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The (a, b)-Status Index of Exponential Domination Graphs

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ABSTRACT

The sum of degree between u and all other vertices of a connected graph determines the status of a vertex u. The (a, b)-status index of exponential domination graphs is analyzed and discussed. The (a, b)-status index of path, cycle, complete graph, and complete bipartite graphs is also computed. We also compute precise formulas for path, cycle, complete graph, and complete bipartite graphs, as well as the F_1-status index, first and second status Gourava indices, and symmetric division status index of a graph.

Keywords: Distance, (a, b)-status index, F_1-status index, symmetric division status index.

Introduction

The well-known topic of graph domination is a valuable tool for assessing situations that may be represented by networks and in which a vertex has control or domination over all vertices in its general area. In some real life, a vertex can influence not only the vertices in its immediate vicinity, but also all vertices within a certain distance. This circumstance is known as distance domination. Domination can take many different forms. Some of them take the distance between a vertex and the set into consideration. For example, in distance domination, a vertex dominates all other vertices within a specified distance. When a vertex's dominance reduces as distance grows, Dankelmann et.al.[1] investigated into it. The dominating power of a vertex decreases exponentially with distance, by a factor of $\frac{1}{2}$, with a distance . As a result, a vertex v can be dominated by a neighbour or a group of vertices that are close to v. A model such as this could be used to study the spread of information in social networks, where the importance of the information decreases with each transmission. Obtaining the exponential domination number in this application includes identifying the smallest number of sources required to provide each person with trustable information. Domination comes in a variety of forms. Some of these take account the distance between a vertex and the set. In distance domination, for example, a vertex dominates all vertices within a certain distance.

Let G be a finite, simple, connected graph. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of G. The degree $d_G(u)$ of vertex u is the number of vertices adjacent to u. The distance, denoted by $d(u,v)$, between any two vertices u and v is the length of shortest path connecting u and v. The status $\sigma(u)$ of vertex u in G is the sum of degree of all other vertices from u in G. For undefined terms and notations, we refer [2]. A graph index is a numerical parameter mathematically

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derived from the graph structure. The graph indices have their applications in various disciplines of Science and Technology [3,4]. Some of the graph indices can be found in [5, 6, 7, 8, 9, 10, 11]. In chemical graph theory, we have many invariant polynomials and topological indices for a molecular graph. A topological index is a numeric value for correction of chemical structure with various physical properties, chemical reactivity, or biological activity. The first and second status connectivity indices were introduced by Ramane at al.in [12].Recently the first and second Gourava indices were studied in [13].Recently, some status indices were introduced and studied such as multiplicative vertex status index [14], multiplicative first and second status indices [15], multiplicative (a, b)-status index, F-status index [16], In his paper, the (a, b)-status index of path, cycle, complete graph and complete bipartite graphs are determined. V. R. Kulli introduce the (a, b)-status index of a graph and it is defined as

$$S_{a,b}(G) = \sum_{u,v \in V} \left[\sigma(u)^a \sigma(v)^b + \sigma(u)^b \sigma(v)^a \right]$$

Definition 1. For the graph G

1. The first status index $S_1(G) = S_{1,0}(G)$
2. The second status index $S_2(G) = \frac{1}{2} S_{1,1}(G)$
3. The product connectivity status index $PS(G) = \frac{1}{2} S_{\frac{-1}{2},\frac{-1}{2}}(G)$
4. The reciprocal product connectivity status index $RPS(G) = \frac{1}{2} S_{\frac{1}{2},\frac{1}{2}}(G)$
5. The general second status index $S_2^a(G) = \frac{1}{2} S_{a,a}(G)$
6. The $F_1 - Status$ index $F_1S(G) = S_{2,0}(G)$
7. The second status Gourava index $SGO_2(G) = S_{2,1}(G)$
8. The symmetric division status index $SDS(G) = S_{1.-1}(G)$

Definition 2.

Let G be a graph and $D \subseteq V(G)$ we denote by $\langle D \rangle$ the sub graph of G induced by D. For each vertex $v \in D$ and for each vertex $v \in V(G) - D$. We define $\bar{d}(u,v) = \bar{d}(v,u)$ to be a length of shortest path in $\langle V(G) - (D - \{u\}) \rangle$ if such a path exists , and ∞ otherwise. Let $v \in V(G)$. The definition is

$$w_D(v) = \begin{cases} \sum_{u \in D} \left(\frac{1}{2} \right)^{\bar{d}(u,v)-1} & , if \;\; v \in D \\ 2, & if \;\; v \notin D \end{cases}$$

We refer to $w_D(v)$ as the weight of D at v then D is exponential dominating set.

