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A Study on new Solving Procedure for Fuzzy Time Cost Trade-off Problems using decomposition and aggregated techniques

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ABSTRACT

This research article share out precise structure and procedure for solving a fuzzy time cost trade-off problems using linear programming problem in a fuzzy environment. The decomposition method for work out of fuzzy linear programming problem has been used. In this method a several number of linear programming problems can be aggregated into a unique linear programming problem which gives the finest solution. The prescribed method is examined through mathematical illustration.

Keywords: Fuzzy theory, fuzzy numbers, fuzzy time cost trade-off problems, fuzzy linear programming problem, decomposition techniques, aggregation of m-LPPs.

1. Review of literature

Project Management is one of the important fields in business and industry. The tradeoff between the project cost and the project completion time and the uncertainty of the environment are considerable issues for all real life project decision makers. An important aspect of project management is to schedule the time accurately. In the literature, there are several approaches proposed over the past years for finding the optimum duration with minimum cost. Zadeh [11] introduced the concept of fuzzy sets and today almost all research areas have depended on the development of the minimal cost with optimal duration. James E. Kelley[6] first introduced the critical path planning and scheduling and followed by that Ghazanfari et al.[5] presented the new optimal model for time cost trade off problem in a fuzzy environment using goal programming problem.P. Pandian et al. [8] proposed a new method called decomposition method to solve integer linear programming problems by using triangular fuzzy variables and also a new approach to fuzzy network crashing in a project network whose activity times are uncertain finding an optimal duration without converting the fuzzy activity time to classical number was proposed by Shakeela sathish et al. [9] , Evangeline Jebaseeli et al.[4] formulated a new solution for time cost trade off problems in which both times and costs are fuzzy variables in the same era. Evangeline Jebaseeli et al.[3] proposed an algorithm to solve fully fuzzy time cost trade off models through multi objective linear programming technique. Aggregated techniques of m-LPPs was proposed by Antony raj et al[1].

In this research article we give out a solving approach for fully fuzzy time cost trade off problem using decomposition and aggregated techniques in fuzzy linear programming problem to obtain the perfect solution of the project with numerical illustration.

2. Preliminaries

Definition 1

The characteristic function μ_A in a crisp set A⊆S assigns a value either 0 or 1 for each member in S. The function is generalised to a function $\mu_{\tilde{A}}$ such that the value assigned with the element of S lies within a specified range i.e.

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$\mu_{\tilde{A}} : S \to [0,1]$. The assigned values $\mu_{\tilde{A}}(s)$ for each s∈S denote the membership grade of the element in the set A. The set $\tilde{A} = \{A, \mu_A(x) : x \in X\}$ is called Fuzzy Set.

Definition 2

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: R \to [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $(-\infty, a] \cup [c, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [b,c].
4. $\mu_{\tilde{A}}(x) = 1$ for all x∈b where a$\leq x \leq c$.

Definition 3

Linear pentagonal fuzzy number is a fuzzy variable represented with five tuples as follows:
$\tilde{A}_{ls} = (g_1, g_2, g_3, g_4, g_5)$ This representation is interpreted as membership functions:
We use F(R) to denote the set of all linear pentagonal fuzzy numbers.

$$\mu_{\tilde{A}_{ls}}(x) = \begin{cases} k\dfrac{x-g_1}{g_2-g_1} \quad \text{if} \quad g_1 \leq x < g_2 \\ 1-(1-k)\dfrac{x-g_2}{g_3-g_2} \quad if \quad g_2 \leq x < g_3 \\ 1 \quad x = g_3 \\ 1-(1-k)\dfrac{g_4-x}{g_4-g_3} \quad if \quad g_3 \leq x < g_4 \\ k\dfrac{g_5-x}{g_5-g_4} if \quad g_4 \leq x < g_5 \\ 0 \quad x > g_5 \end{cases}$$

