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Laplace transform and Homotopy perturbation method for nonlinear Fokker-Planck equations

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ABSTRACT
Another procedure dependent on Laplace change and Homotopy annoyance strategies has been proposed here to address non straight Fokker–Planck conditions. To exhibit the consistency and capability of the technique, not many models are introduced. The mathematical arrangements outline that the strategy is simple, effective and exceptionally precise to carry out for direct and non straight Fokker-Planck conditions. The strategy gives a promising instrument to tackle direct and non straight fractional differential conditions. Diagrams are given to notice the arrangements in a superior and exact manner.

KEYWORDS: Fokker-Planck equations, Non linear partial differential equation, Homotopy perturbation method

1. PROLOGUE:

Lately, the growing interest of specialists and originators has been dedicated to logical techniques to deal with straight and non direct issues, and various numerical systems have been applied to these issues. The chance of HPM was first introduced by He (1999) and later a movement of game plans of nonlinear differential circumstances was gained by He (2004, 2005, 2006, 2008). By the homotopy technique in topography, a homotopy can be created with an introducing limit $p \in [0, 1]$, which is thought of as somewhat one. HPM is a mix of the irritation and homotopy procedures. This procedure can take the potential gains of the common inconvenience methodology while abstaining from its limits. He's HPM has been presently used by various mathematicians and architects to address different down to earth conditions. In this strategy, the nonlinear issue is moved to an unending number of sub-issues and, then, the game plan is approximated by how much the courses of action of the underlying a couple of sub-issues. This procedure has in like manner been used to handle the nonlinear game plan of second solicitation limit regard issues (Yusufoglu, 2007), immediate and nonlinear states of hotness move (Ganji, 2007, Rajabi and Ganji, 2007 and Ganji and Sadighi, 2007), non straight Schrödinger conditions (Biazar and Ghazvini, 2007) and irreplaceable circumstances (Abbasbandy, 2008). The Fokker-Planck condition arises in different fields like speculative science, engineered actual science, circuit speculation, quantum optics and solid state actual science.

Most as of late, Jafari et al. right off the bat applied Laplace change in the iterative technique and proposed another immediate strategy called iterative Laplace change strategy (Jafari et al, 2013) to look for mathematical arrangements of an arrangement of fragmentary incomplete differential conditions. By utilizing the strategy for Laplace Transform, Jafari and Seifi (2009) effectively got the mathematical arrangements of two frameworks of room time fragmentary differential conditions. It has been shown that, with this technique, one can find a few arrangements found by the current strategies, for example, homotopy annoyance strategy, Laplace Adomian disintegration strategy, and variational iterative strategy (Ongun, 2011). By utilizing the Laplace change procedure, Arda and Sever (2012) concentrated on careful bound state arrangements and comparing standardized Eigen elements of the outspread Schrödinger condition for the pseudo-symphonious and Mie-type possibilities. It is shown the variety of the initial six standardized wave elements of the above possibilities. It is additionally given mathematical outcomes for the bound conditions of two diatomic sub-atomic possibilities, and contrasted the outcomes and the ones acquired in writing. Yin et al (2014) introduced a coupled strategy for Laplace change and Legendre wavelets for the estimated

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arrangements of nonlinear Klein–Gordon conditions. By utilizing Laplace administrator and Legendre wavelets functional networks, the Klein–Gordon condition is changed over into an arithmetical framework. Then, at that point, the obscure Legendre wavelets coefficients are determined as series whose parts are processed by applying a recursive connection.

Tsaur and Wang (2014), Das and Arda (2015), Nogueira et al (2016) and Zarrinkamar et al (2017) has effectively applied the Laplace fundamental change to Fokker-Planck condition and different wave conditions of quantum mechanics including Schrödinger, Dirac and Klein-Gordon conditions for various possibilities including consonant, Morse, and so forth Hemeda and Eladdad (2018) proposed the new iterative strategy and acquaint the essential iterative technique with address straight and nonlinear Fokker-Planck conditions and some comparative conditions. The outcomes got by the two strategies are contrasted and those acquired by both Adomian deterioration and variational cycle techniques. By applying Laplace Transforms, Gupta et al (2019a) tracked down the overall arrangements of one dimensional Schrodinger's time free wave condition for a molecule in a limitless square well potential. In this paper, we will examine the Eigen energy esteems and Eigen elements of a molecule in a boundless square well potential. Gupta et al (2019b) acquired the quantum mechanical reflection and transmission coefficients for a molecule through a one-dimensional vertical advance potential. De Castro (2020) asserted the quantum issue of a molecule in a boundless square well potential to be tackled by means of Laplace change and recommended that the right arrangements were found for an Eigen esteem issue with variable characterized on a limited reach and called attention to painstakingly and unmistakably those missteps that may happen with the utilization of improper strategies for a given Eigen esteem issue.

