JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2465 - 2469
https://publishoa.com
ISSN: 1309-3452

A Short Note on Distance2-domination for some family of graphs

B. Suganya[1], Dr. J. Saral[2]
[1]*Department of Mathematics*
Kalasalingam Academy of Research and Education
Srivilliputhur, India
[2]*Department of Mathematics*
SRM Institute of Science and Technology
Chennai, India
[1]*suganyaptj@gmail.com*, [2]*saralj@srmist.edu.in*

ABSTRACT

The aim of this paper is to study the importance of graph theoretical concepts and the applications of distance - 2domination of some graphs. A set D is a distance – 2 dominating set iff or every vertex $u \in V-D, d(u,D) \leq 2$ andisdenotedby $\gamma_{\leq 2}(G)$.Also we discuss about many bounds and exact values for some standard graphs.

Keywords: *Dominating set ,connected dominating set, Mobile adhoc network, distance- 2dominatingset.*

1. Introduction

Mathematically, graph theory is a branch of mathematics. It is actively used in biochemistry, chemistry, communication networks and coding theory, computer science (algorithms and computation), and operations research (scheduling), as well as many applications including coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, and data base management. The concept of partitioning vertices or edges of a graph into sets based on certain properties is applicable to a variety of practical problems such as coding theory, logic partitioning in digital computers, state reduction in sequential machines, and so on. Problems of this type include chromatic partitioning, domination, matching, and labelling. Euler's concept of planarity was later nurtured by Kuratowski in 1930. Kuratowski defined planar graphs with a theorem he devised. The concept of planarity has been used in a variety of applications, including electrical and road networks. In 1936, psychologist Kurt Lewin proposed that an individual's "life space" can be represented by a planar graph. A planar graph's regions resemble an atlas. Mobius proposed the four-color conjecture, which states that "four colours are sufficient to colours any atlas," in 1840, while attempting to colours the map of the countries of England, and it is still a conjecture today.

This paper defines and discusses the distance - 2 dominating set for some graph families. All of the graphs under consideration here are simple and finite.

The graph $G = (V, E)$, where V is a finite set of elements known as vertices and E is a set of unordered pairs of distinct vertices of G known as edges. The number of edges incident on a vertex v in G determines its degree. Every pair of adjacent vertices in G is said to be complete, and the complete graph on$n$vertices is denoted by K_n. Let $u$and v be the vertices of a graph G, and an uv walk of G is an alternating sequence of vertices and edges beginning with vertex u and ending with vertex v such that $e_{i=}u_{i-1}v_i$for all $i = 1,2, \ldots, n$. The number of edges in a walk is referred to as its length. A path is a walk in which all of the vertices are separated by a distance. P_n represents a path with$n$ vertices. A closed path is referred to as a cycle, and a cycle with n vertices is denoted by C_n. Let $G = (V, E)$ be a simple connected graph, and for any vertex $v \in V$, the open neighborhood is the set $N(v) = \{u \in V/uv \in E\}$, and the closed neighborhood is the set $N[v] = N(v) \cup \{u\}$. For a set $S \subset V$, the open neighbourhood of S is $N(S) = \bigcup N(v)$, $v \in S$, and the closed neighbourhood of S is $N[S] \cup S$.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2465 - 2469
https://publishoa.com
ISSN: 1309-3452

Definition 1.1:

Aset $D \subset V$ ofverticesinagraph $G = (V, E)$ iscalleda dominating set if every vertex $v \in V$ is either an elementof D or it is adjacent to an element of D. The dominationnumberof G istheminimumcardinalityofadominatingsetanditisdenotedby $\gamma(G)$.

Definition 1.2:

A dominating set is said to be a connecteddominating set D if the inducedsubgraph $\langle D \rangle$ is connected.The connected dominationnumber $\gamma_c(G)$ of a connected graph G is the minimumcardinality of a connected dominating set of G.

Definition 1.3:

A set D ofverticesinagraph $G = (V, E)$ isadistance-2dominatingset if every vertex in $V - D$ is within distance 2 of at least onevertexin D .Thedistance–2dominationnumber $\gamma_{\leq 2}(G)$ of G equalstheminimumcardinalityofadistance2 –dominating setG.

