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Split Liar Domination on Intuitionistic Fuzzy Graphs

S. Roseline Mary[1], L. Selvam[2]

[1]Department of Mathematics, Loyola College Vettavalam, Tiruvannamalai
[2]Department of Computer Science, Loyola College Vettavalam, Tiruvannamalai

ABSTRACT

A set S is said to be a liar dominating set if it can identify the location node x of an intruder when any one of the nodes which is closed neighborhood of x lie or wrongly identify the intruder's location. In other words, Let $G = (V,E,\mu)$ be a fuzzy graph. A set S is called a liar dominating set of a fuzzy graph G if it satisfies the following two constraints.

1. Each node of $V(G)$ is dominated by at least two nodes of $V(G)$
2. Each pair of nodes of $V(G)$ is dominated by at least three nodes of $V(G)$

Liar dominating set lies between double dominating set and triple dominating set since triple dominating set persists a liar dominating set and every liar dominating set double dominates. In this paper, we introduce the split liar dominating set for intuitionistic fuzzy graphs and also discuss some theorems and results with suitable examples.

Introduction

Assume that each vertex of a graph G is the probable location for an "intruder", for instance, a thief or a saboteur, a fire in a facility or some imaginable processor fault in a computer network. A device at a vertex u could be expected to sense the intruder at any vertex in its closed neighbourhood $N[u]$ and to recognize at which vertex in $N[u]$ the intruder is positioned. Liar's dominating sets can identify an intruder's position even if the other device in the neighbourhood of the intruder vertex fails, that is, if device in the neighbourhood of the intruder vertex misidentifies the presence of other vertex in its closed neighbourhood as the intruder location. P.J. Slater [1] introduced liar's dominating sets in graph theory. P.J. Slater [2] introduced Locating sets and it was carried forward by F. Harary and R.A. Melter[3] where they termed it as metric bases. D. Manuel Paul [4] premeditated on both locating-domination and liar domination in circulant networks to narrate the characterization of locating-dominating set and liar dominating set of circulant networks and sharp lower and upper bounds. N. D. Soner[5] discussed split domination number of fuzzy graphs.[6] A.Nagoorgani studied Fuzzy Independent dominating sets.[7] A.Nagoorgani studied double domination on intuitionistic fuzzy graphs.

1 Basic Definitions

Definition 1 : [7] An edge uv is said to be strong if $\alpha_2{}^{\infty}(u,v) = \alpha_2(u,v)$ and $\beta_2{}^{\infty}(u,v) = \beta_2(u,v)$, where $\alpha_2{}^{\infty}(u,v)$ is maximum weight of weakest arc and $\beta_2{}^{\infty}(u,v)$ is minimum weight of weakest arc.
That is,
$\alpha_2{}^{\infty}(u_i,u_j)= \sup\{\alpha_2{}^{k}(u_i,u_j)/k=1,2,\ldots n\}$
$\beta_2{}^{\infty}(u_i,u_j)= \inf\{\alpha_2{}^{k}(u_i,u_j)/k=1,2,\ldots n\}$

Definition 2 : [7] Open neighbourhood of a vertex u is defined as, $N(u) = \{v \in V(G)\backslash\mu^{\infty}(u,v) = \mu(u,v)\}$
Definition 3 : [7] Closed neighbourhood of a vertex $N[u]$ is defined as, $N[u] = \{u\} \cup \{v \in V(G) : \mu^{\infty}(u,v) = \mu(u,v)\}$
Definition 4 : [7] The vertex u is said to be dominated by the vertex v if $u \in N[v]$, where, $N[v] = \{v\} \cup \{u \in V,(u,v)$ *is strong edge*$\}$.
Definition 5 : [7] Let $G =< \sigma,\mu >$ be a fuzzy graph on V. A set $A \subseteq V$ is called a liar dominating or domination set if it satisfies the following conditions.

1. Each vertex $u \in V(G)$ is dominated by at least two vertices in A.
2. Every pair of vertices $u,v \in V(G)$ is dominated by at least three vertices in A.

