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Symmetric Presentations of Finite Groups

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Abstract

In abstract algebra, a symmetric group defined over a set is the group for which the composition of functions is the group operation, and whose members are all the bijections from the set to itself. Galois theory, invariant theory, lie group representation theory, and combinatorics are just a few of the many branches of mathematics that rely on the symmetric group. The mathematical field of group representation theory studies the effects that groups have on predetermined structures. Particular attention is paid here to group operations on vector spaces. Yet, we also take into account groups that operate upon other groups or sets. The primary goal of this study is to offer an alternate approach of describing group elements of finite simple groups that is both brief and informative. In this research, we show how we found some new symmetric constructions for significant finite groups. As the orders of our photographs are becoming more and bigger, we've started using Magma to help us out with some of the math.

Keywords: Symmetric, Presentation, Finite Group, Magma, Double Coset Enumeration

Introduction

Throughout a long length of time, presentations with symmetric relations have been the subject of much research. Curtis takes a different tack, focusing on symmetric generating sets. Let's pretend that G is a finite 2-generator group with generators x_1 and x_2 and an automorphism with $x_1\theta = x_2\theta$ and $\theta^2 = 1$. Then, G's appearance is perfectly symmetrical. It is important to remember that such a symmetric presentation exists for every finite non-abelian simple group. Therefore G = 〈a, b if and only if G is a finite non-abelian simple group with $a^2 = 1$. Here we focus on H = 〈b, b^a〉 ≤ G. Thus, if G = H or |G: H| = 2. Yet, since G is elementary, G = H = 〈b, ba〉, and G has a symmetrical presentation. The conclusions and conjectures of a recent paper by Miklos Abert (unpublished) have been expanded to the case where, for every non-trivial group G, there exists a presentation of G such that, when symmetrized, the presentation yields an image of G that is not trivial. What was said above about symmetric presentations for simple groups is a fallout of Abert's findings.

Many branches of mathematics make use of representation theory, as do the fields of quantum chemistry and physics. It has several applications in algebra, including the study of group structure. Harmonic analysis and number theory also benefit from this. With the current method, representation theory is utilized to learn new things about automorphic forms, for instance.

Literature Review

Allocca, et al (2015) An established concept of length for an element $g in G$ in a finite group G with a generating subset S is expressed in terms of the minimum length expression for g as a product of elements from S. A precise measure of a group's stability against specific sorts of tiny perturbations in the generating expressions for the elements of the group has recently been described by the values $lambda 1(G,S)$ and $\lambda {2}(G,S)$. This work establishes limits for the functions $\lambda {1}(G,S)$ and $\lambda {2}$ for $G=D {n}$, a dihedral group, to further reveal their essential features. By fully characterizing the so-called symmetric presentations of the groups $D {n}$, we get insight into the relationship between the finite group presentations $\lambda {1}(G,S)$ and $lambda 2(G,S)$. This is interesting in its own right, apart from any investigation of the values $\lambda_{1}(G,S),\; \lambda_{2}(G,S)$. Lastly, we end with a discussion of many hypotheses and unanswered problems.

Yaghmaie, Aboutorab. (2012) classification, there are two schools of thought when it comes to theories of scientific representation. Informational theories focus on the objective link between the represented and the representing, whereas cognitive-functionalist perspectives center on the intents of actors. The first section presents a revised structuralist theory that incorporates the goals of the actors. In the second section, we refute an objection to the structural account of representation, which holds that the use of similarity as representation's foundation creates problems because similarity possesses certain logical features—reflexivity, symmetry, and transitivity—that representation does not. In this paper, I show that scientific representation exhibits these logical properties, using the representational link between quantum and statistical field theories.

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Bishop, Marcus & Pfeiffer, Götz. (2012) We explain a presentation for the descent algebra of the symmetric group $symn$ in 2012, where it is shown as a quiver including relations. This display is the result of a recently developed method for constructing the descent algebra, which treats it as a homomorphic image of an algebra of forests of binary trees that maps onto a region of the free Lie algebra. In this context, we provide a novel concise demonstration of the well-established fact that limited partition refinement provides the quiver of the descent algebra of $symn$. More specifically, we characterize some families of relations and conjecture that, for fixed $ninmathbbN$, the finite set of relations from these families that are relevant for the descent algebra of $symn$ generates the ideal of relations, which in turn yields an explicit presentation by generators and relations of the algebra.

