

DOM-CHRO Number In Operations On Intuitionistic Fuzzy Graphs

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ABSTRACT

The concept of Domination-Chromatic set (Dom-Chro) and Dom-Chro number of an IFG is introduced in this paper. In Addition to study the Dom-Chro number of a complete IFG and complete bipartite IFG and identify some more bounds of Dom-Chro number are studied. Finally he Dom-Chro number of a join IFG’s and a Cartesian product of IFG’s are examined.

Keywords: Intuitionistic fuzzy graphs,

1. Introduction

For a given χ -coloring of an IFG G , a dominating set $S \subseteq V(G)$ is named to be Dom-Chro set if it covers at least single vertex from each colour of G . The Dom-chro number $\chi_{ifd}(G)$ is the minimum cardinality taken among the Dom-Chro sets of IFG G .

The concept of Dom-Chro set and Dom-Chro number of an IFG is introduced in this paper. In Addition to study the Dom-Chro number of a complete IFG and complete bipartite IFG and identify some more bounds of Dom-Chro number are studied. Finally he Dom-Chro number of a join IFG’s and a Cartesian product of IFG’s are examined.

2. Domination chromatic number of IFG

In this section we develop the concept of domination chromatic (Dom-Chro) number in IFG and investigate the bounds of Dom-Chro number in different IFG

Theorem 2.1: In a complete IFG $G(V, E)$, then the Dom-Chro number $\chi_{ifd}(G) = O(G)$.

Proof: Assume $G(V, E)$ remain a complete IFG. Therefore there exist an effective edge among all couple of vertices in V clearly $G(V, E)$ n-color IFG. Assume D be a γ_{if} set of complete IFG $G(V, E)$. Clearly $D = \{v_i | v_i = \Delta_N(G)\}$ this implies D did not cover all color in $G(V, E)$. This implies the set V is a Dom-Chro set and it cover all the colours in complete IFG $G(V, E)$. Hence $\chi_{ifd}(G) = |V| \Rightarrow \chi_{ifd}(G) = O(G)$.

Example 2.1:

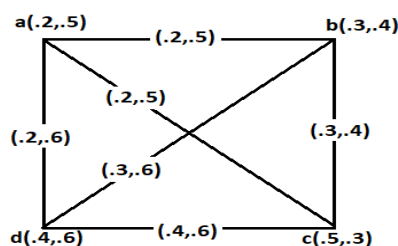


Fig 2.1: Complete IFG $G(V, E)$

In figure 2.1, the Dom-Chro set and Dom-Chro number of the complete bipartite IFG $G(V, E)$ are $\{a, b, c, d\} = V$ & $\chi_{ifd}(G) = 1.8 = O(G)$ respectively.

Theorem 2.2: In a complete bipartite IFG $G(V, E)$, then the Dom-Chro number $\chi_{ifd}(G) = \gamma_{if}(G)$.

Proof: Undertake $G(V, E)$ remain a complete bipartite IFG. This implies the vertex set V_1 & V_2 are disjoint sets clearly $G(V, E)$ 2-color IFG. Let D stands a γ_{if} set of complete bipartite IFG. Clearly $D = \{v_1, v_2 | v_1 \in V_1 \& v_2 \in V_2\}$ this implies v_1 & v_2 having the different color, since $v_1 \in V_1$ & $v_2 \in V_2$. Therefore the set D cover all the colors in complete bipartite IFG $G(V, E)$. Hence $\chi_{ifd}(G) = |D| \Rightarrow \chi_{ifd}(G) = \gamma_{if}(G)$.