Definition 6 : [7] An intuitionistic fuzzy graph (IFG) is of the form $G = (V,E)$ where

1. $V = \{u_1,u_2,u_3,...,u_n\}$ *such that* $\alpha_1 : V \to [0,1]$ *and* $\beta_1 : V \to [0,1]$ denote the degree of membership and non membership of the vertex $u_i \in V$ respectively and $0 \le \alpha_1(u_i) + \beta_1(u_i) \le 1$, for every $u_i \in V$ $(1 = 1,2,..n)$.
2. $E \subset V \times V$ *where* $\alpha_1 : V \times V \to [0,1]$ *and* $\beta_1 : V \times V \to [0,1]$ such that

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$$\alpha_{2ij} = \alpha_2(u_i, u_j) \leq \min(\alpha_1(u_i), \alpha_1(u_j))$$
$$\beta_{2ij} = \beta_2(u_i, u_j) \leq \max(\beta_1(u_i), \beta_1(u_j))$$
And $0 \leq \alpha_2(u_i) + \beta_2(u_i) \leq 1$ for all $(u_i, u_j) \in E$

Definition 7 : [7] An intuitionistic fuzzy graph G = (V,E) is called strong if
$$\alpha_{2ij} = \alpha_2(u_i, u_j) = \min(\alpha_1(u_i), \alpha_1(u_j))$$
$$\beta_{2ij} = \beta_2(u_i, u_j) = \max(\beta_1(u_i), \beta_1(u_j))$$
for all $(u_i, u_j) \in E$.
Definition 8 : [7] An intuitionistic fuzzy graph G = (V,E) is called complete if
$$\alpha_{2ij} = \alpha_2(u_i, u_j) = \min(\alpha_1(u_i), \alpha_1(u_j))$$
$$\beta_{2ij} = \beta_2(u_i, u_j) = \max(\beta_1(u_i), \beta_1(u_j))$$
for all $u_i, u_j \in V$.
Definition 9: [7] Let G =(V,E) be an IFG. Then the cardinality of G is defined to be
$|G|=\left|\sum_{u_i \in V} \frac{1+\alpha_1(u_i)-\beta_1(u_i)}{2} + \sum_{u_i, u_j \in E} \frac{1+\alpha_2(u_i,u_J)-\beta_2(u_i,u_J)}{2}\right|$
Definition 10:[7]
Let G =(V,E) be an IFG. Then the vertex cardinality of G is defined to be
$|V|=\sum_{u_i \in V} \frac{1+\alpha_1(u_i)-\beta_1(u_i)}{2}$, for all $u_i \in V$
Definition 11:[7]
Let G =(V,E) be an IFG. Then the edge cardinality of G is defined to be
$|E|=\sum_{u_i, u_j \in V} \frac{1+\alpha_2(u_i,u_J)-\beta_2(u_i,u_J)}{2}$, for all $u_i, u_j \in E1$
Definition 12 : [7] A path in an intuitionistic fuzzy graph G is a sequence of vertices and edges $u_1 e_1 u_2 e_2$... such that either one of the following conditions is satisfied.

1. $\alpha_{2ij} > 0$ *and* $\beta_{2ij} = 0$ *for* some *i, j.*
2. $\alpha_{2ij} = 0$ *and* $\beta_{2ij} > 0$ *for* some *i, j.*
3. $\alpha_{2ij} > 0$ *and* $\beta_{2ij} > 0$ *for* some *i,j.*

Definition 13 : [7] Let *u* be a vertex in intuitionistic fuzzy graph. Then, $N[u] = \{u\} \cup \{v : v \in V$ and (u,v) is strong edge in IFG $\}$ is called closed neighbourhood of the vertex *u* in *G*.
Definition 11 : [6] A vertex *u* is said to be a isolated vertex if $N(u) = \emptyset$. An isolated vertex *u* is dominated by itself.
Definition 12 : [6] We say that *u* dominates *v* if $v \in N[u]$. This implies that *u* dominates itself. Obviously, domination satisfies symmetric relation on *V* . That is, *u* dominates *v* iff *v* dominates $u, \forall u,v \in V$.
If no edge is strong edge in IFG, then we can not form liar dominating set for intuitionistic fuzzy graphs, since every node is single dominant node in this case.

Split Liar Domination in Intuitionistic Fuzzy graphs

Definition: A set $I \subseteq V$ in a IFG is called a liar dominating set if *I* satisfies the following two conditions:

1. every $u \in v$ is dominated by minimum two nodes in IFG.
2. every pair $u,v \in V$, is dominated by minimum three nodes in IFG.