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Basic Results

Theorem 2.1. For every positive integer n , $\gamma_e(P_n) = \left\lceil \frac{n+1}{4} \right\rceil$

Theorem 2.2. For every positive integer $n \geq 3$, $\gamma_e(C_n) = \begin{cases} 2, & if \;\; n=4 \\ \left\lceil \frac{n}{4} \right\rceil, & otherwise \end{cases}$

Main Results

Theorem 3.1.

The (a,b)- status index of a Exponential Path graph $ex(P_n)$ is
$S_{a,b}(\gamma_e(P_n)) = 2\left[1^a\,2^b + 1^b\,2^a\right] + (n-2)\left[2(n-2)^{a+b}\right] + 2\left[\,2^a\,1^b + 2^b\,1^a\right]$……..(i)

Proof :

An exponential path graph P_n , $n \geq 2$, is a graph that can be constructed by joining 3 copies of $V_1, V_2\, and\, V_3$ with a common vertices.

Let $\gamma_e(P_n)$ be an exponential path graph with n vertices and $n-1$ edges. By calculation we obtain that there are three types of vertices as follows:

$V_1 = \{u,v \in V(\gamma_e(P_n)) / d(u) = 1, d(v) = 2\}$, $|V_1| = 2$

$V_2 = \{u,v \in V(\gamma_e(P_n)) / d(u) = n-2 = d(v)\}$, $|V_2| = n-2$

$V_3 = \{u,v \in V(\gamma_e(P_n)) / d(u) = 2, d(v) = 1\}$, $|V_3| = 2$

Therefore there are three types of status vertices as given below

Table 1: Edge Partition of exponential path graph

$\sigma(u),\sigma(v) \setminus u,v \in V(\gamma_e(P_n))$	$(1,2)$	$(n-2,n-2)$	$(2,1)$
$Number\ of\ vertices$	2	$n-2$	2

By Using definition , we deduce

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$$S_{a,b}\left(\gamma_e\left(P_n\right)\right)=\sum_{u,v\in V\left(ex\left(P_n\right)\right)}\left[\sigma(u)^a\,\sigma(v)^b+\sigma(u)^b\,\sigma(v)^a\right]$$

$$=2\left[1^a\,2^b+1^b\,2^a\right]+\left(n-2\right)\left[\left(n-2\right)^a\left(n-2\right)^b+\left(n-2\right)^b\left(n-2\right)^a\right]+2\left[2^a\,1^b+2^b\,1^a\right]$$

$$=2\left[1^a\,2^b+1^b\,2^a\right]+\left(n-2\right)\left[2\left(n-2\right)^{a+b}\right]+2\left[2^a\,1^b+2^b\,1^a\right]$$

Theorem 3.2. $SGO_1\left(\gamma_e\left(P_n\right)\right)=n^3-4n^2+4n+20$

Proof. By definition first Gourava index , we have

$$SGO_1\left(\gamma_e\left(P_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(P_n\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\,\sigma(v)\right]$$

Using above table, we note that

$$SGO_1\left(\gamma_e\left(P_n\right)\right)=2\left[\left(1+2\right)+\left(1.2\right)\right]+\left(n-2\right)\left[\left(n-2\right)+\left(n-2\right)+\left(n-2\right)\left(n-2\right)\right]+2\left[\left(2+1\right)+\left(2.1\right)\right]$$

$$=10+\left(n^3-4n^2+4n\right)+10$$

$$=n^3-4n^2+4n+20$$

Corollary 3.1. From observations and by using equation (i), we establish the following results.