Example 1.1

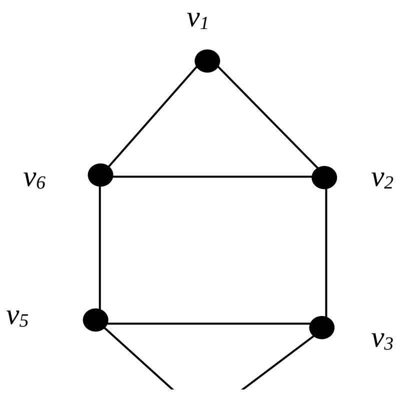

Figure 1.1

In the above example the set D is $\{v_1, v_2\}$. Hence the distance – 2 dominating number $\gamma_{\leq 2}(G)$ is 2.

2. Origin and Growth

The study of dominance in graphs has grown rapidly in recent years. It is a burgeoning area of study in graph theory. Hundreds of research articles have been published on this topic of research as a result of its growing relevance in real life. The game of 'Chess,' invented by Indians during the first millennium AD, was a driving force in the development of the theory of domination in graphs. In the 1850s, European chess fans debated the Queens problem, which involved determining the minimum number of queens required to attack every square on the chess board. It was correctly assumed to be five. According to the literature review, Jaenisch was the first researcher to conduct a mathematical analysis of the Queens problem in his article. W.W.Rouse Ball described three types of Queen's problems related to domination in 1982: independent domination, N-queens problem, and multi-queens problem (queens attack all the squares but not themselves). The book written by Claude Berge in 1958 dealt for the first time with the concept of domination, and he used the term 'coefficient of external stability' to refer to the domination number of a graph. Oystein Ore published another book on graph theory in 1962, in which he studied the domination concept for the first time, using the terms dominating set' and 'domination number' with the notation $d(G)$. Cockayne and Hedetniemi published a survey paper on the few available domination results in 1977. They were the first to use the symbol $\gamma(G)$ for a

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 2, 2022, p. 2465 - 2469
https://publishoa.com
ISSN: 1309-3452

graph's dominance number. The tremendous growth of the field of study, as well as its underlying practical applications, encourages this research work.

3. Preliminaries

Proposition1.2: For any graph $G, \gamma_{\leq 2}(G) \leq \gamma(G)$.

But the converse of the above result is not true.

Proposition1.3: For any Grid graph $G = G_{2,k}$, for $k \geq 1, \gamma_{\leq 2}(G) \leq \left\lfloor \frac{2+k}{3} \right\rfloor$.

Proposition1.4: Let D be a distance-2 dominating set of a graph G. Then D is a minimal distance – 2 dominating set if and only if each vertex $u \in D$ satisfies at least one of the following conditions:

1) there exist a vertex $v \in V - D$ for which $N_{\leq 2}(v) \cap D = \{u\}$.
2) $d(u, w) > 2$ for every vertex $w \in D - \{u\}$.

Theorem1.5: If G is a connected graph with at least 3 vertices and $diam(G) \geq 2$, then G has a minimum distance-2 dominating set D such that every vertex $u \in D$ satisfies condition 2 and has a private 2 neighbor $v' \in V - D$ for which $d(u, u') = 2$.

4. Nordhaus – Gaddum Type Results

Theorem1.6: For any graph G and $\bar{G}$ with $n \geq 3$, then

1) $2 \leq \gamma_{\leq 2}(G) + \gamma_{\leq 2}(\bar{G}) \leq n + 1$,
2) $1 \leq \gamma_{\leq 2}(G).\gamma_{\leq 2}(\bar{G}) \leq n$.

Theorem1.7: For any graph G and $\bar{G}$ with $n \geq 3$, then

1) $2 \leq \gamma_{\leq 2}(G) + \gamma_{\leq 2}(\bar{G}) \leq \frac{n}{3} + 1$.
2) $1 \leq \gamma_{\leq 2}(G).\gamma_{\leq 2}(\bar{G}) \leq \frac{n}{3}$.

5. Main Results

In this section we discuss about the distance - 2 domination number for strong product of graphs, H join graph.

Definition 2.1: The strong product of graphs G and H is a graph such that whose vertex set is $V(G) \times V(H)$ and edge set can be defined by as follows: if (u, u') and (v, v') are distinct vertices are adjacent in strong product if and only if $u = v$ and u' is adjacent to v' ,or $u' = v'$ and u is adjacent to v ,or u is adjacent to v and u' is adjacent to v'. And it is denoted by $G \otimes H$.