In the current examination, a scientific guess to the arrangement of the nonlinear Fokker–Planck condition is acquired utilizing the Laplace Transform and Homotopy Perturbation Method (LTHPM). The outcomes got by means of LTHPM affirm the legitimacy of the proposed strategy and are contrasted and those acquired by Adomian decay technique.

2. NON-LINEAR FOKKER-PLANCK EQUATION

For Variable *x*, generalized Fokker–Planck equation is stated as follows:

$$\frac{\partial f}{\partial t} = \left[-\frac{\partial}{\partial x} g(x) + \frac{\partial^2}{\partial x^2} h(x) \right] f \tag{1}$$

under the condition that:

$$f(x,0) = u(x) \tag{2}$$

$f(x, t)$ obscure capacities in this situation and x is a genuine number, the float coefficient $g(x)$, the dissemination coefficient $h(x)$, and more prominent than nothing. Since the float and dissemination coefficients are time-reliant, the condition above can likewise be composed as follows:

$$\frac{\partial f}{\partial t} = \left[-\frac{\partial}{\partial x} g(x,t) + \frac{\partial^2}{\partial x^2} h(x,t) \right] f \tag{3}$$

As a dissemination condition with a first-request subordinate of x, Equation 2 is a second-request straight halfway differential condition.

Pattern formation, laser physics, surface physics, population dynamics, plasma physics, polymer physics and biophysics, as well as nonlinear hydrodynamics, neurosciences and psychology, require the Nonlinear Fokker–Planck equation to be nonlinear. It is stated as follows:

$$\frac{\partial f}{\partial t} = \left[-\sum_{i=1}^{N} \frac{\partial}{\partial x_i} g_i(X,t,f) + \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i x_j} h_{i,j}(X,t,f) \right] f \tag{4}$$

Linearization of the nonlinear Fokker–Planck equation previously stated when $g_i(X,t,f) = g_i(X)$ and $h_{i,j}(X,t,f) = h_{i,j}(X)$.

3. HOMOTOPY PERTURBATION METHOD AND APPLICATION OF LAPLACE TRANSFORM METHOD

Take a look at a nonlinear differential equation like follows:

$$B(f)-\phi(y)=0, \quad y\in D \tag{5}$$

only if certain conditions are met:

$$f(0)=\alpha_0, \quad f'(0)=\alpha_1, f''(0)=\alpha_2,..., f^{n-1}(0)=\alpha_{n-1} \tag{6}$$

Where $\phi(y)$ is a known scientific capacity and B is an overall differential administrator. The administrator, B, would be isolated into two sections, *l* and *n*, where *l* is a straight and *n* is a nonlinear administrator. Thusly, Eq. (5) can be reworked as:

$$l(f)+n(f)-\phi(y)=0 \tag{7}$$

In this case, a homotopy is constructed $F(y,p) : D \times [0,1] \to \square$ so that it fulfils

$$H(F,p)=(1-p)[l(F)-f_0]+p[B(F)-\phi(y)]=0, \quad p\in[0,1], \quad r\in D \tag{8}$$

which can take the form

$$H(F,p)=l(F)-f_0+pf_0+p[n(F)-\phi(y)]=0 \tag{9}$$

where f_0 is an underlying guess to arrangement of condition (5) and p alludes to the implanting boundary. From conditions (8) and (9), we get

$$H(F,0)=l(F)-f_0=0 \tag{10}$$

$$H(F,1)=B(F)-\phi(y)=0 \tag{11}$$

At the point when we apply the Laplace transform to condition (9), we get

$$L\{l(F)-f_0+pf_0+p[n(F)-\phi(y)]\}=0 \tag{12}$$

Here, the Transform of Derivative Property of Laplace Transform is employed to illustrate the concept.

$$s^nL\{F\}-s^{n-1}F(0)-s^{n-2}F'(0)-...-F^{n-1}(0)=L\{f_0-pf_0-p[n(F)-\phi(y)]\} \tag{13}$$

Then, using the inverse Laplace transform, we arrive to

$$F=L^{-1}\left\{\frac{1}{s^n}\left[s^{n-1}F(0)+s^{n-2}F'(0)+...+F^{n-1}(0)+L\{f_0-pf_0-p[n(F)-\phi(y)]\}\right]\right\} \tag{14}$$