1. The first status index $S_1\left(\gamma_e\left(P_n\right)\right)=2\left(n-2\right)^2+12$
2. The second status index $S_2\left(\gamma_e\left(P_n\right)\right)=\left(n-2\right)^3+8$
3. The product connectivity status index $PS\left(\gamma_e\left(P_n\right)\right)=\frac{1}{2}S_{\frac{-1}{2},\frac{-1}{2}}\left(\gamma_e\left(P_n\right)\right)=1+2\sqrt{2}$
4. The reciprocal product connectivity status index $RPS\left(\gamma_e\left(P_n\right)\right)=\frac{1}{2}S_{\frac{1}{2},\frac{1}{2}}(\gamma_e\left(P_n\right))=\left(n-2\right)^2+4\sqrt{2}$
5. The general second status index $S_2^a\left(\gamma_e\left(P_n\right)\right)=\frac{1}{2}S_{a,a}\left(\gamma_e\left(P_n\right)\right)=4.2^a+\left(n-2\right)^{2a+1}$
6. The $F_1-Status$ index $F_1S\left(\gamma_e\left(P_n\right)\right)=S_{2,0}\left(\gamma_e\left(P_n\right)\right)=2\left(n-2\right)^3+20$
7. The second status Gourava index $SGO_2\left(\gamma_e\left(P_n\right)\right)=S_{2,1}\left(\gamma_e\left(P_n\right)\right)=2\left(n-2\right)^4+24$
8. The symmetric division status index $SDS\left(\gamma_e\left(P_n\right)\right)=S_{1,-1}\left(\gamma_e\left(P_n\right)\right)=2n+6$

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Theorem 3.3.

The (a,b)- status index of a Exponential Cycle graph $ex(C_n)$ is

$$S_{a,b} = \begin{cases} 2\left(\frac{n^2}{4}\right)^{a+b}, & if\ \ n\ is\ even \\ 2\left(\frac{n^2-1}{4}\right)^{a+b}, & if\ \ n\ is\ odd \end{cases} \quad \text{………(ii)}$$

Proof.

An exponential cycle graph C_n , is a graph that can be constructed by joining n copies of V_1 *and* V_2 with a common vertices.

Let $\gamma_e(C_n)$ be an exponential cycle graph with n vertices and n edges. By calculation we obtain that there are two types of vertices as follows:

$$V_1 = \{u,v \in V(\gamma_e(C_n)) / d(u) = 2, d(v) = 2\},\ |V_1| = \frac{n^2}{4}\ if\ \ n\ is\ even$$

$$V_2 = \{u,v \in V(\gamma_e(C_n)) / d(u) = 2, d(v) = 2\},\ |V_1| = \frac{n^2-1}{4}\ if\ \ n\ is\ odd$$

Therefore there are three types of status vertices as given below

Table 2: Edge Partition of exponential cycle graph

$\sigma(u),\sigma(v) \setminus u,v \in V(\gamma_e(C_n))$	$(2,2)$	$(2,2)$
Number of vertices	$\frac{n^2}{4}$	$\frac{n^2-1}{4}$

By Using definition , we deduce

$$S_{a,b}(\gamma_e(C_n)) = \sum_{u,v\in V(\gamma_e(C_n))} \left[\sigma(u)^a\,\sigma(v)^b + \sigma(u)^b\,\sigma(v)^a\right]$$

$case(i)$: Suppose n is even. Then $\sigma(u) = \frac{n^2}{4}$ for any vertex u of C_n. Thus

$$S_{a,b}(\gamma_e(C_n)) = \sum_{u,v\in V(\gamma_e(C_n))} \left[\sigma(u)^a\,\sigma(v)^b + \sigma(u)^b\,\sigma(v)^a\right]$$

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$$=\left[\left(\frac{n^2}{4}\right)^a\left(\frac{n^2}{4}\right)^b+\left(\frac{n^2}{4}\right)^b\left(\frac{n^2}{4}\right)^a\right]$$

$$=\left[\left(\frac{n^2}{4}\right)^{a+b}+\left(\frac{n^2}{4}\right)^{b+a}\right]$$

$$=\left[2\left(\frac{n^2}{4}\right)^{a+b}\right]$$

$case(ii)$: Suppose n is odd. Then $\sigma(u)=\frac{n^2-1}{4}$ for any vertex u of C_n. Thus

$$S_{a,b}\left(\gamma_e\left(C_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(C_n\right)\right)}\left[\sigma(u)^a\,\sigma(v)^b+\sigma(u)^b\,\sigma(v)^a\right]$$

$$=\left[\left(\frac{n^2-1}{4}\right)^a\left(\frac{n^2-1}{4}\right)^b+\left(\frac{n^2-1}{4}\right)^b\left(\frac{n^2-1}{4}\right)^a\right]$$

$$=\left[\left(\frac{n^2-1}{4}\right)^{a+b}+\left(\frac{n^2-1}{4}\right)^{b+a}\right]$$

$$=\left[2\left(\frac{n^2-1}{4}\right)^{a+b}\right]$$

Theorem 3.4. $SGO_1\left(\gamma_e\left(C_n\right)\right)=\begin{cases}2n^2, & if\ \ n\ is\ even\\ 2\left(n^2-1\right), & if\ \ n\ is\ odd\end{cases}$

