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Properties of Polynomial Quasi Orthogonal of Type I Matrices

R. Gajalakshmi [1], G. Ramesh [2]
[1]Research Scholar, Department of Mathematics, Govt Arts College (Autonomous), Kumbakonam, Tamil Nadu, India. Affiliated to Bharathidasan University, Tiruchirappalli-620024.
[2]Associate Professor and Head, Department of Mathematics, Govt Arts College (Autonomous), Kumbakonam, Tamil Nadu, India. Affiliated to Bharathidasan University, Tiruchirappalli-620024.

Abstract:

This paper is concerned with a new type of polynomial matrix. The concept of polynomial quasi orthogonal of type I matrices are introduced. We define the index of quasi orthogonal of type Imatrices and we extended some results of quasi orthogonalof type I matrix to polynomial quasi orthogonal of type I matrices

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Keywords: Quasi orthogonal matrix, orthogonal of type I matrix,quasi orthogonal of type I matrix, determinant, inverse, transpose.

1. Introduction

In linear algebra, a matrix A is orthogonal if its transpose is equal to its inverse, which entails $AA^T=A^TA=I$, where I is the identity matrix. The term orthogonal matrix was used in 1854 by Charles Hermite (1822-1901) in the Cambridge and Dublin mathematical journal, although it was not until 1878 that the formal definition of an orthogonal matrix was published by Frobenius [6]. The orthogonal Matrixwas defined by Sylvester in 1867 [5]. A polynomial matrix $A(\lambda)=\sum_{j=0}^{n}\lambda^j A_j$ is a polynomial orthogonal matrix whose coefficient matrix $A_i's$ are orthogonal matriceswhich was referred by [1] [2] [3][4].Polynomial matrices arise naturally as modeling tools in several areas of applied mathematics, systems theory, sciences and engineering.

2. Polynomial quasi orthogonal of type I matrices:

Definition:2.1

A square matrix A is called an orthogonal of type I matrix if $A^k(A^T)^k=I_n$ *and* $(A^T)^k(A\ )^k=I_n$, for some $k\epsilon R$

Example:2.2

Let $A=\begin{bmatrix}-i & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -i\end{bmatrix}$ is orthogonal of type I matrices.

Definition: 2.3

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A real square matrix A of order n is called quasi orthogonal matrix if it satisfies $AA^T = A^T A = cI_n$ .Where, I_n is $n \times n$ identity matrix and c is a constant real number.

Example:2.4

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$ is quasi orthogonal matrix.

Definition:2.5

Let Abe an quasi orthogonal of type I matrix andthere exists positive integer k with $A^k (A^T)^k = c^k I_n$ is called the index of A. We say that A is quasi orthogonal of typeI of period k .

Example: 2.6

$$A = \begin{bmatrix} 5i & 0 \\ 0 & 5i \end{bmatrix}$$ is quasi orthogonal of type I matrix.

Solution:

Given that $A = \begin{bmatrix} 5i & 0 \\ 0 & 5i \end{bmatrix}$

If A is quasi orthogonal of type I matrix.

Thus, $A^k (A^T)^k = c^k I_n$

Put k=1, $AA^T = \begin{bmatrix} 5i & 0 \\ 0 & 5i \end{bmatrix} \begin{bmatrix} 5i & 0 \\ 0 & 5i \end{bmatrix} = 5 \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Put k =2, $A^2 (A^T)^2 = 5^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Put k=3, $A^3 (A^T)^3 = 5^3 \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Put k=4, $A^4 (A^T)^4 = 5^4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Put$k$=5, $A^5 (A^T)^5 = 5^5 \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Put k=6, $A^6 (A^T)^6 = 5^6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The index of A is 2, then $A^k(A^T)^k=I_2$,k=2,4,6,….. and A is of period 2.

Definition: 2.7

Let $A(\lambda) = A_0 + A_1\lambda + A_2\lambda^2 + ... + A_n\lambda^n$ be polynomial quasi orthogonal of type I matrix. Here coefficient matrices $A_i's$ are quasi orthogonal of type I matrices andthere exists positive integer k with $A_i^k (A_i^T)^k = c^k I_n$ is called the index of $A_i's$. We say that $A_i's$ is quasi orthogonal of typeI of period k.