Definition 4

Let $(g_1, g_2, g_3, g_4, g_5)$ and $(h_1, h_2, h_3, h_4, h_5)$ be two linear pentagonal fuzzy numbers. Then

$$(g_1, g_2, g_3, g_4, g_5) \oplus (h_1, h_2, h_3, h_4, h_5) = (g_1+h_1, g_2+h_2, g_3+h_3, g_4+h_4, g_5+h_5)$$

$$(g_1, g_2, g_3, g_4, g_5) - (h_1, h_2, h_3, h_4, h_5) = (g_5-h_1, g_4-h_2, g_3-h_3, g_2-h_4, g_1-h_5)$$

$$c(g_1, g_2, g_3, g_4, g_5) = (cg_1, cg_2, cg_3, cg_4, cg_5), \quad for\, c \geq 0.$$

$$c(g_1, g_2, g_3, g_4, g_5) = (cg_5, cg_4, cg_3, cg_2, cg_1), \quad for \;\; c < 0.$$

$$\frac{(g_1, g_2, g_3, g_4, g_5)}{(h_1, h_2, h_3, h_4, h_5)} = \left(\frac{g_1}{h_5}, \frac{g_2}{h_4}, \frac{g_3}{h_3}, \frac{g_4}{h_2}, \frac{g_5}{h_5}\right)$$

Definition 5

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A fuzzy project network is an acyclic digraph, where the points represent events and the oriented lines represents activities. Let us represent the fuzzy project network by $\tilde{P} = \langle \mathrm{N}, \mathrm{L}, \tilde{\mathrm{O}} \rangle$. Let $N = \{n_1, \mathrm{n}_2, \ldots, \mathrm{n}_m\}$ be the set of all points (events), n_m and n_1 are the head and tail events of the project. Let $L \subset N \times N$ be the set of all oriented lines $L = \{l_{ij} = (n_i, \mathrm{n}_j) / \mathrm{n}_i, \mathrm{n}_j \in N\}$, which denote the activities to be represented in the project. A critical path is a longest path between initial event n_1 and terminal event n_m and an activity l_{ij} on a critical path is known as critical activity.

Definition 6

Linear programming problem is one among the most habitually applied operations research technique by assuming that all variables and parameters are real numbers. But in real life circumstance we do not have proper data. So, the fuzzy variables and fuzzy numbers are used in Linear programming problem. The standard form fully fuzzy linear programming problems with n fuzzy variables and m fuzzy constants are given below:

$$Maximize\ or\ (Minimize)\ (\tilde{\mathrm{A}}^T \otimes \tilde{Y})$$

Subject to $\tilde{B}\tilde{Y} = \tilde{d}$

$\tilde{Y}$ is a non-negative fuzzy number.

$$\tilde{A}^T = \tilde{a}_{j_{1xn}} \tilde{Y} = \tilde{y}_{i_{nx1}}, \tilde{\mathrm{B}} = [\tilde{\mathrm{b}}_{ij}]_{mxn}, \tilde{d} = [\tilde{d}_i]_{mx1} and$$

Where $\tilde{c}_j, \tilde{y}_j, \tilde{b}_{ij}, \tilde{d}_i \in F(R)$

$$where i = 1, 2, \ldots \mathrm{m} \,\&\, j = 1, 2, \ldots \mathrm{n}$$

Definition 7

A fuzzy project network can be defined by an activity-on-activity arc network P=(N,L) where N={1,2,...,m} is the set of nodes(points) and A is the set of arcs(oriented lines) represents the activities. In the fuzzy project network, node 1 and n denotes the initial and terminal of the project respectively. The **complete fuzzy Mathematical** model for fully fuzzy time cost trade-off problems is given as follows:

$$\mathrm{Min}\tilde{\mathrm{Z}} = \sum_k \sum_l A_{kl}$$

subject to

$$\tilde{D}_1 = 0, \tilde{\mathrm{D}}_l - \tilde{D}_k - \tilde{y}_{kl} \geq 0, \tilde{D}_m \leq \tilde{D}; \tilde{a}_{kl} = \tilde{s} * (N\tilde{D}_{kl} - \tilde{y}_{kl}), A\tilde{D}_{kl} \leq \tilde{y}_{kl} \leq N\tilde{D}_{kl}$$

$$\forall (\mathrm{k}, \mathrm{l}) \in P, \tilde{A}_{kl} = \sum_k \sum_l \tilde{a}_{kl} + \tilde{I} * (\tilde{D}_m - \tilde{D}_1) + \sum m\tilde{K}_m; \quad Where\, a = (1, 2, \ldots m)\ and\ b = (1, 2, \ldots m).$$