Example 2.2:

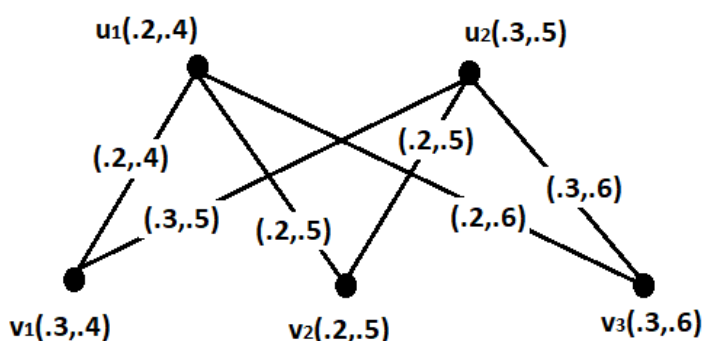


Fig 2.2: Complete bipartite IFG $G(V, E)$

In figure 2.2, the Dom-Chro set and number of the complete bipartite IFG $G(V, E)$ are $\{u_1, v_2\}$ & $\chi_{ifd}(G) = 0.75 = \gamma_{if}(G)$ respectively.

Theorem 2.3: In an IFG's $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$, then the Dom-Chro number $\chi_{ifd}(G_1 \cup G_2) = \chi_{ifd}(G_1) + \chi_{ifd}(G_2)$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be an IFG with Dom-Chro number $\chi_{ifd}(G_1)$ and $\chi_{ifd}(G_2)$ respectively. Let $D_c \subseteq V(G)$ be a minimal Dom-Chro number of an IFG $G_1 + G_2$. In $G_1 + G_2$ edges are in the following form

- (i). $xy \in G_1$
- (ii). $xy \in G_2$

This implies there is an effective edge among vertices in G_1 and G_2 . This indicates minimal Dom-Chro sets D_{c1} and D_{c2} of G_1 and G_2 are dominating set of $G_1 + G_2$. Therefore the different set of colours used in G_1 and G_2 . Clearly D_{c1} and D_{c2} are not a Dom-Chro set of $G_1 \cup G_2$. Since some of the vertex color in $G_1 \cup G_2$ are not covered by D_{c1} and D_{c2} . The set $D_{c1} \cup D_{c2}$ cover all the colors in $G_1 \cup G_2$. Hence the set $D_c = (D_{c1} \cup D_{c2})$ is a dom-chro set of $G_1 \cup G_2$.

$$D_c = (D_{c1} \cup D_{c2})$$

$$|D_c| = |D_{c1}| + |D_{c2}|$$

$$\chi_{ifd}(G_1 \cup G_2) = \chi_{ifd}(G_1) + \chi_{ifd}(G_2)$$

Example 2.3:

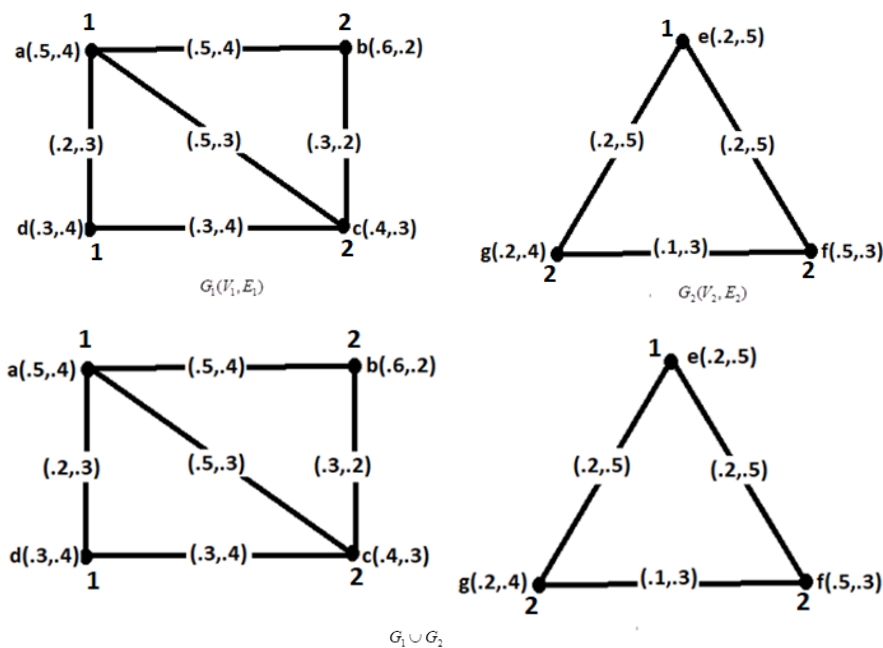


Fig 2.3: $G_1 \cup G_2$

In figure 2.3, the dom- chro sets of an IFG's G_1 and G_2 are $\{a, c\}$ & $\{e, g\}$ respectively. Note the dom- chro number of the IFG's G_1 and G_2 are $\chi_{ifd}(G_1) = 1.1$ & $\chi_{ifd}(G_2) = .75$. The dom- chro set and number of the IFG $G_1 \cup G_2$ are $\{a, c, e, g\}$ & $\chi_{ifd}(G_1 \cup G_2) = 1.85$ respectively.

Theorem 2.4: In an IFG $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$, then the Dom-Chro number $\chi_{ifd}(G_1 + G_2) = \chi_{ifd}(G_1) + \chi_{ifd}(G_2)$.

Proof: Assume $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be an IFG with Dom-Chro number $\chi_{ifd}(G_1)$ and $\chi_{ifd}(G_2)$ respectively. Let $D_C \subseteq V(G)$ be a minimal Dom-Chro number of an IFG $G_1 + G_2$. In $G_1 + G_2$ edges are in the following form

- (i). $xy \in G_1$
- (ii). $xy \in G_2$
- (iii). $xy \in (G_1 + G_2)$

This implies there exist a strong edge among vertices in G_1 and G_2 . This indicates minimal Dom-Chro sets D_{C_1} and D_{C_2} of G_1 and G_2 are dominating set of $G_1 + G_2$. Therefore the different set of colours used in G_1 and G_2 . Clearly D_{C_1} and D_{C_2} are not a Dom-Chro set of $(G_1 + G_2)$. Since some of the vertex colour in $(G_1 + G_2)$ are not covered by D_{C_1} and D_{C_2} . The set $D_{C_1} \cup D_{C_2}$ cover all the colours in $(G_1 + G_2)$. Hence the set $D_C = (D_{C_1} \cup D_{C_2})$ is a Dom-Chro set of $(G_1 + G_2)$.

$$D_C = (D_{C_1} \cup D_{C_2})$$

$$|D_C| = |D_{C_1}| + |D_{C_2}|$$

$$\chi_{ifd}(G_1 + G_2) = \chi_{ifd}(G_1) + \chi_{ifd}(G_2)$$

Example 2.4

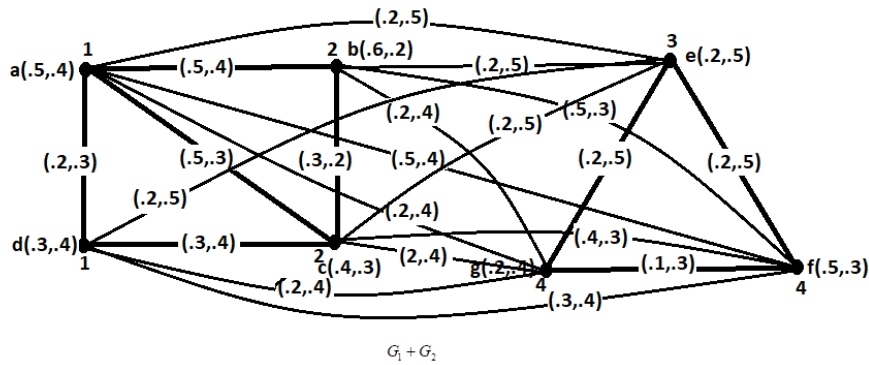


Fig 2.4: $(G_1 + G_2)$

In figure 2.4, the Dom-Chro set of the IFG's G_1 and G_2 are $\{a, c\}$ & $\{e, g\}$ respectively. Note the Dom-Chro number of the IFG's G_1 and G_2 are $\chi_{ifd}(G_1) = 1.1$ & $\chi_{ifd}(G_2) = .75$. The Dom-Chro set and number of the IFG $G_1 + G_2$ are $\{a, c, e, g\}$ & $\chi_{ifd}(G_1 + G_2) = 1.85$ respectively.