Definition: A liar dominating set *I* in IFG is called a minimum liar dominating set, if there is no liar dominating set I^0 such that $|I^0| < |I|$.
Definition: The minimum fuzzy cardinality of liar dominating set *I* in IFG is called liar dominating number and is denoted by $\omega(IFG)$.
In Figure 1, $\omega(IFG) = \{v_1,v_2,v_4\}$ forms liar dominating set.
Definition: A liar dominating set I^0 in IFG, is called a minimal liar dominating set if no proper subset of I^0 is a liar dominating set.
Definition: A liar dominating set *I* in IFG is called a maximum liar dominating set if there is no liar dominating set I^0 such that $|I^0| > |I|$.
Definition
A dominating set I in a intuitionistic fuzzy graph is said to be split liar dominating set if < V-I> is disconnected.
Definition:
The split liar dominating set which have minimum fuzzy cardinality is said to be minimum split liar domination set.
Definition:

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The fuzzy cardinality of minimum split liar domination set is called split liar domination number.it is denoted by $I_S(IFG)$. Split liar domination set does not exist for complete intuitionistic fuzzy graphs and complete bipartite intuitionistic fuzzy graphs.

Theorem(1)

A liar domination set I in a Intuitionistic fuzzy graph is a split liar dominations set if there exists two nodes u,v $\in V\setminus I$ such that

$(\alpha_2)^{\infty}$(u,v) = 0

$(\beta_2)^{\infty}$(u,v) = 0 in G\I

Proof:

Let I be split liar domination set in a intuitionistic fuzzy graph. Let u,v$\in V\setminus I$.

By the definition of split liar domination set, the removal of I from G will disconnect

The fuzzy graph G.

Then,

$(\alpha_2)^{\infty}$(u,v) = 0

$(\beta_2)^{\infty}$(u,v) = 0.

Conversely, Let $(\alpha_2)^{\infty}(u,v)$= 0 and $(\beta_2)^{\infty}$(u,v) = 0 in G\I, where u,v$\in V\setminus I$.

This means that the vertices u and v are not connected in G\I.

Therefore I is split liar domination set of G.

Theorem (2)

A liar domination set I in a Intuitionistic fuzzy graph is a split liar domination set iff for some nodes x_i, x_j, x_k, x_l of I, there exists two vertices $d_1, d_2 \in$ V\I such that N(d_1) $\cap$ I={x_i, x_j}, N(d_2) $\cap$ I={x_k, x_l} and $\alpha_2(c_{1,}\ c_2)$=0 and $\beta_2(c_{1,}\ c_2)$=0

Proof:

Let I be a split liar domination set of a intuitionistic fuzzy graph G.

Since I is a liar domination set of G, every node of V(G) is dominated by at least two vertices in I,

N(d_1) $\cap$ I={x_i, x_j},

N(d_2) $\cap$ I={x_k, x_l}for some $x_i, x_j, x_k, x_l \in I$, $d_1, d_2 \in V - I$

Since I is a split liar domination set, removal of vertices from I will disconnect the fuzzy graph. Therefore,

$\alpha_2(d_1, d_2)$= 0 and $\beta_2(d_1, d_2)$= 0.

Conversely, Let I be a liar domination set of G.

If for some vertices $x_i, x_j, x_k, x_l \in I$, there exists some $d_1, d_2 \in V - I$ such that

N(d_1) $\cap$ I={x_i, x_j},

N(d_2) $\cap$ I={x_k, x_l} and

$\alpha_2(d_1, d_2)$=0 , $\beta_2(d_1, d_2)$=0.

Then there is no connectedness between any two vertices of V-I.

This implies that I is a split liar domination set.

Theorem (3)

If I is a minimal split liar domination set of an intuitionistic fuzzy graph G, then there is a fuzzy path between every two vertices of V\I at least three vertices of S.

Proof:

Let I be a split liar domination set of an intuitionistic fuzzy graph G. Since I is a liar domination set, at least two vertices of I dominate every vertex of G and at least three vertices of I dominate every pair of vertices of G. Particularly, every pair of vertices in I is dominated by at least three vertices in I, name such three vertices as u,v,w. Then there will be a u-v-w fuzzy path in I. Since I is minimal, u dominates a vertex in V-I and also w dominates a vertex in V-I.