Decomposition Theorem

A triangular fuzzy number $\tilde{y} = (\tilde{y}_1, \tilde{\mathrm{y}}_2, \tilde{\mathrm{y}}_3)$ is an optimal result of the problem (Q) if and only if $\tilde{\mathrm{y}}_1, \tilde{\mathrm{y}}_2$ *and* $\tilde{\mathrm{y}}_3$ are optimal results of the prescribed crisp linear programming problems (Q2), (Q1) and (Q3) respectively where:

(Q) Maximize $\tilde{Z} = Ay$ Subject to $\tilde{B}y \leq \tilde{d}, y \geq 0$

(Q2) Maximize $Z_2 = Ay_2$ Subject to $By_2 \leq d_2, y_2 \geq 0$

(Q1) Maximize $Z_1 = Ay_1$ Subject to $By_1 \leq d_1, \mathrm{y}_1 \geq 0, \mathrm{y}_1 \leq y_2$

(Q3) Maximize $Z_3 = Ay_3$ Subject to $By_3 \leq d_3, \mathrm{y}_3 \geq 0, \mathrm{y}_3 \geq y_2$

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Aggregation of m-LPPs [1]

Notations

k	:	*k^{th} problem (k=1,2,…m)*
l	:	*l^{th} problem (l=1,2,…n_k)*
y_{kl}	:	*l^{th} variable of the k^{th} problem*
a_{kl}	:	*constant coefficient of the l^{th} variable of the k^{th} problem*
n_k	:	*Number of variables in the k^{th} problem*
r_k	:	*Number of constraints in the k^{th} problem*
d_{kr_k}	:	*RHS value of the r_k^{th} constraints of the k^{th} problem*

General LPP structure of the k^{th}- problem (k=1,2,...m) can be given as:

$$Max\, Z_k = a_{k1}\, y_{k1} + a_{k2} y_{k2} + \ldots + a_{kn_k} y_{kn_k}$$

Subject to the constraints:

$$b_{k11} y_{k1} + b_{k12} y_{k2} + \ldots + b_{k1n_k} y_{kn_k} \{\le,=,\ge\}\, d_{k1}$$

$$b_{k21} y_{k1} + b_{k22} y_{k2} + \ldots + b_{k2n_k} y_{kn_k} \{\le,=,\ge\}\, d_{k2}$$

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$$b_{ki_k1} y_{k1} + b_{ki_k2} y_{k2} + \ldots + b_{ki_kn_k} y_{kn_k} \{\le,=,\ge\}\, d_{ki_k}$$

$$y_{kl} \ge 0, \{k = 1, \ldots m\ ,\ l = 1, 2, \ldots, n_k\}$$

Aggregated structure of m-LPPs together

$$Max\, Z = \sum_{k=1}^{m} \sum_{l=1}^{n_k} a_{kl}\, y_{kl}$$

Subject to the constraints:

$$b_{11} y_{11} + b_{12} y_{12} + \ldots + b_{11n1} y_{1n1} \{\le,=,\ge\} d_{11}$$

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$$b_{1i_{11}} y_{11} + b_{1i_{12}} y_{12} + \ldots + b_{1i_{1n1}}\, y_{1n1} \{\le,=,\ge\} d_{1k_1}$$

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$$b_{m_{11}} y_{11} + b_{m_{12}} y_{12} + \ldots + b_{m_{1n1}} y_{1n1} \{\le,=,\ge\} d_{m_1}$$

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$$b_{m_{km1}} y_{11} + b_{m_{km2}} y_{12} + \ldots + b_{m_{kmn1}} y_{mn1} \{\le,=,\ge\} d_{m_{km}}$$

$$x_{kl} \ge 0,\ \{k = 1, \ldots m, \boldsymbol{l} = 1, 2, \ldots, n_k\}.$$

3 Algorithm

Step 1 Determine the direct cost and the cost slope of the fuzzy project using pentagonal fuzzy variable

Step 2 Transform the fuzzy time cost trade-off problem into fuzzy linear programming problem with the use of fully fuzzy mathematical model.

Step 3 Decomposition procedure is used to split up fuzzy linear programming problem into crisp linear programming problems.

Step 4 Utilize the aggregation of m-LPPs to aggregate the crisp linear programming problems into unique linear programming problem.

Step 5 The optimum result of the crash cost and crash duration for all the activities can be found in the respective variables.