Theorem 2.5: In an IFG's $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$, then the Dom-Chro number $\chi_{ifd}(G_1 \times G_2) = \gamma_{if}(G_1 \times G_2)$

Proof: Assume $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be an IFG with γ_{if} set D_1 and D_2 individually. Let $D_C \subseteq V(G)$ be a minimal Dom-Chro number of an IFG $G_1 \times G_2$. In $G_1 \times G_2$ edges are in the following form

- (i). $(x_1 y_1)(x_1 y_2) \in G_1 \times G_2$
- (ii). $(x_1 y_1)(x_2 y_1) \in G_1 \times G_2$

Let D be a γ_{if} set of $(G_1 \times G_2)$. Assume D is not a minimal Dom-Chro set of $(G_1 \times G_2)$. There is a Colouring set of $G_1 \times G_2$ are not covered by the set D . This implies there exist a vertex $(x_1 y_1)$ in $(G_1 \times G_2)$ is not an dominated by D . This is contradict to our assumption D be a γ_{if} set of $(G_1 \times G_2)$. Henceforth D is a minimal Dom-Chro set of $G_1 \times G_2$. Therefore we get $D_C = D \Rightarrow |D_C| = |D| \Rightarrow \chi_{ifd}(G_1 \times G_2) = \gamma_{if}(G_1 \times G_2)$.

Example 2.5:

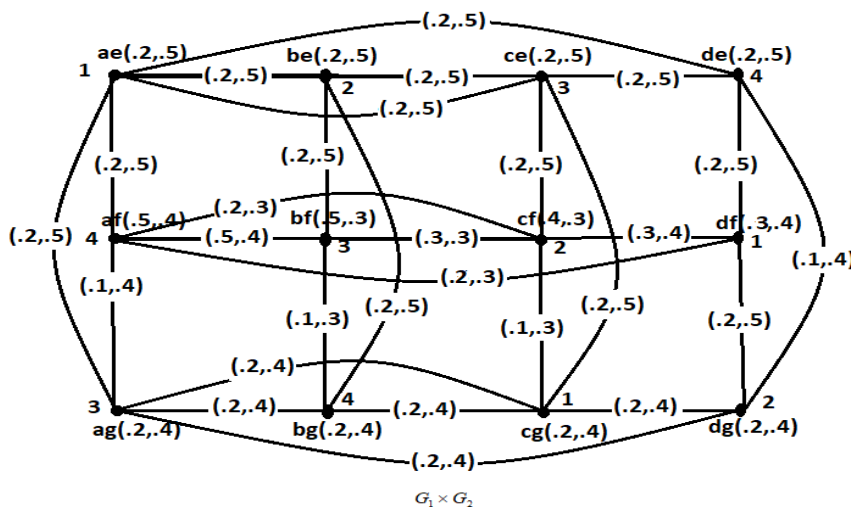


Fig 2.5: $G_1 \times G_2$

In figure 2.5, the Dom-Chro set and number of the IFG $G_1 \times G_2$ are $\{ae, be, ce, de\}$ & $\chi_{ifd}(G_1 \times G_2) = 1.4 = \gamma_{if}(G_1 \times G_2)$ respectively.

3. Conclusion

The concept of Dom-Chro set and Dom-Chro number of an IFG is introduced in this paper. In Addition to study the Dom-Chro number of a complete IFG and complete bipartite IFG and identify some more bounds of Dom-Chro number are studied. Finally he Dom-Chro number of a join IFG's and a Cartesian product of IFG's are examined. In future we will develop the domination chromatic (Dom-Chro) number for various Dominating set.

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