Hence

$(\alpha_2)^{\infty}$(u,v) > 0

$(\beta_2)^{\infty}$(u,v) $<$ 0, $\forall$ u,v ϵ V-I

Theorem (4)

ω_s(IFG)$\geq$ $\omega(IFG)$

Proof:

Suppose ω_s(IFG)$<$ $\omega(IFG)$, then $\omega(IFG)$ will not be a liar domination number.

If ω_s(IFG)$=$ $\omega(IFG)$ liar domination number can be a split liar domination number in an intuitionistic fuzzy graph. Therefore, this is possible.

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Suppose ω_s(IFG)> $\omega(IFG)$, the removal of liar domination set may not disconnect the intuitionistic fuzzy graph, unless some vertices are added in the liar domination set. In this case,
ω_s(IFG)> $\omega(IFG)$. Hence Proved.

Theorem (5)

A split liar dominating set is a vertex cover.

Proof:

A vertex cover of a fuzzy graph is a set of vertices such that each strong edge of a fuzzy graph is incident to at least one vertex of the set.

All the vertices in V(G) have strong neighbours in a liar dominating set. Therefore, liar dominating set is a vertex cover.

Note:

Converse need not be true.

Theorem (6)

A se S is a split liar domination set iff the set satisfies the following two conditions.

1. S is a liar domination set
2. S is a vertex cut

Proof:

Proof follows the definitions of liar domination set, vertex cut and split liar domination set.

Theorem A set I ⊆ V is fuzzy independent set of G, iff V-I is fuzzy vertex covering of G.

Theorem (7)

A set I ⊆ V is fuzzy independent set of G, iff V-I is fuzzy vertex covering of G.

Proof:

I⊆ V is called a fuzzy independent set of G if there is no strong arc between any two vertices of I.

This implies that no strong edge of G has both ends in I.

Then each strong edge of G has at least one end in V\I.

This means that V\I fuzzy vertex covering of G.

Conversely, V\I is a fuzzy vertex covering of G. Then each edge of G is incident with at least one vertex in G.

I⊆ V is the set of vertices such that no strong edge of G both end vertices in I. Then one of the end vertices of all strong edges are the vertices of I.

That is, strong edge of G has both end vertices in I. And therefore, I is a fuzzy independent set.

Corollary (7.1)

$\alpha_0 + \beta_0$=p

Proof:

Let $\alpha_0 \ and \ \beta_0$ be the fuzzy independent number and fuzzy vertex covering number of fuzzy graph G.

Let M be the maximum fuzzy independent set of G and N be the minimum fuzzy vertex covering of G. If M is the maximum independent set, then V\M is fuzzy vertex covering of G.

Then

$$|V \setminus M| \geq |N|$$

p-$\alpha_0 \geq \beta_0$

p≥ $\alpha_0 + \beta_0$ …………(1)

If N is minimum fuzzy vertex covering of G, then V\N is fuzzy independent set of G.

Therefore,

$$|V \setminus N| \leq |M|$$

p-$\beta_0 \leq \alpha_0$

p≤ α_0+β_0 …………..(2)

From (1) and (2)

α_0+β_0 = p

Theorem (8)

Let G be a fuzzy graph without isolated vertices, then the sum of fuzzy independent number and fuzzy edge covering number is order of G.

Proof:

Let E_0 be a minimum fuzzy edge covering of G. Let H=G[E_0] be the strong edge induced subgraph of G. Let M_0 be the maximum fuzzy matching in H. We denote the set M_0 unsaturated vertices in H by w_0. H[w_0] has no strong edges.

