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Homomorphism Of Intuitionistic Multi-Anti Fuzzy Ideal Of Aring

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ABSTRACT

In this paper, We discuss the properties of anti image and anti pre image of an intuitionistic multi-anti fuzzy ideal of a ring under homomorohism and anti homomorphism.

Keywords : Intuitionistic multi-anti fuzzy ideal, homomorphism and anti homomorphism of an intuitionistic multi-anti fuzzy ideal of a ring.

1. INTRODUCTION

The idea of fuzzy sets introduced byL.A. zadeh 1965[18] is an approach to mathematical representation of vagueness in everyday curriculum, The idea of fuzzy set is welcome because it handles uncertainty and vagueness which ordinary set could not address. In fuzzy set theory membership function of an element is single value between 0 and 1. Therefore, a generalization of fuzzy set was introduced by Attanassov[1], 1983 called intuitionistic fuzzy set (IFS) which deals with the degree of non-membership function and the degree of hesistation. After several year, Sabu Sebastian [13]introduced the theory of multi-fuzzy sets in terms of multi-dimensional membership function. R. Muthuraj and S. Balamurugan [15] introduced the concept of multi-anti fuzzy subgroup and discussed some of its properties. R. Muthuraj and S. Balamurugan [17] introduced the concept of multi-anti fuzzy ideal of a ring under homomorphism.

In this paper, We discuss the properties of anti image and anti pre image of an intuitionistic multi-anti fuzzy ideal of a ring under homomorohism and anti homomorphism.

1.1 BASIC CONCEPTS

The theory of an intuitionistic multi-anti fuzzy set is an extension of theories of multi-fuzzy sets. The membership function of a multi-fuzzy set is an ordered sequence of membership functions of a fuzzy set. The notion of an intuitionistic multi-fuzzy sets provides a new method to represent some problems which are difficult to explain in other extentions of fuzzy set theory.

Throughout this paper, we will use the following notations (i) R_1 and R_2 for ring. (ii) G and H for an ideal.

1.2 Definition [6]

Let R be a ring. Let $G = \{\langle x, A(x), B(x)\rangle / x\in R\}$be an intuitionistic multi-fuzzy set defined on a ring R. Then G is said to be an intuitionistic multi-anti fuzzy ring on R if the following conditions are satisfied. For all $x, y\in R$,

i. $A(x-y) \leq \max\{A(x), A(y)\}$,
ii. $A(xy) \leq \max\{A(x), A(y)\}$,
iii. $B(x-y) \geq \min\{B(x), B(y)\}$,
iv. $B(xy) \geq \min\{B(x), B(y)\}$.

1.3 Remark

For an intuitionisticmulti-anti fuzzy subring $G = \{\langle x, A(x), B(x)\rangle / x\in R\}$of a ring R, the following result is obvious.For all $x, y\in R$,

i. $A(x) \geq A(0)$ and $A(x) = A(-x)$,
ii. $A(x-y) = 0$ implies that $A(x) = A(y)$.
iii. $B(x) \leq B(0)$ and $B(x) = B(-x)$,
iv. $B(x-y) = 0$ implies that $B(x) = B(y)$.

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1.4 Definition[6]

A mapping f from a ring R_1 to a ring R_2 (both R_1 and R_2 not necessarily commutative) is called an anti-homomorphism if for all x ,y∈R

i f(x + y) = f(x) + f(y) and

ii f(xy) = f(x) f(y).

A subjective anti- homomorphism is called an anti- epimorphism.

1.5 Defition

Let f be a mapping from a set R_1 to a set R_2 and let A be an intuitionistic multi- anti fuzzy subset in R_1. Then A is called f – invariant if f(x) = f(y) implies A(x) = A(y) for all x ,y∈R. Clearly, if A is f- invariant, then $f^{-1}(f(A)) = A$.

II Properties of an intuitionistic multi- anti fuzzy Ideal of a Ring

In this section, we discuss some results on intuitionistic multi-anti fuzzy ideal of a ring under homomorphism and anti homomorphism.