After that, we suppose that the solution of equation (14) can be found as a power series in *p*, which is consistent with the Homotopy Perturbation Method.

$$F(x)=\sum_{r=0}^{\infty}p^rF_r \tag{15}$$

We reach the following result by incorporating equation (15) into equation (14).

$$\sum_{r=0}^{\infty}p^rF_r=L^{-1}\left\{\frac{1}{s^n}\left[s^{n-1}F(0)+s^{n-2}F'(0)+...+F^{n-1}(0)+L\left\{f_0-pf_0-p[n(\sum_{r=0}^{\infty}p^rF_r)-\phi(y)]\right\}\right]\right\}$$

(16)

Putting the multiple powers of p on both sides of the equation in the same equation (16)

$$F_0=L^{-1}\left\{\frac{1}{s^n}\left[s^{n-1}F(0)+s^{n-2}F'(0)+...+F^{n-1}(0)+L\{f_0\}\right]\right\} \tag{17}$$

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$$F_1 = L^{-1}\left\{\frac{1}{s^n} L\left\{n\left(F_0\right) - f_0 - \phi\left(y\right)\right\}\right\} \tag{18}$$

$$F_2 = L^{-1}\left\{\frac{1}{s^n} L\left\{n\left(F_0,\ F_1\right)\right\}\right\} \tag{19}$$

$$F_3 = L^{-1}\left\{\frac{1}{s^n} L\left\{n\left(F_0,\ F_1,\ F_2\right)\right\}\right\} \tag{20}$$

In the same way, we have

$$F_j = L^{-1}\left\{\frac{1}{s^n} L\left\{n\left(F_0,\ F_1,\ F_2,\ldots,\ F_{j-1}\right)\right\}\right\} \tag{21}$$

and so forth.

Consider for a moment that the initial estimate had the following form:

$$F(0) = f_0 = \alpha_0,\ \mathrm{F}'(0) = \alpha_1,\ F''(0) = \alpha_2,\ldots,\ F^{n-1}(0) = \alpha_{n-1}$$

Then, at that point, the specific arrangement may be obtained as:

$$f = \lim_{p\to 1} F = F_0 + F_1 + F_2 + \ldots \tag{22}$$

4. METHOD APPLIED TO NON-LINEAR FOKKER-PLANCK EQUATION

Since then, the Fokker-Planck equation can be expressed in nonlinear form as follows:

$$\frac{\partial f}{\partial t} = -\sum_{i=1}^{N} \frac{\partial}{\partial x_i} g_i^*(X,t,f) + \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i x_j} h_{i,j}^*(X,t,f) \tag{23}$$

where $g_i^*(X,t,f) = g_i(X,t,f)f$ and $h_{i,j}^*(X,t,f) = h_{i,j}(X,t,f)$

After that, the following homotopy is formed in order to answer the previous equation:

$$H\left(F\left(X,t\right),p\right) = F_t\left(X,t\right) - f_0\left(X,t\right) +$$
$$p\left\{f_0\left(X,t\right) + \left(\sum_{i=1}^{N} \frac{\partial}{\partial x_i} g_i^*(X,t,F) - \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i x_j} h_{i,j}^*(X,t,F)\right)\right\} = 0 \tag{24}$$

where $f_0\left(X,t\right)$ is an underlying estimate of the answer for condition (23). Also, from condition (24), we can get:

$$H\left(F\left(X,t\right),0\right) = F_t\left(X,t\right) - f_0\left(X,t\right) = 0 \tag{25}$$

$$H\left(F\left(X,t\right),1\right) = F_t\left(X,t\right) + \left(\sum_{i=1}^{N} \frac{\partial}{\partial x_i} g_i^*(X,t,F) - \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i x_j} h_{i,j}^*(X,t,F)\right) = 0 \tag{26}$$

Taking Laplace transform and utilizing the property 'Change of Derivatives' in condition (24), we get

$$sL\left\{F\left(X,t\right)\right\} - f\left(X,0\right) = L\left\{f_0\left(X,t\right) - p\left(f_0\left(X,t\right) + \left(\sum_{i=1}^{N} \frac{\partial}{\partial x_i} g_i^*(X,t,F) - \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i x_j} h_{i,j}^*(X,t,F)\right)\right)\right\} \tag{27}$$