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$|E_0|$-$|M_0|$ = $|E_0\backslash M_0| \geq |w_0|$=p-2$|M_0|$
$\beta_0{}'$-$\alpha_0{}'$+2$\alpha_0{}' \geq p$
$\beta_0{}'$+$\alpha_0{}' \geq p$ ………….(1)
Let M be a fuzzy maximum matching in G and let W be the set of M – unsaturated
Vertices in G. Then,there is a set E of $|w|$ stong edges and one of its strong edges in incident with a vertex in W.
This implies that M∪ E' is an fuzzy edge covering of G.
Here,

$$|E'| \leq |W|$$

Now,
$\beta_0{}' \leq |M \cup E'| \leq |M|$+$|W|$
$\beta_0{}' \leq \alpha_0{}' + p - 2\alpha_0{}'$, ……………(2)
Since number of edges in E\M is equal to the number of vertices in w, where w is the set of M –unsaturated vertices.
From(2)
$\alpha_0{}'$+$\beta_0{}' \leq p$ ……(3)
From (1) and (3),
$\alpha_0{}'$+$\beta_0{}' = p$

Theorem (9)

Let $K_{\sigma_{1,}\sigma_2}$ be a fuzzy bipartite graph without isolated vertices.Then the fuzzy cardinality of the vertices in a fuzzy maximum independent set is less than or equal to fuzzy cardinality of the edges in fuzzy minimum edge covering.
Proof:
Let α_0, β_0,$\alpha_0{}'$, $\beta_0{}'$ be the fuzzy vertex independent number , fuzzy vertex covering number, fuzzy edge independence number and fuzzy edge covering number respectively. We prove that
$\alpha_0 \leq \beta_0{}'$
We have already proved in corollary(1) that
$\alpha_0 + \beta_0$= p ……...(1)
Similarly from the previous theorem,
$\alpha_0{}' + \beta_0{}' = p$ …......(2)
We recall that, a subset $|M_0|$ of E of a fuzzy graph G is called fuzzy matching in G if its members are strong edges and no two strong edges are adjacent in G.
α_0 is fuzzy cardinality of the edges in the maximum fuzzy matching of G. Therefore in a fuzzy bipartite graph, fuzzy cardinality in fuzzy matching is less than or equal to fuzzy cardinality of fuzzy minimum covering.
Therefore,
$\alpha_0{}' \leq \beta$ ………..(3)
From (1) and (2)
$\alpha_0 + \beta_0$=$\alpha_0{}' + \beta_0{}'$
From equation (3),

$$\alpha_0{}' \leq \beta_0$$

Then
$\alpha_0 + \beta_0$=$\alpha_0{}' + \beta_0{}' \leq \beta_0$+$\beta_0{}'$

$$\alpha_0 \leq \beta_0+\beta_0{}' - \beta_0$$
$$\alpha_0 \leq \beta_0{}'$$

Theorem (10)

$$\beta_o(G) \leq \lambda_S(G)$$

Proof:

$\beta_o(G)$ is vertex covering number of fuzzy graph G.
Here, There are possibilities that some vertices in V(G) are not dominated by two vertices in $\beta_o(G)$. To make it, some more vertices must be added to $\beta_o(G)$, then only $\beta_o(G)$ will be a split liar domination set of G.
Therefore,

$$\beta_o(G) \leq \lambda_S(G)$$

Theorem (11)

For any fuzzy graph G, Split liar domination number is
1. $\lambda_S(G) \geq \Delta(G)$-2p
2. $\lambda_S(G) \geq$ q-2p

Proof:
p = O(G) = $\sum_{u\epsilon V} \sigma(u)$
q = S(G) = $\sum_{u,v\epsilon E(G)} \mu(u,v)$
$\sum_{uv\in E(G)} \mu(u,v) \geq \vee(d(v)/v\epsilon V$, Since $\Delta(G)$= $\vee$(d(v)/vϵV)
That is,

$$\sum_{uv\in E(G)} \mu(u,v) \geq \Delta(G)$$

$\sum_{u\in V} d(v)$= 2$\sum \mu(u,v)$
This implies that,

$$\sum d(v) \geq \sum \mu(u,v)$$

Thus,
2p$\geq \sum_{v\in V} d(v) \geq \sum_{uv\in E(G)} \mu(u,v) \geq \Delta(G)$, since $\mu(u,v) \leq (\sigma(u)\wedge\sigma(v))$S
2p+$\omega_S(G) \geq \Delta(G)$
$\omega_S(G) \geq \Delta(G) - 2$p
Also
2p+$\omega_S(G) \geq$q
$\omega_S(G) \geq$q-2p

Conclusion
In this paper, we introduced the split liar dominating set for intuitionistic fuzzy graphs and also discussed some theorems and results with suitable examples.

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