Campbell, Colin & Havas, George & Ramsay, Colin & Robertson, E. (2014) Finding concise descriptions of the finite simple groups is an area of active research as of 2014. It has been argued that technically speaking, all of these collectives are efficient. For all but one of the simple groups of order less than a million, we have constructed attractive efficient presentations in prior articles. Here, we provide efficient presentations for all simple groups with order between 1 million and 5 million, proving that they are all efficient. Apart from a few linear groupings, all of these findings are all new. To locate and validate our presentations, we heavily rely on systems for computational group theory and, in particular, computer implementations of coset enumeration. We also demonstrate that certain covering groups and some bigger simple groups are efficient.

Constructions of Finite Groups

Relevant Definitions and Theorems

By our method of double coset enumeration, we build a number of different groups. Many reliable copies of the original 2^{*24}: ($2{\bullet}S_4$) are analyzed here. As their index is too high, we build our groups over maximum subgroups. Here, we detail the process for DCE, or double coset enumeration. Consider the group G and the subgroup N of G with the equations N =< x, y >. For a word w in the t_i's, the Double Coset, NwN, is provided by {Nwn|n ∈ N} = {Nnn^{-1}wn|n ∈ N} = {Nw^n |n ∈ N}.

Double Coset Enumeration Algorithm

a. If w is a word in the symmetric generators, then the number of right cosets $\frac{|N|}{|N^{(w)}|}$is equal to $N^{(w)}$ = {n ∈ N|Nw^n = Nw} is the coset stabilizer of the right coset and n is the number of elements in N.

b. All double cosets and the single cosets included inside them must be computed since G = Nw$_1$N ∪ Nw$_2$N ∪ … ∪ NwnN.

c. The $N^{(w)}$ orbits on the symmetric generators must be computed for every NwN. For each orbit, one needs just locate the double cosets of the appropriate coset Nwt_i for a single representative t_i.

Right multiplication by t_i will conclude the set of right cosets, indicating that the DCE is complete. After finishing the DCE for each set, the number of correct cosets may be shown using a Cayley diagram. To begin, we will review what a double coset is and how it is defined.

Definition 1. (Double Coset) A subset of G of the form SgT, where S and T are subgroups of G; g ∈ G.

Construction of S4 over A3

i. Elements of Control Group, N

Here is a picture of a 2^{*3}: A3 = t1t2t3t1 (1, 2, 3). With N =< xx > and xx = (1, 2, 3), we get a control group of N ≅ A3 =< (1, 2, 3) >. In this case, xx ∼ x and t ∼ t1. We may deduce that t1t2 = (1, 2, 3)t1t3 → N t1t2 = N(1, 2, 3)t1t3 from the provided relation. Hence, we have the relationship 1, 2 ∼1, 3. These are some Magma codes that may be used to locate the elements of N inside Magma.

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$> S := Alt(3);$
$> xx := S!(1,2,3);$
$> N := sub < S|xx >;$
$> Set(N);$
$> \#N;$
3
$> G < x,t > := Group < x,t|x^3, t^2, x = t * t^x * t^{x^2} * t >;$
$> \#G;$
24

So |N| = 3, where N = e,(1, 2, 3),(3, 2, 1) and |G| = 24.

ii. Isomorphism Type of Permutation Representation of G (G1)

To find out what kind of isomorphism G1 exhibits, we look at its composition components and normal lattice. Our Magma output is shown below.

$> f, G1, k := CosetAction(G, sub < G|Id(G) >);$
$> CompositionFactors(G1);$
$> NL := NormalLattice(G1);$
$> IsIsomorphic(G1, Sym(4));$
$true$

This means that G1 ≅ S4. We then put our DCE algorithm into action.

iii. First Double Coset, [∗]

Cosplay's Ne = N Stabilizer

To begin, let's have a look at the equation $N^{(e)} = \{n \in N|Ne^n = Ne\}$. If so, then N(e) contains all three of N. Hence, the Ne coset stabilizer is $N^{(e)} = \{e,(1, 2, 3),(3, 2, 1)\}$. Thus $|N^{(e)}| = 3$.

Right Cosets of [∗]

Then, we count the number of correct cosets in [*] by $\frac{|N|}{|N^{(w)}|}$. So $\frac{|N|}{|N^{(w)}|} = \frac{3}{3} = 1$.