Then we'll be able to get

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$$F(X,t) = L^{-1}\left\{\frac{1}{s}\left\{f(X,0) + L\left\{f_0(X,t) - p\left(f_0(X,t) + \left(\sum_{i=1}^{N}\frac{\partial}{\partial x_i}g_i^*(X,t,F) - \sum_{i,j=1}^{N}\frac{\partial^2}{\partial x_i x_j}h_{i,j}^*(X,t,F)\right)\right)\right\}\right\}\right\} \tag{28}$$

As indicated by Homotopy Perturbation strategy, we expect *p* as a little amount and arrangement of condition of above condition can be taken as

$$F(X,t) = \sum_{r=0}^{\infty} p^r F_r(X,t) \tag{29}$$

Equation (29) is used in equation (28) and we obtain by comparing various powers of *p* that

$$F_0(X,t) = L^{-1}\left\{\frac{1}{s}\left(f(X,0) + L\left\{f_0(X,t)\right\}\right)\right\} \tag{30}$$

$$F_1(X,t) = -L^{-1}\left\{\frac{1}{s}L\left\{f_0(X,t) + \sum_{i=1}^{N}\frac{\partial}{\partial x_i}g_i^*(X,t,F_0) - \sum_{i,j=1}^{N}\frac{\partial^2}{\partial x_i x_j}h_{i,j}^*(X,t,F_0)\right\}\right\} \tag{31}$$

$$F_2(X,t) = -L^{-1}\left\{\frac{1}{s}L\left\{\sum_{i=1}^{N}\frac{\partial}{\partial x_i}g_i^*(X,t,F_0,F_1) - \sum_{i,j=1}^{N}\frac{\partial^2}{\partial x_i x_j}h_{i,j}^*(X,t,F_0,F_1)\right\}\right\} \tag{32}$$

We have something comparable to this.

$$F_j(X,t) = -L^{-1}\left\{\frac{1}{s}L\left\{\sum_{i=1}^{N}\frac{\partial}{\partial x_i}g_i^*(X,t,F_0,F_1,...,F_{j-1}) - \sum_{i,j=1}^{N}\frac{\partial^2}{\partial x_i x_j}h_{i,j}^*(X,t,F_0,F_1,...,F_{j-1})\right\}\right\} \tag{33}$$

and so forth.

Allow us to consider that that the underlying estimation got the structure: . Then, at that point, the specific arrangement may be gained as:

$$f(X,t) = \lim_{p\to 1} F(X,t) = F_0(X,t) + F_1(X,t) + F_2(X,t) + ... \tag{34}$$

5. Illustrations

Illustration 1. To simplify things, the following nonlinear Fokker-Planck equation can be written as

$$f(x,0) = x^2, \quad g(x,t,f) = \frac{1}{3x}\left(12f - x^2\right), \quad h(x,t,f) = f, \quad x \in \mathbb{R} \tag{35}$$

When solving equation (35) it was observed that the exact result was as follows: $f(x,t) = x^2 e^t$

It is necessary to establish the following homotopy in order to solve problem (35) using LTHPM.

$$F_t(x,t) - f_0(x,t) + p\left\{f_0(x,t) + \frac{8FF_x}{x} - \frac{4F^2}{x^2} - \frac{F}{3} - \frac{xF_x}{3} - 2F_x^2 - 2FF_{xx}\right\} = 0 \tag{36}$$

With the Laplace transform and the 'Transform of Derivatives' property in equation (36), we may get the following result:

$$sL\left\{F(x,t)\right\} - F(x,0) = L\left\{f_0(x,t) - p\left\{f_0(x,t) + \frac{8FF_x}{x} - \frac{4F^2}{x^2} - \frac{F}{3} - \frac{xF_x}{3} - 2F_x^2 - 2FF_{xx}\right\}\right\} \tag{37}$$

Then there's the matter of

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$$F(x,t)=L^{-1}\left\{\frac{1}{s}\left(F(x,0)+L\left\{f_0(x,t)-p\left(f_0(x,t)+\frac{8FF_x}{x}-\frac{4F^2}{x^2}-\frac{F}{3}-\frac{xF_x}{3}-2F_x^2-2FF_{xx}\right)\right\}\right)\right\} \quad (38)$$