2.1 Definition[10]

Let R be a ring. Let G = {⟨x, A(x), B(x)⟩ / x∈R } be an intuitionistic multi-fuzzy set defined on a ring R. Then G is said to be an intuitionistic multi-anti fuzzy left ideal on R if the following conditions are satisfied. For all x, y ∈R,

i. $A(x-y) \leq \max\{A(x), A(y)\}$,
ii. $A(xy) \leq A(y)$,
iii. $B(x-y) \geq \min\{B(x), B(y)\}$,
iv. $B(xy) \geq B(y)$

2.2 Definition [17]

Let R be a ring. Let G = {⟨x, A(x), B(x)⟩ / x∈R} be an intuitionistic multi-fuzzy set defined on a ring R. Then G is said to be an intuitionistic multi-anti fuzzy right ideal on R if the following conditions are satisfied. For all x, y ∈R,

i. $A(x-y) \leq \max\{A(x), A(y)\}$,
ii. $A(xy) \leq A(x)$,
iii. $B(x-y) \geq \min\{B(x), B(y)\}$,
iv. $B(xy) \geq B(x)$.

2.3 Definition[17]

Let R be a ring. Let G = {⟨x, A(x), B(x)⟩ / x∈R } be an intuitionistic multi-fuzzy set defined on a ring R. Then G is said to be an intuitionistic multi-anti fuzzy ideal on R if the following conditions are satisfied. For all x, y ∈R,

i. $A(x-y) \leq \max\{A(x), A(y)\}$,
ii. $A(xy) \leq \min\{A(x), A(y)\}$,
iii. $B(x-y) \geq \min\{B(x), B(y)\}$,
iv. $B(xy) \geq \max\{B(x), B(y)\}$.

2.4 Example

Consider the ring Z_9 = {0, 1, 2, …, 8 } with respect to the operations $+_9$ and $\times_9$. Let G = { ⟨x, A(x), B(x)⟩ / x∈R } be an intuitionistic multi-fuzzy set defined on a ring Z_9, where,

$$A(x) = (A_1(x), A_2(x)) = \begin{cases} (0.3, 0.5) & \text{if } x = 0 \\ (0.4, 0.6) & \text{if } x = 3 \text{ or } 6 \\ (0.5, 0.6) & \text{otherwise} \end{cases} \text{ and}$$

$$B(x) = (B_1(x), B_2(x)) = \begin{cases} (0.6, 0.5) & \text{if } x = 0 \\ (0.6, 0.4) & \text{if } x = 3 \text{ or } 6 \\ (0.4, 0.4) & \text{otherwise} \end{cases}$$

Clearly, G is an intuitionistic multi-anti fuzzy ideal on Z_9.

2.5 Remark

An intuitionistic multi-anti fuzzy ring need not be an intuitionistic multi-anti fuzzy ideal of a ring R.

2.6 Example

Let R be the ring of real numbers under the usual operation of addition and multiplication. Let G = { ⟨x, A(x), B(x)⟩ / x∈R } be an intuitionistic multi-fuzzy set of dimension 2 on R is defined as,

$$A(x) = (A_1(x), A_2(x)) = \begin{cases} (0.1, 0.2) & \text{if x is rational} \\ (0.4, 0.6) & \text{otherwise} \end{cases} \quad \text{and}$$

$$B(x) = (B_1(x), B_2(x)) = \begin{cases} (0.6, 0.5) & \text{if x is rational} \\ (0.5, 0.3) & \text{otherwise} \end{cases}$$

Consider 1 and $\sqrt{2}$.

Clearly, A (1) = (0.1, 0.2), A($\sqrt{2}$) = (0.4, 0.6) and A(1 · $\sqrt{2}$) = A($\sqrt{2}$) = (0.4, 0.6).

min { A(1), A($\sqrt{2}$) } = (0.1, 0.2) and max { A(1), A($\sqrt{2}$) } = (0.4, 0.6).