Proof. By definition first Gourava index, we have

$$SGO_1\left(\gamma_e\left(C_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(C_n\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\,\sigma(v)\right]$$

Using above table, we note that

$case(i)$: Suppose n is even.

$$SGO_1\left(\gamma_e\left(C_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(C_n\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\,\sigma(v)\right]$$

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$$SGO_1\left(\gamma_e\left(C_n\right)\right)=\frac{n^2}{4}\left[\left(2+2\right)+\left(2.2\right)\right]$$

$$=\frac{n^2}{4}[8]$$

$$=2n^2$$

$case(ii):$ Suppose n is odd.

$$SGO_1\left(\gamma_e\left(C_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(C_n\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)\right]$$

$$SGO_1\left(\gamma_e\left(C_n\right)\right)=\frac{n^2-1}{4}\left[\left(2+2\right)+\left(2.2\right)\right]$$

$$=\frac{n^2-1}{4}[8]$$

$$=2\left(n^2-1\right)$$

Therefore , $SGO_1\left(\gamma_e\left(C_n\right)\right)=\begin{cases}2n^2, & if\ \ n\ is\ even\\ 2\left(n^2-1\right), & if\ \ n\ is\ odd\end{cases}$

Corollary 1.2. From observations and by using equation (ii), we establish the following results.

1. The first status index $S_1\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2}{4}\right)^{1+0}=\left(\frac{n^2}{2}\right)$ for n is even

$S_1\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2-1}{4}\right)^{1+0}=\left(\frac{n^2-1}{2}\right)$ for n is odd

2. The second status index

$S_2\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{1,1}(\gamma_e\left(C_n\right))=\frac{1}{2}2\left(\frac{n^2}{4}\right)^{1+1}=\left(\frac{n^2}{4}\right)^2=\left(\frac{n^4}{16}\right)$ for n is even

$S_2\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{1,1}(\gamma_e)=\frac{1}{2}2\left(\frac{n^2-1}{4}\right)^{1+1}=\left(\frac{n^2-1}{4}\right)^2=\left(\frac{(n^2-1)^2}{16}\right)$ for n is odd

3. The product connectivity status index $PS\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{\frac{-1}{2},\frac{-1}{2}}\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}2\left(\frac{n^2}{4}\right)^{-\frac{1}{2}-\frac{1}{2}}=\frac{4}{n^2}$ for n is even

$$PS\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{\frac{-1}{2},\frac{-1}{2}}\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}2\left(\frac{n^2-1}{4}\right)^{-\frac{1}{2}-\frac{1}{2}}=\frac{4}{n^2-1} \text{ for } n \text{ is odd}$$

4. The reciprocal product connectivity status index $RPS\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{\frac{1}{2},\frac{1}{2}}(\gamma_e\left(C_n\right))=\frac{1}{2}2\left(\frac{n^2}{4}\right)^{\frac{1}{2}+\frac{1}{2}}=\left(\frac{n^2}{4}\right)$ for n is even

$$RPS\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{\frac{1}{2},\frac{1}{2}}(\gamma_e\left(C_n\right))=\frac{1}{2}2\left(\frac{n^2-1}{4}\right)^{\frac{1}{2}+\frac{1}{2}}=\left(\frac{n^2-1}{4}\right) \text{ for } n \text{ is odd}$$

5. The general second status index

$$S_2^a\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{a,a}\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}2\left(\frac{n^2}{4}\right)^{a+a}=\left(\frac{n^2}{4}\right)^{2a} \text{ for } n \text{ is even}$$

$$S_2^a\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}S_{a,a}\left(\gamma_e\left(C_n\right)\right)=\frac{1}{2}2\left(\frac{n^2-1}{4}\right)^{a+a}=\left(\frac{n^2-1}{4}\right)^{2a} \text{ for } n \text{ is odd}$$

6. The $F_1-Status$ index

$$F_1S\left(\gamma_e\left(C_n\right)\right)=S_{2,0}\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2}{4}\right)^{2+0}=\frac{n^4}{8} \text{ for } n \text{ is even}$$

$$F_1S\left(\gamma_e\left(C_n\right)\right)=S_{2,0}\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2-1}{4}\right)^{2+0}=\left(\frac{\left(n^2-1\right)^2}{8}\right) \text{ for } n \text{ is odd}$$