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4 Numerical illustration

List of activities for construction of house is given below with the details. Table 1 gives out the description of the project. In this fuzzy project, time and costs of the project are considered in pentagonal fuzzy number form. Indirect cost of the project per day is (100,100,100,100,100). The project manager wishes to complete within (78,79,80,81,82) days. Activities information is given in table 2.

Table 1 Project explanation

ACTIVITY	DESCRIPTION
1-2(E)	To Acquire Land or plat
1-3 (F)	Prepare Estimate and Budget
1-4 (G)	Approach a Builder
2-3 (H)	Site preparation and levelling work
3-5 (I)	Foundation Plinth Beam, column ,slab
4-5 (J)	Brisk masonry Work

Table 2 Fuzzy data of the project

Activity	Crash Duration	Normal Duration	Normal cost	Crash cost
1-2(E)	(17,18,19,20,21)	(25,25,25,25,25)	(1200,1200,1200,1200,1200)	(1500,1500,1500,1500,1500)
1-3 (F)	(20,20 20,20,20)	(25,25,25,25,25)	(1500,1500,1500,150015 00)	(1600,1600,1600,1600,1600)
1-4 (G)	(56, 57,58,59,60)	(60,60,60,60,60)	(1100,1100,1100,1100,1100)	(1500,1500,1500,15001500)
2-3 (H)	(10,11,12,1314)	(15,15,15,15,15)	(900,1000,1100,1200,1300)	(1300,1300,1300,1300,1300)
3-5 (I)	(20,20,20,20,20)	(25,25,25,25,25)	(2000,2000,2000,2000,2000)	(2500,2500,2500,2500,2500
4-5 (J)	(18,18,18,18,18)	(20,20,20,20,20)	(1600,1700,1800,1900,2000)	(2000,2000,2000,2000,2000)

Table: 3 Crash slope of the project

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Activity	ΔT	ΔC	Slope cost ΔS =ΔC/ΔT
1-2(E)	(4,5, 6,7,8)	(300,300,300,300,300)	(37.5,42.8,50,60,75)
1-3 (F)	(5,5,5,5,5)	(100,100,100,100,100)	(20,20,20,20,20)
1-4 (G)	(1,2,3,4,5)	(400,400,400,400,500)	(80,100,133.3,200,400)
2-3 (H)	(1,2,3,4,5)	(100,200,300,400 ,500)	(20,50,100 ,200,500)
3-5 (I)	(5,5,5,5,5)	(500,500,500,500,500)	(100,100,100,100,100)
4-5 (J)	(2,2,2,2,2)	(100,200,300,400,500)	(50,100,150,200,250)

Direct Cost = (8300, 8500, 8700, 8900,9100)

Total Cost =(16300,16500,16700,16900,17100)

Hence we have: $Min\tilde{Z} = [\sum_k \sum_l \tilde{a}_{kl} + \tilde{I} * \left(\tilde{D}_m - \tilde{D}_1\right) + \sum_m \tilde{K}_m]$

Subject to the constraints:

$\tilde{D}_1 = 0 \; ; \tilde{D}_2 = 0 ; \tilde{D}_3 = 0 ; \tilde{D}_4 = 0 ; \tilde{D}_5 = 0$

$\tilde{D}_{21} - \tilde{D}_{11} - \tilde{y}_{E1} \geq 0 \; ; \; \tilde{D}_{22} - \tilde{D}_{12} - y_{E2} \geq 0 ; \tilde{D}_{23} - \tilde{D}_{13} - \tilde{y}_{E3} \geq 0 \; ; \tilde{D}_{24} - \tilde{D}_{14} - \tilde{y}_{E4} \geq 0 \; \tilde{D}_{25} - \tilde{D}_{15} - \tilde{y}_{E5} \geq 0$

$\tilde{D}_{31} - \tilde{D}_{11} - \tilde{y}_{F1} \geq 0 \; ; \tilde{D}_{32} - \tilde{D}_{12} - \tilde{y}_{F2} \geq 0 ; \tilde{D}_{33} - \tilde{D}_{13} - \tilde{y}_{F3} \geq 0 ; \tilde{D}_{34} - \tilde{D}_{14} - \tilde{y}_{F4} \geq 0 ; \tilde{D}_{35} - \tilde{D}_{15} - \tilde{y}_{F5} \geq 0$