Let's assume that equation (38)'s solution is:

$$F(X,t)=F_0(X,t)+pF_1(X,t)+p^2F_2(X,t)+... \quad (39)$$

Substituting (39) into (38) and looking at the two sides' force coefficients, we get:

$$F_0(x,t)=L^{-1}\left\{\frac{1}{s}\left\{F(x,0)+L\left\{f_0(x,t)\right\}\right\}\right\} \quad (40)$$

$$F_1(x,t)=-L^{-1}\left\{\frac{1}{s}L\left\{f_0(x,t)+\frac{8F_0F_{0x}}{x}-\frac{4F_0^2}{x^2}-\frac{F_0}{3}-\frac{xF_{0x}}{3}-2F_{0x}^2-2F_0F_{0xx}\right\}\right\} \quad (41)$$

$$F_2(x,t)=-L^{-1}\left\{\frac{1}{s}L\left\{\frac{8F_0F_{1x}}{x}+F_{0x}F_1-\frac{4F_0F_1}{x^2}+F_0F_1-F_{0x}F_{1x}-2F_0F_{1xx}+F_{0xx}F_1-\frac{F_1}{3}-\frac{xF_{1x}}{3}\right\}\right\} \quad (42)$$

We have something comparable to this.

$$F_j(x,t)=-L^{-1}\left\{\frac{1}{s}L\left\{\frac{8}{x}\sum_{k=0}^{j-1}F_kF_{j-k-1x}-\frac{4}{x^2}\sum_{k=0}^{j-1}F_kF_{j-k-1}-2\sum_{k=0}^{j-1}F_{kx}F_{j-k-1x}-2\sum_{k=0}^{j-1}F_kF_{j-k-1x}-\frac{F_{j-1}}{3}-\frac{xF_{j-1x}}{3}\right\}\right\} \quad (43)$$

etc.

Expecting $f_0(x,t)=F(x,0)=x^2$ and addressing conditions $F_j(x,t),\ j=0,1,...$ (40)- (43) for yields

$$F_0(x,t)=x^2(1+t) \quad (44)$$

$$F_1(x,t)=\frac{x^2t^2}{2} \quad (45)$$

$$F_2(x,t)=\frac{x^2t^3}{3!} \quad (46)$$

$$F_3(x,t)=\frac{x^2t^4}{4!} \quad (47)$$

$$F_4(x,t)=\frac{x^2t^5}{5!} \quad (48)$$

and so forth.

Subsequently, the answer for condition (35) can be composed as

$$f(x,t)=F_0(x,t)+F_1(x,t)+F_2(x,t)+...=x^2\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\frac{t^4}{4!}+\frac{t^5}{5!}+...\right)=x^2e^t$$

which is same as definite arrangement.