7. The second status Gourava index

$$SGO_2\left(\gamma_e\left(C_n\right)\right)=S_{2,1}\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2}{4}\right)^{2+1}=\frac{n^6}{32} \text{ for } n \text{ is even}$$

$$SGO_2\left(\gamma_e\left(C_n\right)\right)=S_{2,1}\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2-1}{4}\right)^{2+1}=\frac{\left(n^2-1\right)^3}{32} \text{ for } n \text{ is odd}$$

8. The symmetric division status index

$$SDS\left(\gamma_e\left(C_n\right)\right)=S_{1,-1}\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2}{4}\right)^{1-1}=2 \text{ for } n \text{ is even}$$

$$SDS\left(\gamma_e\left(C_n\right)\right)=S_{1,-1}\left(\gamma_e\left(C_n\right)\right)=2\left(\frac{n^2-1}{4}\right)^{1-1}=2 \text{ for } n \text{ is odd}$$

Theorem 3.5.

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The (a,b) - status index of a Exponential complete graph $\gamma_e\left(K_n\right)$ is $S_{a,b}\left(\gamma_e\left(K_n\right)\right)=2n\left[\left(n-1\right)^{a+b}\right]$……..(iii)

Proof :

An exponential complete graph K_n , is a graph that can be constructed by joining one copy of V_1 with a common vertices.

Let $\gamma_e\left(K_n\right)$ be an exponential complete graph with n vertices and $\frac{n(n-1)}{2}$ edges. By calculation we obtain that there is one type of vertices as:

$$V_1=\left\{u,v\in V\left(\gamma_e\left(K_n\right)\right)/\,d(u)=n-1,d(v)=n-1\right\},\ \left|V_1\right|=n$$

Therefore there is type of status vertices as given below

Table 3: Edge Partition of exponential complete graph

$\sigma(u),\sigma(v)\setminus u,v\in V\left(\gamma_e\left(K_n\right)\right)$	$(n-1,n-1)$
$Number\ of\ vertices$	n

By Using definition , we deduce

$$S_{a,b}\left(\gamma_e\left(K_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(K_n\right)\right)}\left[\sigma(u)^a\,\sigma(v)^b+\sigma(u)^b\,\sigma(v)^a\right]$$

$$=n\left[\left(n-1\right)^a\left(n-1\right)^b+\left(n-1\right)^b\left(n-1\right)^a\right]$$

$$=2n\left[\left(n-1\right)^{a+b}\right]$$

Theorem 3.6. $SGO_1\left(\gamma_e\left(K_n\right)\right)=n^3-n$

Proof. By definition first Gourava index , we have

$$SGO_1\left(\gamma_e\left(K_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(K_n\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)\right]$$

Using above table, we note that

$$SGO_1\left(\gamma_e\left(K_n\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(K_n\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\sigma(v)\right]$$

$$SGO_1\left(\gamma_e\left(K_n\right)\right)=n\left[\left(n-1\right)+\left(n-1\right)+\left(n-1\right)\left(n-1\right)\right]$$

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$$=n\left[2n-2+n^2-2n+1\right]$$

$$=n\left[n^2-1\right]$$

$$=n^3-n$$

Corollary 1.3. From observations and by using equation (iii), we establish the following results.