A(1 · $\sqrt{2}$) = A($\sqrt{2}$) = (0.4, 0.6) ≤ (0.4, 0.6) = max { A(1), A($\sqrt{2}$) }, but

A(1 · $\sqrt{2}$) = A($\sqrt{2}$) = (0.6, 0.4) $\nleq$ (0.1, 0.2) = min { A(1), A($\sqrt{2}$) }.

Hence, A is a multi-anti fuzzy ring of R and A is not a multi-anti fuzzy ideal of R.

2.7 Theorem

Let R_1 and R_2 be any two rings. Let f : $R_1 \to R_2$ be a homomorphism onto rings. Let G = { ⟨x, A(x), B(x)⟩ / x∈R_1 } be an intuitionistic multi-anti fuzzy left ideal of R_1 then f (G) is an intuitionistic multi-anti fuzzy left ideal of R_2, if G has a inf property and G is f-invariant.

Proof

Let G = { ⟨x, A(x), B(x)⟩ / x∈R_1 } be an intuitionistic multi-anti fuzzy left ideal of R_1

Then, f(G) = { ⟨f(y), f(A)(f(y)), f(B)(f(y))⟩ / y∈R_2 }.

There exist x,y∈ R_1 such that f(x) , f(y) ∈ R_2,

i. (f (A)) (f(x) – f(y)) = (f(A))(f(x–y)) ,
= A (x–y)
≤ max {A (x) , A (y)}
= max {(f (A)) (f(x)) , (f (A)) (f(y))}

(f (A)) (f(x) – f(y)) ≤ max {(f (A)) (f(x)), (f (A)) (f(y))}.

ii. (f (A)) (f(x) f(y)) = (f (A)) (f(xy))
= A (xy)
≤ A (y)
= (f (A)) (f(y))

(f (A)) (f(x)f(y)) ≤ (f (A)) (f(y))

iii. (f (B)) (f(x) – f(y)) = (f(B))(f(x–y)) ,
= B(x–y)
≥ min {B (x) , B (y)}
= min {(f (B)) (f(x)) , (f (B)) (f(y))}

(f (B)) (f(x) – f(y)) ≥ min {(f (B)) (f(x)), (f (B)) (f(y))}

iv. (f (B)) (f(x) f(y)) = (f (B)) (f(xy))
= B (xy)
≥ B (y)
= (f (B)) (f(y))

(f (B)) (f(x)f(y)) ≥ (f (B)) (f(y))

Hence, f (G) is an intuitionistic multi-anti fuzzy left ideal of R_2.

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2.8 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be a homomorphism onto rings. Let $G = \{ \langle x, A(x), B(x)\rangle / x \in R_1 \}$ be an intuitionistic multi-anti fuzzy right ideal of R_1 then f (G) is an intuitionistic multi-anti fuzzy right ideal of R_2, if G has a inf property and G is f-invariant.

Proof

Let $G = \{ \langle x, A(x), B(x)\rangle / x \in R_1 \}$ be an intuitionistic multi-anti fuzzy right ideal of R_1

Then, $f(G) = \{ \langle f(y), f(A)(f(y)), f(B)(f(y))\rangle / y \in R_2 \}$.

There exist $x,y \in R_1$ such that $f(x)$, $f(y) \in R_2$,

i. $(f (A)) (f(x) – f(y)) = (f(A))(f(x–y))$,
$= A (x–y)$
$\leq \max \{A (x) , A (y)\}$
$= \max \{(f (A)) (f(x)) , (f (A)) (f(y))\}$
$(f (A)) (f(x) – f(y)) \leq \max \{(f (A)) (f(x)), (f (A)) (f(y))\}$.