Theorem:2.8

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If $A(\lambda)$ is an polynomial quasiorthogonal of type I matrix is a polynomial quasiorthogonal of type I matrix whose coefficient matrices arequasi orthogonal of type I matrices. Then $\det(A_i^{\,k}) = \pm c^{kn/2}$

Example:2.9

Let $A(\lambda) = A_0 + A_1\lambda + A_2\lambda^2 + ...+ A_n\lambda^n$ be polynomial quasi orthogonal of type I matrix. Here coefficient matrices $A_i's$ are quasi orthogonal of type I matrices. Where i = 1, 2, 3,........n. If $A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is quasi orthogonal of type I matrix , then $\det A_1 = \pm 4^{n/2}$.

Solution:

Given that $A$is quasi orthogonal of type I matrix

$$A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Since $A_1A_1^{\,T} = A_1^{\,T}A_1 = cI_n$

Thus , $A_1A_1^T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 4I_n$

ie) $A_1A_1^{\,T} = 4I_n$

$$A_1A_1^T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 4I_n$$

ie) $A_1^{\,T}A_1 = 4I_n$. Hence $A_1A_1^{\,T} = A_1^{\,T}A_1 = 4I_n$

So that , $A_1A_1^{\,T} = 4I_n$

Taking determinant on both sides

$\det(A_1A_1^{\,T}) = \det(4I_n)$ ($\because$ $\det(aI) = a^n \det(I)$)

$\det A_1.\det A_1^{\,T} = 4^n \det(I_n)$ ($\because$ $\det(I) = 1$)

Since, $\det A_1 = \det A_1^{\,T}$

That is $\det A_1 \ \det A_1 = \det(4I_n)$

$(\det A_1)^2 = 4^n$

$\det A_1 = \pm 4^{n/2}$

Hence $\det A_1 = \pm 4^{n/2}$

Theorem:2.10

If $A(\lambda) = A_0 + A_1\lambda + A_2\lambda^2 + ...+ A_n\lambda^n$ be polynomial quasiorthogonal of type I matrix. Here coefficient matrices $A_i's$ are quasiorthogonal of type I matrices.

The following statements are equivalent.

(i) $A(\lambda)$ is an polynomial quasi orthogonal of type I matrix.

(ii) $A(\lambda)^{-1}$is an polynomialquasiorthogonal of type I matrix.

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(iii) $A(\lambda)^T$ is an polynomial quasiorthogonal of type I matrix.

(iv) $\overline{A(\lambda)}$ is an polynomial quasiorthogonal of type I matrix.

(v) $A(\lambda)^*$ is an polynomial quasiorthogonal of type I matrix.

Proof:

Let $A(\lambda) = A_0 + A_1\lambda + A_2\lambda^2 + ... + A_n\lambda^n$ be polynomial quasiorthogonal of type I matrix. Here coefficient matrices $A_i's$ are quasiorthogonal of type I matrices. Where i = 1, 2, 3,........n.

To prove (i) ⇒ (ii)

Suppose that $A_i's$ is an quasi orthogonal of type I matrix.

To prove A_i^{-1} is an quasi orthogonal of type I matrix.

Since , $A_i^k (A_i^T)^k = c^k I_n$, for some $k \epsilon N$

Taking inverse on both side

$$(A^k (A^T)^k)^{-1} = (c^k I_n)^{-1}$$

$$(A_i^k)^{-1}((A_i^T)^k)^{-1} = (c^k)^{-1} I_n$$

$$(A_i^{-1})^k ((A_i^{-1})^T)^k = (c^{-1})^k I_n$$

Hence A_i^{-1} is an quasi orthogonal of type I matrix.

Hence all the coefficients of $A(\lambda)^{-1}$ are quasi orthogonal of type I matrices. Therefore $A(\lambda)^{-1}$ is a polynomial quasi orthogonal of type I matrix.

To prove (ii) ⇒ (iii).

Suppose A_i^{-1} is aquasi orthogonal of type I matrix.

To prove A_i^T is an quasi orthogonal of type I matrix.

