,W,Y}}(w)+\alpha(w)=(N_1,N_2,N_3)$$
$$d_{aG'^{*}_{K,W,Y}}(w)+(c_1,c_2,c_3)=(N_1,N_2,N_3)$$
$$d_{aG'^{*}_{K,W,Y}}(w)=(N_1,N_2,N_3)-(c_1,c_2,c_3)$$
$$d_{aG'^{*}_{K,W,Y}}(w)=(N_1-c_1,N_2-c_2,N_3-c_3) \text{ for all } w\in W$$

Hence, $G'^{*}_{K,W,Y}$ is a pseudo regular picture fuzzy soft graph.

Therefore, (i) and (ii) are equivalent.
Conversely,
Suppose, (i) and (ii) are equivalent.

Let $G'^{*}_{K,W,Y}$ be the pseudo regular picture fuzzy soft graph and totally pseudo regular picture fuzzy soft graph.

Then, $d_{aG'^{*}_{K,W,Y}}(w)=(M_1,M_2,M_3)$ and $td_{aG'^{*}_{K,W,Y}}(w)=(N_1,N_2,N_3)$ for all $w\in W$

$$td_{aG'^{*}_{K,W,Y}}(w)=d_{aG'^{*}_{K,W,Y}}(w)+\alpha(w)$$
$$(N_1,N_2,N_3)=(M_1,M_2,M_3)+\alpha(w)$$
$$\alpha(w)=(M_1,M_2,M_3)-(N_1,N_2,N_3)$$
$$\alpha(w)=(M_1-N_1,M_2-N_2,M_3-N_3) \text{ for all } w\in W$$

Hence, $\alpha(w)$ is constant function.

Theorem:5.2

If $G'^{*}_{K,W,Y}$ is a regular picture fuzzy soft graph. Then $G'^{*}_{K,W,Y}$ is a pseudo regular picture fuzzy soft graph.

Theorem:5.3

Let $G'^{*}_{K,W,Y}$ be totally regular picture fuzzy soft graph and *c* is a constant function. Then $G'^{*}_{K,W,Y}$ is a pseudo regular picture fuzzy soft graph.

Theorem:5.4

Let $G'^{*}_{K,W,Y}$ be totally regular picture fuzzy soft graph and *c* is a constant function. Then $G'^{*}_{K,W,Y}$ is a totally pseudo regular picture fuzzy soft graph.

6. CONCLUSION

In this paper, we introduce the idea of pseudo degree picture fuzzy soft graph, pseudo regular picture fuzzy soft graph, totally pseudo regular picture fuzzy soft graph and also their properties are discussed.

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