Orbits of Ne = N

We can now determine the $N^{(e)}$ orbits on X by solving $O(x) = \{x^n | n \in N^{(e)}\}$. Each member in the coset stabilizer $N^{(e)}$ corresponds to a letter of N, thus we must raise each letter of N to its power.

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$$1^e = 1$$
$$1^{(1,2,3)} = 2$$
$$1^{(3,2,1)} = 3$$
$$\text{Then } \mathcal{O}(1) = \{1, 2, 3\}.$$
$$2^e = 2$$
$$2^{(1,2,3)} = 3$$
$$2^{(3,2,1)} = 1$$
$$\text{Then } \mathcal{O}(2) = \{1, 2, 3\}.$$
$$3^e = 3$$
$$3^{(1,2,3)} = 1$$
$$3^{(3,2,1)} = 2$$
$$\text{Then } \mathcal{O}(3) = \{1, 2, 3\}.$$

Hence, the orbit of $N^{(e)}$ is $O(N^{(e)}) = \{1, 2, 3\}$.

DC [∗]

The last factor for [*] is the destination of each orbit, which can be determined by looking at the Double Cosets of N t_i. N $t_1 \in$ N t_1N = [1].

As a result, [*]'s three orbits are all directed toward [1]. The resulting Cayley Graph (shown in Figure 5.1) is a visual representation of the data.

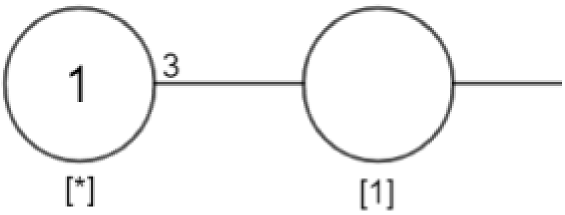

Figure 1: Cayley Graph of [∗] for S4 over A3

Symmetric Presentations

The transitivity of N is required for progenitors of the form m^{*n}: N. Then, we count how many groups there are that contain exactly n letters. Second, we choose a segment i of the sequence at random for further study. To get this group, go to N:= Transitive Group (n, i). Then we may start drafting the progenitor by identifying the N-generating functions.

Symmetric Presentation of 2^{10}: $(5^2: 2^2)$

We use GrindStaff's lemma to build a symmetric presentation for the progenitor located at T(10, 9). We then extend this progenitor with more relations and run it interactively to find finite pictures. Let N = $(5^2: 2^2)$, where N is of order 100 and produced by x ~ (1, 5)(4, 10)(6, 8)(7, 9) and y ~ (1, 5)(4, 10)(6, 8)(7, 9) (1, 2, 7, 6, 3, 10, 9, 4, 5, 8). A precursor to our N may be constructed. For the progenitor, we use symmetric presentation 2^{10}: $(5^2: 2^2)$. To give N a presentation,

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```
G<x,y,t>:=Group<x,y,t|x^2,
(y * x * y)^2, y^10,
t^2 >;
```

The permutations inside N that stabilize 1 are called "stabilizers" of "(N, 1). The following combinations are shown to be stable for 1:

$$(2,8,4,10,6) = y * x * y^{-1} * x * y^2, \text{ and}$$
$$(3,9)(4,10)(5,7)(6,8) = x * y^{-1} * x * y * x * y^{-2}$$

These new terms will be included into our progenitor to make it fully functional.

```
G<x,y,t>:=Group<x,y,t|x^2, (y * x * y)^2, y^10,
t^2,
(t, (y * x * y^-1 * x * y^2)),
(t, (x * y^-1 * x * y * x * y^-2))>;
```

Using Grindstaff's Lemma, in which we examine the orbits of N1 and locate the word (permutation) that takes 1 to a representation of an orbit, we can verify whether or not the progenitor we constructed is accurate.

```
Orbits(N1);
GSet{@ 1 @},
GSet{@ 3, 9 @},
GSet{@ 5, 7 @},
GSet{@ 2, 8, 4, 6, 10 @}
```

The orbits 2, 3, and 5 will be our representatives.

$$1^{(y)};$$
$$2$$
$$1^{(y^4)};$$
$$3$$
$$1^{(x)};$$
$$5$$

The following combinations will be added to our progenitor after being placed in a t-cycle. (t, t(x)), (t, t(y)), (t, t(y 4))

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```
G<x,y,t>:=Group<x,y,t|x^2, (y * x * y)^2, y^10,
t^2,
(t, (y * x * y^-1 * x * y^2)),
(t, (x * y^-1 * x * y * x * y^-2)),
y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1
x * y * x * y * x * y * x * y * x * y^-1 * x,
(t,t^(x)),
(t,t^(y)),
(t,t^(y^4))>;

Index(G,sub<G|x,y>);
1024

#G;
102400

2^10*100;
102400
```

This substantiates the wisdom of our progenitor.