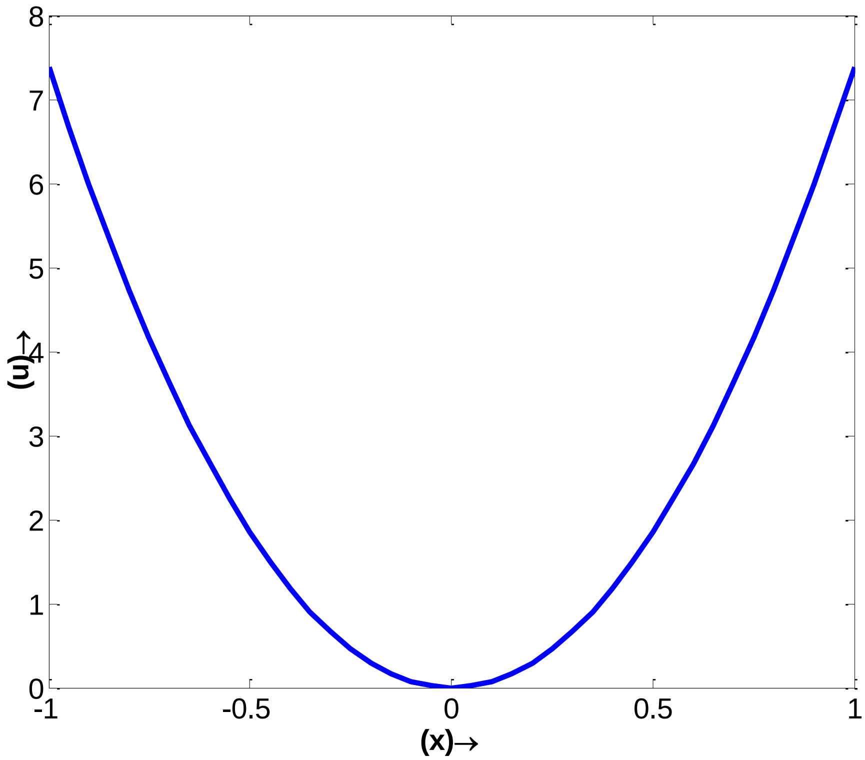

Figure 1 2-D representation of exact solution of the problem in example 1

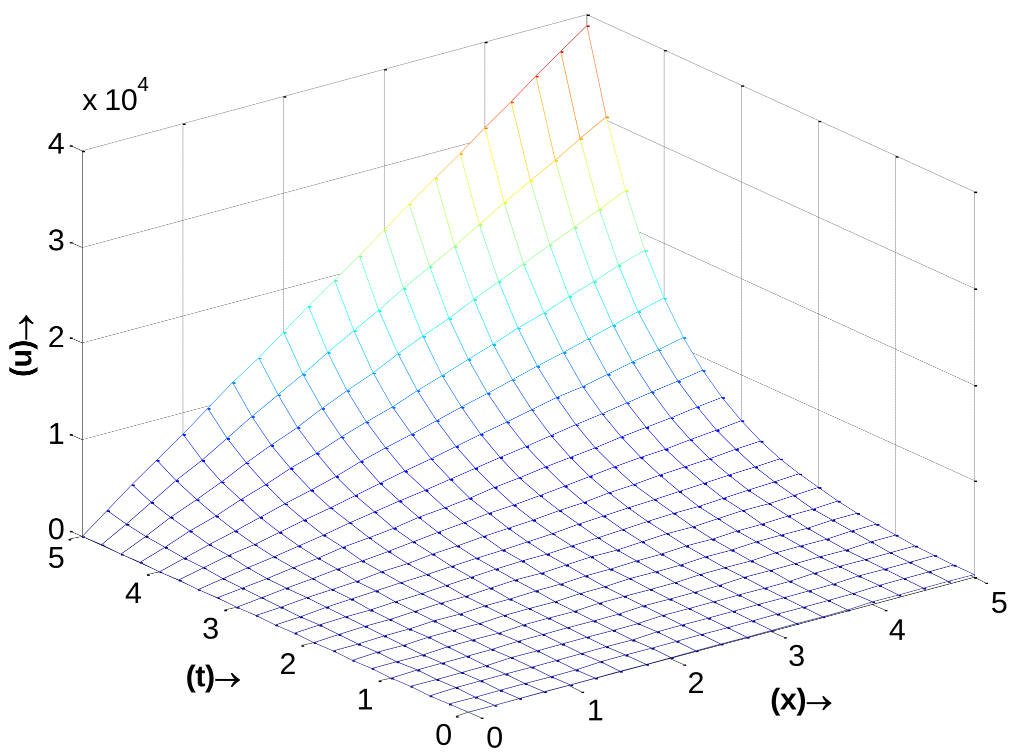

Figure 2 3D representation of exact solution of the problem in example 1

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Illustration 2. It is possible to express the following generalized Fokker-Planck equation in nonlinear form as follows:

$$f(x,0)=x^2,\quad X=(x,z)^t\in\mathbb{R}^2$$

$$\begin{cases} g_1(X,t,f)=\dfrac{4f}{x} \\ g_2(X,t,f)=z \end{cases}\qquad \begin{cases} h_{1,1}(X,t,f)=f \\ h_{1,2}(X,t,f)=1 \\ h_{2,1}(X,t,f)=1 \\ h_{2,2}(X,t,f)=f \end{cases} \tag{49}$$

The specific arrangement of condition (49) was gotten to be: $f(x,t)=x^2e^{-t}$

It is necessary to establish the following homotopy in order to solve problem (49) using LTHPM.

$$F_t(X,t)-f_0(X,t)+p\Big\{f_0(X,t)+\frac{8FF_x}{x}-\frac{4F^2}{x^2}+F \\ +zF_z-2FF_{xx}-2F_x^2-F_{xz}-F_{zx}-2FF_{zz}-2F_z^2\Big\}=0 \tag{50}$$

Using the Laplace transform and the property 'Transform of Derivatives' in equation (50), we may reach the following result:

$$sL\{F(X,t)\}-F(X,0)=L\Big\{f_0(X,t)-p\Big\{f_0(X,t)+\frac{8FF_x}{x}- \\ \frac{4F^2}{x^2}+F+zF_z-2FF_{xx}-2F_x^2-F_{xz}-F_{zx}-2FF_{zz}-2F_z^2\Big\}\Big\} \tag{51}$$