Theorem 2.2: Let D_i be the minimum distance - 2 dominating sets for graphs G_i where $i = 1,2$. Then $D = D_1 \times D_2$ is the minimum distance -2 dominating set for strong product $G_1 \otimes G_2$, then $\gamma_{\leq 2}(G_1 \otimes G_2) \leq \gamma_{\leq 2}(G_1) \times \gamma_{\leq 2}(G_2)$.

Proof: We know that $d_{G_1 \otimes G_2}((u, v), (x, y)) \leq max\left\{d_{G_1}(u, x), d_{G_2}(v, y)\right\}$---(1)

Let $(u, v) \in v - D$. Since D_i is a distance - 2 dominating set, there exist z_i such that $d_{G_1}(u, z_i) \leq 2$ and $d_{G_2}(v, y) \leq 2$. By equation (1), $d_{G_1 \otimes G_2}((u, v), D) \leq 2$. Hence $\gamma_{\leq 2}(G_1 \otimes G_2) \leq \gamma_{\leq 2}(G_1) \times \gamma_{\leq 2}(G_2)$.

Definition 2.3: The H- join of graphs H and $G_1, G_2, \ldots, G_k$ where H and $G_1, G_2, \ldots, G_k$ are connected is a graph obtained from every vertex i of H replacing G_i and every vertex of G_i is adjacent to vertex of G_j if and only if i and j are adjacent. And it is denoted by $V_H[G_1, G_2, \ldots, G_k]$.

Theorem 2.3: $\gamma_{\leq 2}(V_H[G_1, G_2, \ldots, G_k]) \leq \gamma_{\leq 2}(H)$.

Proof: Let $D(H) = \left\{w_i, \ldots, w_{i_m}\right\}$ be a minimum distance – 2 dominating set for each $1 \leq l \leq m$. Choose any vertex say v_{i_l} in G_{i_l} is replaced by w_{i_l}. Let $D = \left\{v_i, \ldots, v_{i_m}\right\}$. We can observe that the distance in G_i has at most 2. Since H is connected. Let $u \in V(V_H[G_1, G_2, \ldots, G_k])$. Then $u \in V(G_j)$ for $1 \leq j \leq k$. Since $j \in V(H)$ there exist $w_{i_l} \in D(H)$ such that $d_H\left(u, v_{i_l}\right) \leq 2$. Hence $d\left(u, v_{i_l}\right) \leq 2$. Therefore $d(u, D) \leq 2$. Thus $\gamma_{\leq 2}(V_H[G_1, G_2, \ldots, G_k]) \leq \gamma_{\leq 2}(H)$.

6. Applications

There are more useful models to many real-world problems when we extend the concept of dominating sets to distance - 2 dominating sets. Indeed, much of the motivation for studying domination stems from issues involving the optimal location of a hospital, police station, fire station, or other emergency service facility.

Almost all schools now have school buses that transport students to and from school. Among the many points to consider are the following:

(i) The time it takes a bus to travel from school to its destination;

(ii) The maximum number of students on a bus at any given time; and

(iii) The maximum distance a student must walk to board a school bus.

Assume that the school is located at the vertex starting point and that the school's management committee decides that no student shall walk more than two blocks to catch a school bus. Create a route for a school bus that departs from the school, travels within two blocks of every child who uses the school bus, and returns to the school. Clearly this bus route forms a distance - 2 dominating set.

Assume we have a group of small villages in a remote part of the world. We'd like to set up radio stations in some of these villages so that messages can be broadcast to all of the villages in the area. However, because radio station installation is costly, we want to locate as few as possible that can cover all other villages. So here we can use distance – 2 domination concept for installation of radio stations.

A mobile ad hoc network (MANET) is a collection of mobile hosts that form a wireless network dynamically without any backbone infrastructure or centralised administration. Mobile hosts in MANETs will be used in situations where there is no fixed backbone infrastructure, such as battlefield scenarios and natural disasters like earthquakes and hurricanes. Because it is made up of heterogeneous hosts, MANET is thought to be adaptable and convenient. MANET is based on the concept of flooding, in which each host broadcasts a message to the entire network after receiving it. This could result in the waste of valuable resources such as network bandwidth and device battery power. One of the most difficult aspects of establishing this type of network is involving the fewest number of hosts in the routing process, because not every node in the network may be required to forward messages. The identification of the minimum connected dominating set among the hosts in a given area is one solution to this problem. We use a distance – 2 dominating set here because it has fewer hosts than a connected dominating set. It helps to reduce communication and storage overhead by keeping its size to a minimum.

7. Conclusion

In this paper, we gave upper bounds for some family of graphs. In future, we will develop an algorithm to find minimum distance - 2 dominating set.