Since , $(A_i^{-1})^k ((A_i^{-1})^T)^k = (c^{-1})^k I_n$, for some $k \epsilon N$

Taking inverse on both side

$$((A_i^{-1})^k ((A_i^{-1})^T)^k)^{-1} = ((c^{-1})^k I_n)^{-1}$$

$$(((A_i^{-1})^T)^k)^{-1}((A_i^{-1})^k)^{-1} = ((c^{-1})^k)^{-1} I_n{}^{-1}$$

$$(((A_i^{-1})^{-1})^T)^k ((A_i^{-1})^{-1})^k = ((c^{-1})^{-1})^k I_n$$

$$(A_i^T)^k A_i^k = c^k I_n$$

Taking transpose on both side

$$((A_i^T)^k A_i^k)^T = (c^k I_n)^T$$

$$(A_i^T)^k ((A_i^T)^T)^k = (c^T)^k I_n$$

Hence A_i^T is an quasi orthogonal of type I matrix.

⇒ $A(\lambda)^T$ is an polynomial quasi orthogonal of type I matrix.

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To prove (iii) ⇒ (iv)

Suppose A_i^T is an quasi orthogonal of type I matrix.

To prove $\overline{A_i}$ is an quasi orthogonal of type I matrix.

Since, $(A_i^T)^k((A_i^T)^T)^k = (c^T)^k I_n$, for some $k\epsilon N$

ie) $(A_i)^k (A_i^T)^k = c^k I_n$

Taking conjugate on both side

$\overline{A_i^k (A_i^T)^k} = \overline{c^k I_n}$

$(\overline{A_i})^k (\overline{A_i^T})^k = (\bar{c})^k I_n$

$(\overline{A_i})^k ((\overline{A_i})^T)^k = (\bar{c})^k I_n$

Hence $\overline{A_i}$ is aquasi orthogonal of type I matrix.

⇒ $\overline{A(\lambda)}$ is an polynomial quasi orthogonal of type I matrix.

To prove (iv) ⇒ (v)

Suppose $\overline{A_i}$ is an quasi orthogonal of type I matrix.

To prove A_i^* is an quasi orthogonal of type I matrix.

Since, $(\overline{A_i})^k ((\overline{A_i})^T)^k = (\bar{c})^k I_n$,for some $k\epsilon N$

Taking transpose on both side

$$((\overline{A_i})^k ((\overline{A_i})^T)^k)^T = ((\bar{c})^k I_n)^T$$

$$(((\overline{A_i})^T)^k)^T ((\overline{A_i})^k)^T = ((\bar{c})^k)^T I_n$$

$$(((\overline{A_i})^T)^T)^k (A_i^*)^k = (c^*)^k I_n \because ((\overline{A_i}))^T = A_i^*)$$

$$((A_i^*)^T)^k (A_i^*)^k = (c^*)^k I_n$$

Hence A_i^* is an quasi orthogonal of type I matrix .

⇒ $A(\lambda)^*$ is an polynomial quasi orthogonal of type I matrix.

To prove (v) ⇒ (i)

Suppose A_i^* is an quasi orthogonal of type I matrix.

To prove A is an quasi orthogonal of type I matrix .

Since, $((A_i^*)^T)^k (A_i^*)^k = (c^*)^k I_n$,for some $k\epsilon N$

So that, $((A_i^*)^T)^k (A_i^*)^k = (c^*)^k I_n$. Taking $*$ on both side

$(((A_i^*)^T)^k (A_i^*)^k)^* = ((c^*)^k I_n)^*$

$(((A_i^*)^T)^k)^* ((A_i^*)^k)^* = ((c^*)^k)^* I_n$

$(((A_i^*)^*)^T)^k A_i^k = c^k I_n$ s

$(A_i^T)^k A_i^k = c^k I_n$

Hence A_i is aquasi orthogonal of type I matrix.

$\Rightarrow A(\lambda)$ is an polynomial quasi orthogonal of type I matrix.

3.Conclusion

In this paper some properties of polynomial quasi orthogonal of type I matrices are derived and also we extended to some results of polynomial quasi orthogonal of type I matrices.

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