1. The first status index $S_1\left(\gamma_e\left(K_n\right)\right)=2n\left(n-1\right)^{1+0}=2n^2-2n$
2. The second status index $S_2\left(\gamma_e\left(K_n\right)\right)=\frac{1}{2}2n\left(n-1\right)^{1+1}=n\left(n-1\right)^2=n^3-2n^2+n$
3. The product connectivity status index $PS\left(\gamma_e\left(K_n\right)\right)=\frac{1}{2}S_{\frac{-1}{2},\frac{-1}{2}}\left(\gamma_e\left(K_n\right)\right)=\frac{1}{2}2n\left(n-1\right)^{-\frac{1}{2}-\frac{1}{2}}=\frac{n}{n-1}$
4. The reciprocal product connectivity status index $RPS\left(\gamma_e\left(K_n\right)\right)=\frac{1}{2}S_{\frac{1}{2},\frac{1}{2}}(G)=\frac{1}{2}2n\left(n-1\right)^{\frac{1}{2}+\frac{1}{2}}=n\left(n-1\right)=n^2-n$
5. The general second status index $S_2^a\left(\gamma_e\left(K_n\right)\right)=\frac{1}{2}S_{a,a}\left(\gamma_e\left(K_n\right)\right)=\frac{1}{2}2n\left(n-1\right)^{a+a}=n\left(n-1\right)^{2a}$
6. The $F_1-Status$ index $F_1S\left(\gamma_e\left(K_n\right)\right)=S_{2,0}\left(\gamma_e\left(K_n\right)\right)=2n\left(n-1\right)^{2+0}=2n\left(n-1\right)^2=2n^3-4n^2+2n$
7. The second status Gourava index $SGO_2\left(\gamma_e\left(K_n\right)\right)=S_{2,1}\left(\gamma_e\left(K_n\right)\right)=2n\left(n-1\right)^{2+1}=2n\left(n-1\right)^3=2n^4-6n^3+6n^2-2n$
8. The symmetric division status index $SDS\left(\gamma_e\left(K_n\right)\right)=S_{1,-1}\left(\gamma_e\left(K_n\right)\right)=2n\left(n-1\right)^{1-1}=2n$

Theorem 3.7.

The (a,b)- status index of a Exponential complete bipatite graph $\gamma_e\left(K_n\right)$ is $S_{a,b}\left(\gamma_e(K_{m,n}\right)=\left(m+n\right)\left[\left(n+2\left(m-1\right)\right)^a\left(m+2\left(n-1\right)\right)^b+\left(n+2\left(m-1\right)\right)^b\left(m+2\left(n-1\right)\right)^a\right]$ ……(iv)

Proof :

An exponential complete bipartite graph $K_{m,n}$, is a graph that can be constructed by joining one copy of V_1 with a common vertices.

Let $\gamma_e\left(K_{m,n}\right)$ be an exponential complete bipartite graph with $n+m$ vertices and mn edges. By calculation we obtain that there is one type of vertices as:

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$$V_1=\left\{u,v\in V\left(\gamma_e\left(K_{m,n}\right)\right)/\,d(u)=m,d(v)=n\right\},\ \left|V_1\right|=m+n$$

Therefore there is type of status vertices as given below

Table 4: Edge Partition of exponential bipartite graph

$\sigma(u),\sigma(v)\setminus u,v\in V\left(\gamma_e\left(K_{m,n}\right)\right)$	$\left(n+2(m-1),m+2(n-1)\right)$
Number of vertices	$m+n$

By Using definition , we deduce

$$S_{a,b}\left(\gamma_e\left(K_{m,n}\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(K_{m,n}\right)\right)}\left[\sigma(u)^a\,\sigma(v)^b+\sigma(u)^b\,\sigma(v)^a\right]$$

$$=\left(m+n\right)\left[\left(n+2\left(m-1\right)\right)^a\left(m+2\left(n-1\right)\right)^b+\left(n+2\left(m-1\right)\right)^b\left(m+2\left(n-1\right)\right)^a\right]$$

Theorem 3.8. $SGO_1\left(\gamma_e\left(K_{m,n}\right)\right)=\left(m+n\right)\left[\left(n+2(m-1)\right)+\left(m+2(n-1)\right)+\left(n+2(m-1)\right)\left(m+2(n-1)\right)\right]$

Proof. By definition first Gourava index , we have

$$SGO_1\left(\gamma_e\left(K_{m,n}\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(K_{m,n}\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\,\sigma(v)\right]$$

Using above table, we note that

$$SGO_1\left(\gamma_e\left(K_{m,n}\right)\right)=\sum_{u,v\in V\left(\gamma_e\left(K_{m,n}\right)\right)}\left[\sigma(u)+\sigma(v)+\sigma(u)\,\sigma(v)\right]$$

$$SGO_1\left(\gamma_e\left(K_{m,n}\right)\right)=\left(m+n\right)\left[\left(n+2(m-1)\right)+\left(m+2(n-1)\right)+\left(n+2(m-1)\right)\left(m+2(n-1)\right)\right]$$

Corollary 1.4. From observations and by using equation (iv), we establish the following results.