Then there's the matter of

$$F(X,t)=L^{-1}\Big\{\frac{1}{S}\Big(F(X,0)+L\Big\{f_0(X,t)-p\Big(f_0(X,t)+\frac{8FF_x}{x}- \\ \frac{4F^2}{x^2}+F+zF_z-2FF_{xx}-2F_x^2-F_{xz}-F_{zx}-2FF_{zz}-2F_z^2\Big)\Big\}\Big)\Big\} \tag{52}$$

Now, we'll assume that the answer to equation (51) has the following representation:

$$F(X,t)=F_0(X,t)+pF_1(X,t)+p^2F_2(X,t)+\ldots \tag{53}$$

Subbing condition (52) into condition (51) ,we get

$$F_0(X,t)=L^{-1}\Big\{\frac{1}{s}\{F(X,0)+L\{f_0(X,t)\}\}\Big\} \tag{54}$$

$$F_1(X,t)=-L^{-1}\Big\{\frac{1}{S}L\Big\{f_0(X,t)+\frac{8F_0F_{0x}}{x}+F_0+zF_{0z}- \\ \frac{4F_0^{\,2}}{x^2}-2F_0F_{0xx}-2F_{0x}^{\,2}-F_{0xz}-F_{0zx}-2F_0F_{0zz}-2F_{0z}^{\,2}\Big\}\Big\} \tag{55}$$

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$$F_2(X,t) = -L^{-1}\left\{\frac{1}{S}L\left\{\frac{8F_0F_{1x}}{x} + F_{0x}F_1 + F_1 + zF_{1z} - 2F_0F_{1zz} - 2F_1F_{0zz} + \left(1-\frac{4}{x^2}\right)F_0F_1 - 2F_0F_{1xx} + F_1F_{0xx} - 4F_{0x}F_{1x} - F_{1xz} - F_{1zx} - 4F_{0z}F_{1z}\right\}\right\} \quad (56)$$

We have something comparable to this.

$$F_j(X,t) = -L^{-1}\left\{\frac{1}{S}L\left\{\frac{8}{x}\sum_{k=0}^{j-1}F_kF_{j-k-1x} - \frac{4}{x^2}\sum_{k=0}^{j-1}F_kF_{j-k-1} + F_{j-1} + zF_{j-1z} - F_{j-1zx} - F_{j-1xz} - 2\sum_{k=0}^{j-1}F_0F_{j-k-1xx} - 2\sum_{k=0}^{j-1}F_{kx}F_{j-k-1x} - 2\sum_{k=0}^{j-1}F_kF_{j-k-1zz} - 2\sum_{k=0}^{j-1}F_{kz}F_{j-k-1z}\right\}\right\} \quad (57)$$

and so forth.

Assuming $f_0(X,t) = F(X,0) = x^2$ and solving equations (54)-(57) for $F_j(X,t),\ j = 0,1,\ldots$ the purpose of obtaining

$$F_0(X,t) = x^2\left(1+t\right) \quad (58)$$

$$F_1(X,t) = -\frac{x^2}{2}\left(4t+t^2\right) \quad (59)$$

$$F_2(X,t) = \frac{x^2}{3!}\left(6t^2+t^3\right) \quad (60)$$

$$F_3(X,t) = -\frac{x^2}{4!}\left(8t^3+t^4\right) \quad (61)$$

$$F_4(X,t) = \frac{x^2}{5!}\left(10t^4+t^5\right) \quad (62)$$

and etc.

Accordingly, the answer for condition (49) can be composed as

$$f(x,t) = F_0(X,t) + F_1(X,t) + F_2(X,t) + \ldots = x^2\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{t^5}{5!} + \ldots\right) = x^2e^{-t} \quad (63)$$