ii. $(f (A)) (f(x) f(y)) = (f (A)) (f(xy))$
$= A (xy)$
$\leq A (x)$
$= (f (A)) (f(x))$
$(f (A)) (f(x)f(y)) \leq (f (A)) (f(x))$

iii. $(f (B)) (f(x) – f(y)) = (f(B))(f(x–y))$,
$= B(x–y)$
$\geq \min \{B (x) , B (y)\}$
$= \min \{(f (B)) (f(x)) , (f (B)) (f(y))\}$
$(f (B)) (f(x) – f(y)) \geq \min \{(f (B)) (f(x)), (f (B)) (f(y))\}$

iv. $(f (B)) (f(x) f(y)) = (f (B)) (f(xy))$
$= B (xy)$
$\geq B (x)$
$= (f (B)) (f(x))$
$(f (B)) (f(x)f(y)) \geq (f (B)) (f(x))$

Hence, f (G) is an intuitionistic multi-anti fuzzy right ideal of R_2.

2.9 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be a homomorphism onto rings. Let $G = \{ \langle x, A(x), B(x)\rangle / x \in R_1 \}$ be an intuitionistic multi-anti fuzzy ideal of R_1 then f (G) is an intuitionistic multi-anti fuzzy ideal of R_2, if G has a inf property and G is f-invariant.

Proof

It is clear.

2.10 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be a homomorphism onto rings. Let $H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$be an intuitionistic multi-anti fuzzy left ideal of R_2 then f^{-1} (H) is an intuitionistic multi-anti fuzzy left ideal of R_1.

Proof

$H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$be an intuitionistic multi-anti fuzzy left ideal of R_2.

Then, $f^{-1}(H) = = \{ \langle f^{-1}(x), f^{-1}(C)(x), f^{-1}(D)(x)\rangle / x \in R_1 \}$.

For any $x, y \in R_1$, $f(x)$, $f(y) \in R_2$,

i. $(f^{-1}(C)) (x–y) = C (f(x–y))$
$= C (f(x) – f(y))$
$\leq \max \{C(f(x)) , C(f(y))\}$
$= \max \{(f^{-1}(C)) (x) , (f^{-1}(C)) (y)\}$
$(f^{-1}(C)) (x–y) \leq \max\{(f^{-1}(C)) (x) , (f^{-1}(C)) (y)\}$.

ii. $(f^{-1}(C)) (xy) = C (f(xy))$
$= C (f(x) f(y))$
$\leq C(f(y))$

$= \quad (f^{-1}(C))(y)$

$(f^{-1}(C))(xy) \leq (f^{-1}(C))(y)$.

iii. $(f^{-1}(D))(x-y) = D(f(x-y))$

$= D(f(x) - f(y))$

$\geq \min\{D(f(x)), D(f(y))\}$

$= \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$

$(f^{-1}(D))(x-y) \geq \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$.

iv. $(f^{-1}(D))(xy) = D(f(xy))$

$= D(f(x) f(y))$

$\geq D(f(y))$

$= (f^{-1}(D))(y)$

$(f^{-1}(D))(xy) \geq (f^{-1}(D))(y)$.

Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy left ideal of R_1.

2.11 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be a homomorphism onto rings. Let $H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy right ideal of R_2 then $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy right ideal of R_1.

Proof

$H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy right ideal of R_2.

Then, $f^{-1}(H) = = \{ \langle f^{-1}(x), f^{-1}(C)(x), f^{-1}(D)(x)\rangle / x \in R_1 \}$.

For any $x, y \in R_1$, $f(x), f(y) \in R_2$,

i. $(f^{-1}(C))(x-y) = C(f(x-y))$

$= C(f(x) - f(y))$

$\leq \max\{C(f(x)), C(f(y))\}$

$= \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$

$(f^{-1}(C))(x-y) \leq \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$.

ii. $(f^{-1}(C))(xy) = C(f(xy))$

$= C(f(x) f(y))$

$\leq C(f(x))$

$= (f^{-1}(C))(x)$

$(f^{-1}(C))(xy) \leq (f^{-1}(C))(x)$.

iii. $(f^{-1}(D))(x-y) = D(f(x-y))$

$= D(f(x) - f(y))$

$\geq \min\{D(f(x)), D(f(y))\}$

$= \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$

$(f^{-1}(D))(x-y) \geq \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$.

iv. $(f^{-1}(D))(xy) = D(f(xy))$

$= D(f(x) f(y))$

$\geq D(f(x))$

$= (f^{-1}(D))(x)$

$(f^{-1}(D))(xy) \geq (f^{-1}(D))(x)$.

Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy right ideal of R_1.

2.12 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be a homomorphism onto rings. Let $H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy ideal of R_2 then $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy ideal of R_1.

Proof

It is clear.

2.13 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be an anti homomorphism onto rings. Let $G = \{ \langle x, A(x), B(x)\rangle / x \in R_1 \}$ be an intuitionistic multi-anti fuzzy left ideal of R_1 then

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f (G) is an intuitionistic multi-anti fuzzy right ideal of R_2, if G has a inf property and G is f-invariant.

Proof

Let G = { ⟨x, A(x), B(x)⟩ / x∈R_1 } be an intuitionistic multi-anti fuzzy left ideal of R_1

Then, f(G) = { ⟨f(y), f(A)(f(y)), f(B)(f(y))⟩ / y∈R_2 }.

There exist x, y∈ R_1 such that f(x) , f(y) ∈ R_2,

i. (f (A)) (f(x) – f(y)) = (f(A))(f(y–x)) ,
= A (y–x)
= A (x – y)
≤ max {A (x) , A (y)}
= max {(f (A)) (f(x)) , (f (A)) (f(y))}
(f (A)) (f(x) – f(y)) ≤ max {(f (A)) (f(x)), (f (A)) (f(y))}.

ii. (f (A)) (f(x) f(y)) = (f (A)) (f(yx))
= A (yx)
≤ A (x)
= A (x)
= (f (A)) (f(x))
(f (A)) (f(x)f(y)) ≤ (f (A)) (f(x)).

iii. (f (B)) (f(x) – f(y)) = (f(B))(f(y–x))
= B(y– x)
= B(x–y)
≥ min {B (x), B (y)}
= min {(f (B)) (f(x)) , (f (B)) (f(y))}
(f (B)) (f(x) – f(y)) ≥ min {(f (B)) (f(x)), (f (B)) (f(y))}

iv. (f (B)) (f(x) f(y)) = (f (B)) (f(yx))
= B (yx)
≥ B (x)}
= B (x)
= (f (B)) (f(x))
(f (B)) (f(x)f(y)) ≥ (f (B)) (f(x))

Hence, f (G) is an intuitionistic multi-anti fuzzy right ideal of R_2.

2.14 Theorem

Let R_1 and R_2 be any two rings. Let f : $R_1 \rightarrow R_2$ be an anti homomorphism onto rings. Let G = { ⟨x, A(x), B(x)⟩ / x∈R_1 } be an intuitionistic multi-anti fuzzy right ideal of R_1 then f (G) is an intuitionistic multi-anti fuzzy left ideal of R_2, if G has a inf property and G is f-invariant.

Proof

Let G = {⟨x, A(x), B(x)⟩ / x∈R_1} be an intuitionistic multi-anti fuzzy right ideal of R_1.

Then, f(G) = { ⟨f(y), f(A)(f(y)), f(B)(f(y))⟩ / y∈R_2 }.

There exist x, y∈ R_1 such that f(x) , f(y) ∈ R_2,

i. (f (A)) (f(x) – f(y)) = (f(A))(f(y–x)) ,
= A (y–x)
= A (x – y)
≤ max {A (x) , A (y)}
= max {(f (A)) (f(x)) , (f (A)) (f(y))}
(f (A)) (f(x) – f(y)) ≤ max {(f (A)) (f(x)), (f (A)) (f(y))}.

ii. (f (A)) (f(x) f(y)) = (f (A)) (f(yx))
= A (yx)
≤ A (y)
= A (y)
= (f (A)) (f(y))
(f (A)) (f(x)f(y)) ≤ (f (A)) (f(y)).