$\tilde{D}_{41} - \tilde{D}_{11} - \tilde{y}_{G1} \geq 0 \; ; \; \tilde{D}_{42} - \tilde{D}_{12} - \tilde{y}_{G2} \geq 0 \; ; \tilde{D}_{43} - \tilde{D}_{13} - \tilde{y}_{G3} \geq 0 ; \tilde{D}_{44} - \tilde{D}_{14} - \tilde{y}_{G4} \geq 0 ; \tilde{D}_{45} - \tilde{D}_{15} - \tilde{y}_{G5} \geq 0$

$\tilde{D}_{31} - \tilde{D}_{21} - \tilde{y}_{H1} \geq 0 \; ; \tilde{D}_{32} - \tilde{D}_{22} - \tilde{y}_{H2} \geq 0 \; \tilde{D}_{33} - \tilde{D}_{23} - \tilde{y}_{H3} \geq 0 \; ; \; \tilde{D}_{34} - \tilde{D}_{24} - \tilde{y}_{H4} \geq 0 \; ; \tilde{D}_{35} - \tilde{D}_{25} - \tilde{y}_{H5} \geq 0$

$\tilde{D}_{51} - \tilde{D}_{31} - \tilde{y}_{I1} \geq 0 ; \tilde{D}_{52} - \tilde{D}_{32} - \tilde{y}_{I2} \geq 0 ; \tilde{D}_{53} - \tilde{D}_{33} - \tilde{y}_{I3} \geq 0 ; \tilde{D}_{54} - \tilde{D}_{34} - \tilde{y}_{I4} \geq 0 ; \tilde{D}_{55} - \tilde{D}_{35} - \tilde{y}_{I5} \geq 0$

$\tilde{D}_{51} - \tilde{D}_{41} - \tilde{y}_{J1} \geq 0 ; \tilde{D}_{52} - \tilde{D}_{42} - \tilde{y}_{J2} \geq 0 ; \tilde{D}_{53} - \tilde{D}_{43} - \tilde{y}_{J3} \geq 0 ; \tilde{D}_{54} - \tilde{D}_{44} - \tilde{y}_{J4} \geq 0 ; \tilde{D}_{55} - \tilde{D}_{45} - \tilde{y}_{J5} \geq 0$

$\tilde{D}_{51} \leq 78 \quad ; \tilde{D}_{52} \leq 79 ; \tilde{D}_{53} \leq 80 ; \tilde{D}_{54} \leq 81 ; \tilde{D}_{55} \leq 82$

$\tilde{a}_{12} = \tilde{s}_{12} * \left(N\tilde{D}_{12} - \tilde{y}_{E1}\right) \; ; \; \tilde{a}_{12} = \tilde{s}_{12} * \left(N\tilde{D}_{12} - \tilde{y}_{E2}\right) \; ; \; \tilde{a}_{12} = \tilde{s}_{12} * \left(N\tilde{D}_{12} - \tilde{y}_{E3}\right) \; ; \; \tilde{a}_{12} = \tilde{s}_{12} * \left(N\tilde{D}_{12} - \tilde{y}_{E4}\right) \; ;$

$\tilde{a}_{12} = \tilde{s}_{12} * \left(N\tilde{D}_{12} - \tilde{y}_{E5}\right)$

$\tilde{a}_{13} = \tilde{s}_{13} * \left(N\tilde{D}_{13} - \tilde{y}_{F1}\right) \; ; \; \tilde{a}_{13} = \tilde{s}_{13} * \left(N\tilde{D}_{13} - \tilde{y}_{F2}\right) \; ; \; \tilde{a}_{13} = \tilde{s}_{13} * \left(N\tilde{D}_{13} - \tilde{y}_{F3}\right) \; ; \; \tilde{a}_{13} = \tilde{s}_{13} * \left(N\tilde{D}_{13} - \tilde{y}_{F4}\right) \; ;$

$\tilde{a}_{13} = \tilde{s}_{13} * \left(N\tilde{D}_{13} - \tilde{y}_{F5}\right)$

$\tilde{a}_{14} = \tilde{s}_{14} * \left(N\tilde{D}_{14} - \tilde{y}_{G1}\right) \; ; \; \tilde{a}_{14} = \tilde{s}_{14} * \left(N\tilde{D}_{14} - \tilde{y}_{G2}\right) \; ; \; \tilde{a}_{14} = \tilde{s}_{14} * \left(N\tilde{D}_{14} - \tilde{y}_{G3}\right) \; ; \; \tilde{a}_{14} = \tilde{s}_{14} * \left(N\tilde{D}_{14} - \tilde{y}_{G4}\right) \; ;$