8. Conclusion

[1] E.J.Cockayne, S.T.Hedetniemi, “Towards a Theory of Domination in Graphs”,Networks,7:247–261.

[2] J.Cyman, M.Lemanska and J.Raczek: “Lower bound on the distance k - domination number of a tree”, Math. Slovaca 56 (2006), no. 2, 235–243.

[3] O.Favaron and D.Kratsch: Ratios of domination parameters, “Advances in graph theory, viswa”, Gulbarga (1991), 173–182.

[4] P. Fraisse, “A note on distance dominating cycles”. Discrete Math. 71 (1988), 89-92.

[5] Hansberg, D. Meierling, and L. Volkmann, “Distance domination and distance irredundance in graphs”. Electronic J.Combin. 14 (2007), R35.

[6] Harary, F., “Graph Theory”, Addison – wesley, Massachusetts, 1969.

[7] J.H. Hattingh and M.A. Henning: “Distance irredundance in graphs, Graph Theory, Combinatorics, and Applications”, John Wiley and Sons, Inc, 1(1995) 529–542.

[8] M.A. Henning, O.R. Oellermann and H.C. Swart: “Bounds on distance domination parameters”, J.Combin. Inform. System Sci 16 (1991) 11–18.

[9] D. Meierling and L.Volkmann: “A lower bound for the distance k-domination number of trees”, Result. Math. 47 (2005), 335–339.

[10] Wu,JieMihaelaCardei, Feidai, Shuhui Yang "Extended Dominating Set and its Applications in Ad-Hoc Network using Cooperative Communication". In IEEE Transactions on parallel and Distributed systems, Vol.No.7, Issue No.8, pp 19–39,2009.

[11] V.R. Kulli, "Theory of domination in Graphs", Vishwa International Publications, Gulbarga, India(2010).

[12] V.R. Kulli, ed., "Advances in Domination Theory I", Vishwa International Publications, Gulbarga, India(2012).

[13] NarsinghDeo, "Graph Theory with Applications to Engineering and Computer Science",PHL Learning Pvt.Ltd., New Delhi-1,2009

[14] Sasireka, A.H.Nandhukishore,"Applications of Dominating set of graph in computer networks" International Journal of Engineering Sciences and Research Technology, ISSN:2277–9655 , 3(1), 2014.

[15] K. Ameenal Bibi, A. Lakshmi and R. Jothilakshmi 3, " Applications of Distance - 2 Dominating Sets of Graph in Networks" Advances in Computational Sciences and Technology, ISSN 0973-6107 Volume 10, Number 9 (2017) pp. 2801–2810.

[16] G. Chartrand and L. Lesniak, Graphs and Digraphs, CRC, Fourth Edition (2005).

[17] Teresa W. Haynes, Stephen Hedetniemi, Peter Slater, Fundamentals of Domination in Graphs, CRC,(1998).

[18] Narsingh Deo, 2005. Graph Theory with Applications to Engineering and Computer Science. PHI Learning Pvt. Ltd., New Delhi.

[19] C. Berge, 1962. Theory of Graphs and its Applications. Methuen, London.

[20] O. Ore, 1962. Theory of Graphs. Amer. Math. Soc. Colloq. Publ., 38, Providence.

[21] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, 1998. Fundamentals of domination in graphs. Marcel Dekker Inc., New York.

[22] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, 1997. Advances in the Theory of domination in graph. Marcel Decker Inc., New York.

[23] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, 1998. Domination in graphs – Advanced Topics. Marcel Decker Inc., New York.

[24] F. Harary, T.W. Haynes, 1995. Conditional graph theory IV: Dominating sets. Util. Math., 48, 179–192.

[25] R.B. Allan, R.C. Laskar, 1978. On domination and independent domination numbers of a graph. Discrete Math., 23(2), 73–76.

[26] E.J. Cockayne, R.M. Dawes, S.T. Hedetniemi, 1980. Total domination in graphs. Networks, 10, 211–219. 12.

[27] E. Sampathkumar, H.B. Walikar, 1979. The connected domination number of a graphs. J. Math. Phys. Sci., 13(6), 607–613.

[28] M.B. Cozzens, L.L. Kelleher, 1990. Dominating cliques in graphs. Discrete Math., 86, 101–116.

[29] T.W. Haynes, P. J. Slater, 1998. Paired-domination in graphs. Networks, 32, 199–206.