This is the same as the precise solution

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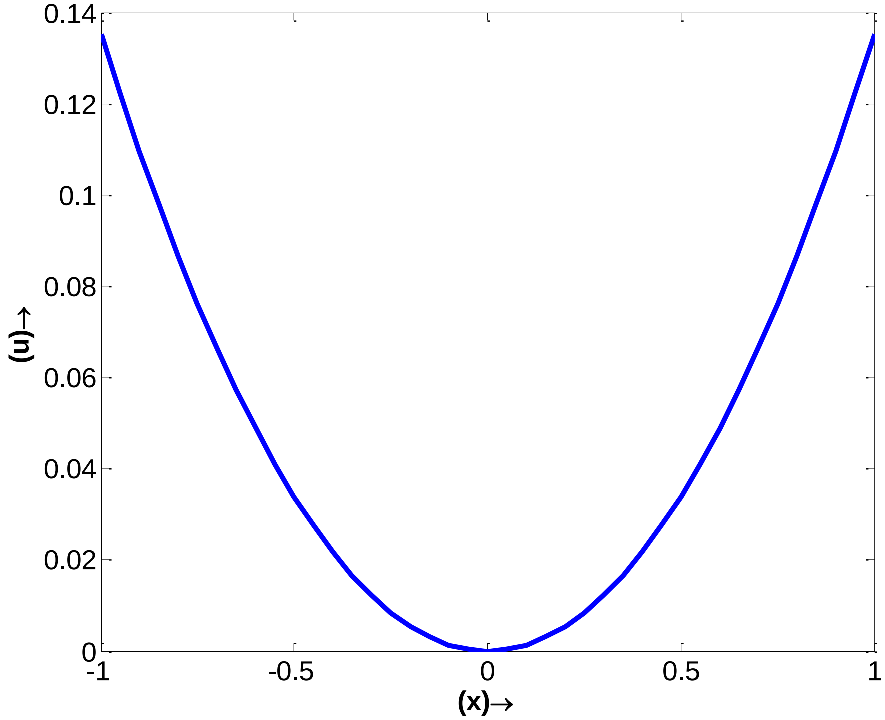

Figure 3 2D representation of exact solution of the problem in example 2

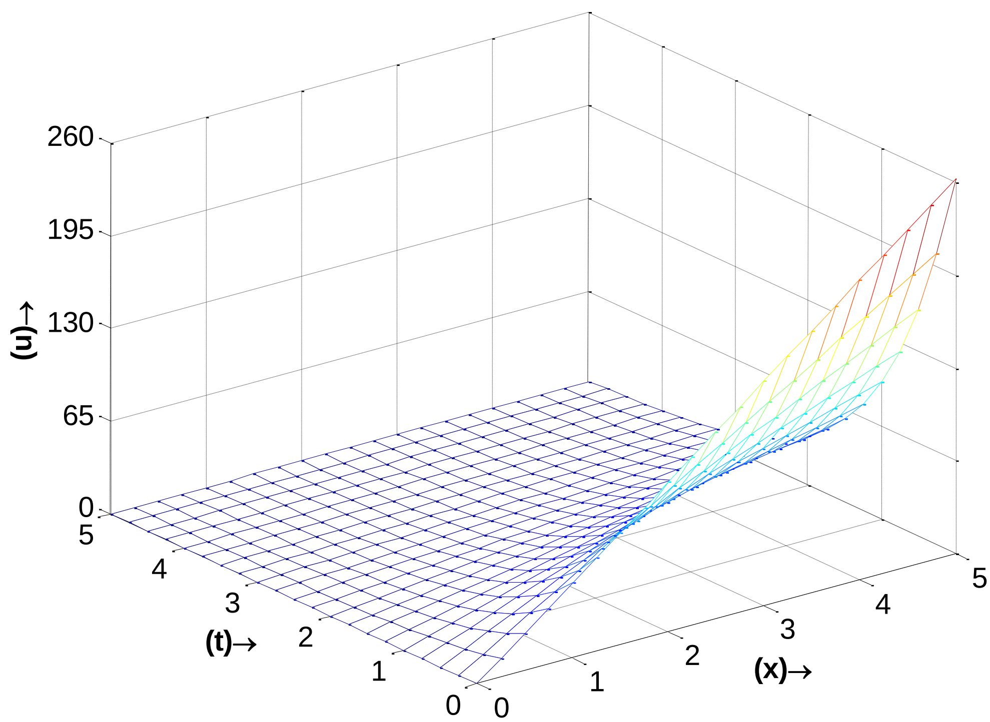

Figure 4 3D representation of exact solution of the problem in example 2

6. CONCLUDING REMARKS

In the ongoing survey, one more technique reliant upon blend of Laplace Transform and Homotopy Perturbation Method is proposed to settle the nonlinear Fokker-Planck condition. The new system, utilized in the ongoing work, is really applied on various frameworks. Not by any stretch of the imagination like the Homotopy Perturbation technique, handling a couple of rehash differential circumstances isn't needed here. Speak Laplace change engages to instantly get the course of action approximations. The guideline advantage of the Laplace Transform and Homotopy Perturbation Method over Adomian Decomposition Method is that there is no great explanation to register Adomian polynomials to acquire the plan. The results gained demonstrate the way that this approach can decipher the issue effectively.

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