$\tilde{a}_{14} = \tilde{s}_{14} * \left(N\tilde{D}_{14} - \tilde{y}_{G5}\right)$

$\tilde{a}_{23} = \tilde{s}_{23} * \left(N\tilde{D}_{23} - \tilde{y}_{H1}\right) \; ; \; \tilde{a}_{23} = \tilde{s}_{23} * \left(N\tilde{D}_{23} - \tilde{y}_{H2}\right) \; ; \; \tilde{a}_{23} = \tilde{s}_{23} * \left(N\tilde{D}_{23} - \tilde{y}_{H3}\right) \; ; \tilde{a}_{23} = \tilde{s}_{23} * \left(N\tilde{D}_{23} - \tilde{y}_{H4}\right) \; ;$

$\tilde{a}_{23} = \tilde{s}_{23} * \left(N\tilde{D}_{23} - \tilde{y}_{H5}\right)$

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$\tilde{a}_{35} = \tilde{s}_{35} * \left(N\tilde{D}_{35} - \tilde{y}_{I1} \right)$; $\tilde{a}_{35} = \tilde{s}_{35} * \left(N\tilde{D}_{35} - \tilde{y}_{I2} \right)$; $\tilde{a}_{35} = \tilde{s}_{35} * \left(N\tilde{D}_{35} - \tilde{y}_{I3} \right)$; $\tilde{a}_{35} = \tilde{s}_{35} * \left(N\tilde{D}_{35} - \tilde{y}_{I4} \right)$; $\tilde{a}_{35} = \tilde{s}_{35} * \left(N\tilde{D}_{35} - \tilde{y}_{I5} \right)$

$\tilde{a}_{45} = \tilde{s}_{45} * \left(N\tilde{D}_{45} - \tilde{y}_{J1} \right)$; $\tilde{a}_{45} = \tilde{s}_{45} * \left(N\tilde{D}_{45} - \tilde{y}_{J2} \right)$; $\tilde{a}_{45} = \tilde{s}_{45} * \left(N\tilde{D}_{45} - \tilde{y}_{J3} \right)$; $\tilde{a}_{45} = \tilde{s}_{45} * \left(N\tilde{D}_{45} - \tilde{y}_{J3} \right)$; $\tilde{a}_{45} = \tilde{s}_{45} * \left(N\tilde{D}_{45} - \tilde{y}_{J5} \right)$

$a\tilde{D}_{12} \le \tilde{y}_E \le N\tilde{D}_{12}$; $a\tilde{D}_{13} \le \tilde{y}_F \le N\tilde{D}_{13}$; $a\tilde{D}_{14} \le \tilde{y}_G \le N\tilde{D}_{14}$; $a\tilde{D}_{23} \le \tilde{y}_H \le N\tilde{D}_{23}$; $a\tilde{D}_{35} \le \tilde{y}_I \le N\tilde{D}_{35}$; $a\tilde{D}_{45} \le \tilde{y}_J \le N\tilde{D}_{45}$

Table 4 Estimated duration for each activity

Activity	Project duration
1-2(E)	(25,25,25,25,25)
1-3(F)	(25,25,25,25,25)
1-4(G)	(60,60,60,60,60)
2-3(H)	(15,15,15,15,15)
3-5(I)	(25,25,25,25,25)
4-5(J)	(20,20,20,20,20)

The minimum values of the fuzzy total cost and planned duration of the project have been determined using LINGO solver package. The estimated duration of each activity is given in table 4. The Optimum Project Cost is Rs.20,450. Hence the Project Manager can able to finish the project within (78,79,80,81,82) days with the estimated duration.

5 Conclusion

In this paper we have proposed a new solving procedure for fuzzy time cost trade-off problem using fuzzy linear programming problem with simple numerical illustration. This method is easier and time consuming compared with the existing methods. As in previous existing methods we can able to get a crisp solution for the fuzzy variables. Using this new method we need not to use ranking or defuzzification method for the pentagonal fuzzy variables, we were able to get a fuzzy solution for the fuzzy variables.

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