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iii. $(f(B))(f(x)-f(y)) = (f(B))(f(y-x))$
$= B(y-x)$
$= B(x-y)$
$\geq \min\{B(x), B(y)\}$
$= \min\{(f(B))(f(x)), (f(B))(f(y))\}$
$(f(B))(f(x)-f(y)) \geq \min\{(f(B))(f(x)), (f(B))(f(y))\}$
iv. $(f(B))(f(x)f(y)) = (f(B))(f(yx))$
$= B(yx)$
$\geq B(y)\}$
$= B(y)$
$= (f(B))(f(y))$
$(f(B))(f(x)f(y)) \geq (f(B))(f(y))$
Hence, f (G) is an intuitionistic multi-anti fuzzy left ideal of R_2.

2.15 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be an anti homomorphism onto rings. Let $G = \{ \langle x, A(x), B(x)\rangle / x \in R_1 \}$ be an intuitionistic multi-anti fuzzy ideal of R_1 then f (G) is an intuitionistic multi-anti fuzzy ideal of R_2, if G has a inf property and G is f-invariant.

Proof

It is clear.

2.16 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be an anti homomorphism onto rings. Let $H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy left ideal of R_2 then $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy right ideal of R_1.

Proof

$H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy left ideal of R_2.

Then, $f^{-1}(H) = \{ \langle f^{-1}(x), f^{-1}(C)(x), f^{-1}(D)(x)\rangle / x \in R_1 \}$

For any $x, y \in R_1$, $f(x), f(y) \in R_2$,

i. $(f^{-1}(C))(x-y) = C(f(x-y))$
$= C(f(y)-f(x))$
$= C(f(x)-f(y))$
$\leq \max\{C(f(x)), C(f(y))\}$
$= \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$
$(f^{-1}(C))(x-y) \leq \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$.
ii. $(f^{-1}(C))(xy) = C(f(xy))$
$= C(f(y)f(x))$
$\leq C(f(x))$
$= C(f(x))$
$= (f^{-1}(C))(x)$
$(f^{-1}(C))(xy) \leq (f^{-1}(C))(x)$.
iii. $(f^{-1}(D))(x-y) = D(f(x-y))$
$= D(f(y)-f(x))$
$= D(f(x)-f(y))$
$\geq \min\{D(f(x)), D(f(y))\}$
$= \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$
$(f^{-1}(D))(x-y) \geq \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$.
iv. $(f^{-1}(D))(xy) = D(f(xy))$
$= D(f(y)f(x))$
$\geq D(f(x))$
$= (f^{-1}(D))(x)$
$(f^{-1}(D))(xy) \geq (f^{-1}(D))(x)$.

Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy right ideal of R_1.

2.17 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be an anti homomorphism onto rings. Let $H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy right ideal of R_2 then f^{-1} (H) is an intuitionistic multi-anti fuzzy left ideal of R_1.

Proof

$H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy right ideal of R_2.

Then, $f^{-1}(H) = \{ \langle f^{-1}(x), f^{-1}(C)(x), f^{-1}(D)(x)\rangle / x \in R_1 \}$

For any $x, y \in R_1$, $f(x), f(y) \in R_2$,

i. $(f^{-1}(C))(x-y) = C(f(x-y))$
$= C(f(y) - f(x))$
$= C(f(x) - f(y))$
$\leq \max\{C(f(x)), C(f(y))\}$
$= \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$
$(f^{-1}(C))(x-y) \leq \max\{(f^{-1}(C))(x), (f^{-1}(C))(y)\}$.

ii. $(f^{-1}(C))(xy) = C(f(xy))$
$= C(f(y) f(x))$
$\leq C(f(y))$
$= C(f(y))$
$= (f^{-1}(C))(y)$
$(f^{-1}(C))(xy) \leq (f^{-1}(C))(y)$.

iii. $(f^{-1}(D))(x-y) = D(f(x-y))$
$= D(f(y) - f(x))$
$= D(f(x) - f(y))$
$\geq \min\{D(f(x)), D(f(y))\}$
$= \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$
$(f^{-1}(D))(x-y) \geq \min\{(f^{-1}(D))(x), (f^{-1}(D))(y)\}$.

iv. $(f^{-1}(D))(xy) = D(f(xy))$
$= D(f(y) f(x))$
$\geq D(f(y))$
$= (f^{-1}(D))(y)$
$(f^{-1}(D))(xy) \geq (f^{-1}(D))(y)$.

Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy left ideal of R_1.

2.18 Theorem

Let R_1 and R_2 be any two rings. Let $f : R_1 \to R_2$ be an anti homomorphism onto rings. Let $H = \{ \langle y, C(y), D(y)\rangle / y \in R_2 \}$ be an intuitionistic multi-anti fuzzy ideal of R_2 then f^{-1} (H) is an intuitionistic multi-anti fuzzy ideal of R_1.

Proof

It is clear.

III CONCLUSION

In this paper, We discuss the properties of anti image and anti pre image of an intuitionistic multi-anti fuzzy ideal of a ring under homomorohism and anti homomorphism.

REFERENCES

1. Attanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets And Systems, 20,87-89,1986.
2. Azan.F.A, A.A.Mamun, and F. Nasrin, Anti fuzzy ideals of a ring, Annals of Fuzzy Mathemathics and Informatics, Volume 5, No.2, pp 349 – 390, March 2013.
3. Biswas.R., fuzzy subgroups and Anti – fuzzy subgroups, Fuzzy sets and Systems, 5, 121 – 124, 1990.
4. Bingxue. Y, Fuzzy semi-ideal and generalized fuzzy quotient ring, Iranian Joural of Fuzzy Systems, Vol. 5, pp 87 – 92 , 2008.
5. Gouguen J.A., L-fuzzy sets, Journal of Mathematics Analysis and Applications, 18, 145 – 174, 1967.

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6. Kumbhojkar. H.V. and Bapat.M.S, Correspondence theorem for fuzzy ideals, Fuzzy Sets and system, 41, 213-219, 1991.
7. Malik.D.S and John N. Moderson, Fuzzy Prime Ideals of Ring, Fuzzy Sets and System 37 , 93-98, 1990.
8. Moderson L.N and D.S. Malik, Fuzzy Commutaive Algebra, World Scientific, Singapore, 1998.
9. Muthuraj.R and Malarselvi.C,Multi-Fuzzy Prime Ideal of a Ring, Discovery, Volume 21, No.65,July 1,2014.
10. Muthuraj. R and Malarselvi. C, Multi-anti Fuzzy Ideal of a Ring, Proceedings of the National converence on Mathematical Modelling,114-120, 2014.ISBN 9380744-7.
11. Rosenfeld, A, Fuzzy groups, Journal of Mathematical Analysis and applications,3, 512- 517,1971.
12. Sabu Sebastian and T.V.Ramakrishnan, Multi-fuzzy Sets, International Mathematical Forum, 5, no. 50, 2471-2476,2010.
13. Sabu Sebastian and T.V.Ramakrishnan, Multi-Fuzzy Topology, International Journal of Applied Mathematics Volume 24, No.1, 117-129,2011.
14. Samit Kumar Majumder and Suit Kumar Sardar, On Cartesian productof fuzzy prime and fuzzy semiprime ideals of semigroups,Journal of Mathematics and Applications, No 32, pp 63-66(2010)
15. Muthuraj.R and Balamurugan. S, Multi-fuzzy group and its level subgroups, Gen.Math.Notes, vol. 17,No. 1, July 2013,pp 74-81.
16. Souriar Sebastian, Mercy K.Jacob and V.M Mary and Divya Mary Daise, On Fuzzy Ideals and Rings, J. Comp. & Math. Sci Vol.3(1), 115-120, 2012
17. Muthuraj.R and Malarselvi.C,
18. Zadeh.L.A,Fuzzy sets, Information and Control 8, 338-353,1965.