Symmetric Presentation of 2^{10}: Sym(6)

Let N = Sym(6), where N is of order 720 and created by the digits x ~ (1, 2, 10)(3, 4, 5)(6, 7, 8), y ~ (1, 3, 2, 6)(4, 5, 8, 7), z ~ (1, 2)(4, 7)(5, 8)(9, 10) and w ~ (3, 6)(4, 7)(5, 8). A precursor to our N may be constructed. For the ancestor 2^{10}:Sym (6), we construct a symmetric presentation. N's presentation is as follows:

```
G<x,y,z,w,t>:=Group<x,y,z,w,t|x^3, y^4,
z^2,w^2,(y^-1 * z)^2,
x^-1 * w * x * w,
(y^-1 * w)^2,(z * w)^2,
y^-2 * x^-1 * y^2 * x^-1,
(z * x^-1)^3,
x^-1 * y^-1 * x^-1 * y^-1 * x^-1 * y * x * y,
(x^-1 * y^-1 * x * z)^3,
t^2>;
```

The permutations inside N that stabilize 1 are called "stabilizers" of "(N, 1). The following combinations are shown to be stable for 1:

(3, 6)(4, 7)(5, 8) = w

(2, 7, 8)(3, 5, 10, 9, 4, 6) = z ∗ x ∗ w ∗ y ∗ x

(2, 7)(4, 10)(5, 6) = w ∗ x^{-1} ∗ y^{-1} ∗ x

To finalize our progenitor, we will include the following extra words:

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```
G<x,y,z,w,t>:=Group<x,y,z,w,t|x^3, y^4,
z^2,w^2,(y^-1 * z)^2,
x^-1 * w * x * w,
(y^-1 * w)^2,(z * w)^2,
y^-2 * x^-1 * y^2 * x^-1,
(z * x^-1)^3,
x^-1 * y^-1 * x^-1 * y^-1 * x^-1 * y * x * y,
(x^-1 * y^-1 * x * z)^3,
t^2,
(t,w),(t,(z * x * w * y * x)),
(t,(w * x^-1 * y^-1 * x))>;
```

Grindstaff's Lemma allows us to determine whether our progenitor is true by traversing all possible orbits of N1 and discovering the word (permutation) that leads to a representation of that orbit.

```
Orbits(N1);
GSet{@ 1 @},
GSet{@ 2, 4, 6, 7, 10, 3, 8, 9, 5 @}
```

The following orbital envoy has been selected: 2

$1^{(x)}$;

2

The following combinations will be added to our progenitor after being placed in a t-cycle.

$(t, t^{(x)})$

```
G<x,y,z,w,t>:=Group<x,y,z,w,t|x^3, y^4,
z^2,w^2,
(y^-1 * z)^2,
x^-1 * w * x * w,
(y^-1 * w)^2,
(z * w)^2,
y^-2 * x^-1 * y^2 * x^-1,
(z * x^-1)^3,
x^-1 * y^-1 * x^-1 * y^-1 * x^-1 * y * x * y,
(x^-1 * y^-1 * x * z)^3,
t^2,
(t,w),(t,(z * x * w * y * x)),
(t,(w * x^-1 * y^-1 * x)),
(t,t^x)>;
```

```
Index(G,sub<G|x,y,z,w>);
1024

#G;
737280

2^10*720;
737280
```

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This substantiates the wisdom of our progenitor.

Conclusion

In this case, the finite symmetric group Sn defined over a finite set of n symbols is the set of all possible permutations of those symbols. The order (number of members) of the symmetric group Sn is n! because there is n! (n factorial) such permutation operations. Although symmetric groups may be formed on infinite sets, the examples, elements, conjugacy classes, finite presentation, subgroups, automorphism groups, and representation theory presented here all pertain to finite symmetric groups. The term "symmetric group" shall hereafter refer to a symmetric group on a finite set. To find the progenitors of some of these groups' symmetric presentations, we shall seek the permutation, wreath product, and monomial progenitors, but first we must define certain key terms. Adding more relations to the semi-direct product allows us to obtain finite pictures of any group.

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