1. The first status index

$$S_1\left(\gamma_e\left(K_{m,n}\right)\right)=\left(m+n\right)\left[\left(n+2(m-1)\right)^1\left(m+2(n-1)\right)^0+\left(n+2(m-1)\right)^0\left(m+2(n-1)\right)^1\right]$$

$$=\left(m+n\right)\left[\left(n+2(m-1)\right)^1+\left(m+2(n-1)\right)^1\right]$$

$$=\left(m+n\right)\left[3m+3n-4\right]$$

2. The second status index

$$S_2\left(\gamma_e\left(K_{m,n}\right)\right)=\frac{1}{2}S_{1,1}\left(\gamma_e\left(K_{m,n}\right)\right)=\frac{1}{2}(m+n)\left[\left(n+2(m-1)\right)^1\left(m+2(n-1)\right)^1+\left(n+2(m-1)\right)^1\left(m+2(n-1)\right)^1\right]$$

$$=(m+n)\left[\left(n+2(m-1)\right)\left(m+2(n-1)\right)\right]$$

3. The product connectivity status index

$$PS\left(\gamma_e\left(K_{m,n}\right)\right)=\frac{1}{2}S_{\frac{-1}{2},\frac{-1}{2}}\left(\gamma_e\left(K_{m,n}\right)\right)=\frac{1}{2}(m+n)\left[\left(n+2(m-1)\right)^{\frac{-1}{2}}\left(m+2(n-1)\right)^{\frac{-1}{2}}+\left(n+2(m-1)\right)^{\frac{-1}{2}}\left(m+2(n-1)\right)^{\frac{-1}{2}}\right]$$

$$=(m+n)\left[\left(n+2(m-1)\right)^{\frac{-1}{2}}\left(m+2(n-1)\right)^{\frac{-1}{2}}\right]$$

4. The reciprocal product connectivity status index

$$RPS\left(\gamma_e\left(K_{m,n}\right)\right)=\frac{1}{2}S_{\frac{1}{2},\frac{1}{2}}(\gamma_e\left(K_{m,n}\right))=\frac{1}{2}\ (m+n)\left[\left(n+2(m-1)\right)^{\frac{1}{2}}\left(m+2(n-1)\right)^{\frac{1}{2}}+\left(n+2(m-1)\right)^{\frac{1}{2}}\left(m+2(n-1)\right)^{\frac{1}{2}}\right]$$

$$=(m+n)\left[\left(n+2(m-1)\right)^{\frac{1}{2}}\left(m+2(n-1)\right)^{\frac{1}{2}}\right]$$

5. The general second status index

$$S_2^a\left(\gamma_e\left(K_{m,n}\right)\right)=\frac{1}{2}S_{a,a}\left(\gamma_e\left(K_{m,n}\right)\right)=\frac{1}{2}(m+n)\left[\left(n+2(m-1)\right)^a\left(m+2(n-1)\right)^a+\left(m+2(n-1)\right)^a\left(n+2(m-1)\right)^a\right]$$

$$=(m+n)\left[\left(n+2(m-1)\right)^a\left(m+2(n-1)\right)^a\right]$$

6. The $F_1-Status$ index

$$F_1S\left(\gamma_e\left(K_{m,n}\right)\right)=S_{2,0}\left(\gamma_e\left(K_{m,n}\right)\right)=(m+n)\left[\left(n+2(m-1)\right)^2\left(m+2(n-1)\right)^0+\left(n+2(m-1)\right)^0\left(m+2(n-1)\right)^2\right]$$

$$F_1S\left(\gamma_e\left(K_{m,n}\right)\right)=S_{2,0}\left(\gamma_e\left(K_{m,n}\right)\right)=(m+n)\left[\left(n+2(m-1)\right)^2+\left(m+2(n-1)\right)^2\right]$$

7. The second status Gourava index

$$SGO_2\left(\gamma_e\left(K_{m,n}\right)\right)=S_{2,1}\left(\gamma_e\left(K_{m,n}\right)\right)=(m+n)\left[\left(n+2(m-1\right)^2\left(m+2(n-1\right)^1+\left(n+2(m-1\right)^1\left(m+2(n-1\right)^2\right]$$

8. The symmetric division status index

$$SDS\left(\gamma_e\left(K_{m,n}\right)\right)=S_{1,-1}\left(\gamma_e\left(K_{m,n}\right)\right)=(m+n)\left[\left(n+2(m-1)\right)^1\left(m+2(n-1)\right)^{-1}+\left(n+2(m-1)\right)^{-1}\left(m+2(n-1)\right)^1\right]$$

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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