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Construct an Empirical Study on the Concept, Modern Portfolio Theory Using Markowitz Model

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Abstract

In 1952, Harry Markowitz outlined a concept to aid financiers. By examining different combinations of the provided securities, the model aids in the selection of the optimal portfolio. He proposed an assumption of Modern Portfolio Theory, which aids in the creation of an optimal portfolio via the analysis of several portfolios. The Markowitz model is a theoretical framework for investigating the connections between different types of investment risk and return. The Modern Portfolio Theory (MPT) may help risk-averse investors create diversified portfolios that maximize returns without taking on undue risk. A forecasted return on each asset is required for current portfolio theory to be applied. There is no assurance that future results can be predicted based on past performance. Several market and investor presuppositions form the basis of MPT. We will first describe our finding on the "Markowitz optimization riddle" and the "Markowitz mean-variance portfolio theory," and then provide the theoretical proofs of our findings. Returns on investments are treated as random variables in the Markowitz mean-variance portfolio theory. So, picking the right portfolio weighting components is essential.

Keywords: Modern, Portfolio Theory, Markowitz Model, Stock, Investment

Introduction

Investments may be chosen using the Modern Portfolio Theory (MPT) to optimize total returns within a predetermined risk budget. To optimize the total anticipated return for a certain degree of risk, this mathematical framework is used to construct a portfolio of assets. According to the notion, diversity is the best way for an investor to maximize anticipated returns while maintaining the appropriate degree of risk. To do this, investors choose assets with a lower correlation to one another and pair positively and negatively moving assets together. In a graph, the 'efficient frontier' refers to the set of portfolios that maximizes expected returns while maintaining a fixed standard deviation. A forecasted return on each asset is required for current portfolio theory to be applied. There is no assurance that future results can be predicted based on past performance.

Markowitz is generally regarded as an early pioneer in the fields of financial economics and corporate finance due to the theoretical implications and practical applications of his work. Markowitz's contributions to these areas, first stated in his 1952 essay "Portfolio Selection" in The Journal of Finance, earned him a share of the Nobel Prize in Economics in 1990. His groundbreaking research paved the way for what is now often called "Modern Portfolio Theory" (MPT). Investors should weigh the potential for loss against the potential for gain, as proposed by Harry Markowitz. The projected rate of return is sensitive to the underlying assumptions. A larger return requires more risk on the part of the investor. Harry Markowitz made a significant contribution to the investment world when he developed modern portfolio theory. Markowitz developed a method for finding the optimal portfolio by balancing an investor's risk aversion and expected return. For a given expected return, MPT predicts that investors would choose a portfolio with lower risk. In accordance with this hypothesis, capital will only be put into risky endeavors if the expected return is high enough.

Literature Review

Sirucek, Martin & Křen, Lukáš (2017) The Markowitz Portfolio Theory is the primary subject of this chapter (MPT). Portfolio security allocations are derived using inputs from the Capital Asset Pricing Model (CAPM). Some of the portfolios that may be calculated are benchmark replication, low and high beta portfolios, and a random portfolio. Of of all the possible asset types, only stocks were included in the analysis. Each portfolio's stocks are selected in accordance with established parameters. The Dow Jones Industrial Average (DJIA) was used to identify all of the stocks. The

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portfolios were evaluated in terms of their potential risks and returns. The research conducted here will provide broad guidelines on how best to choose equities for a portfolio.

Sirucek, Martin & Křen, Lukáš (2015) The Markowitz Portfolio Theory is the theoretical framework for the investing strategies discussed in this study (MPT). Of all the possible asset types, only stocks were included in the analysis. Each portfolio's stocks are selected in accordance with established parameters. The stocks were chosen from the benchmark Dow Jones Industrial Average (DJIA) index. The portfolios were evaluated in terms of their potential risks and returns. The research conducted here will provide broad guidelines on how best to choose equities for a portfolio.

Petters, Arlie & Dong, Xiaoying (2016) Here we provide Harry Markowitz's mathematical model for allocating a fixed amount of money over a set of potentially volatile assets in order to maximize anticipated return while minimizing risk. In this chapter, we will discuss the following topics: modeling security returns, dealing with multivariate normality, determining weights, short selling, portfolio return, portfolio risk, and portfolio log returns; two-security portfolio theory; the efficient frontier for N securities with and without short selling; the global minimum-variance portfolio; the diversified portfolio; the Mutual Fund Theorem; utility functions; and the maximization of utility through diversification; and the Mutual Fund Theorem.

Jones, C Kenneth (2017) This article evaluates three different portfolio optimization strategies. Short-term volatility is the focus of modern portfolio theory (MPT). The interval between samples is the temporal frame of interest. MPT is shortsighted since it suggests that investors are not thinking about variation or mean-reversion over the long run. The goal of intertemporal portfolio choice is to minimize the variation of terminal wealth across the holding period by continually revising portfolios in response to relevant signals. Non-myopic, discrete-time, long-horizon, volatility-free: these are the distinguishing features of digital portfolio theory (DPT).

Fadadu, Purvisha & Mathukiya, Hiral & Parmar, Chetna (2016) To reduce portfolio risk, diversification of scrip is a standard practice in portfolio management. The first section of the article discussed the study's methodology, while the second section explained how investors might benefit from portfolio diversification by making proportional changes to their holdings in response to shifting risk and return trade-offs. Portfolio diversification according to the Markowitz model included the selection of three large industries such fast-moving consumer goods (FMCG), banking, information technology (IT), infrastructure (Infrastructure), and automobiles (Automobiles). Coefficient of association between industrial success and market volatility is very variable, according to the study. It has to generate a profit while reducing the portfolio's overall exposure to risk. Portfolio diversification is a key component of the Markowitz model, and the efficient frontier that is achieved has a positive connection across scrip's, or security types that operate in different industries.

Markowitz Mean-Variance Portfolio Theory

Portfolio Return Rates

The term "asset" is often used to refer to any tradable investment vehicle. Let's pretend we make a purchase of some asset at day x for x0 dollars and subsequently sell it at date x1 for x1 dollars. The ratio is what we refer to as the

$$R = \frac{x_1}{x_0}$$

This is the asset's return. The asset's rate of return is calculated as

$$r = \frac{x_1 - x_0}{x_0} = R - 1.$$

Consequently,

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$x_1 = Rx_0$ and $x_1 = (1 + r)\, x_0$

Assets we do not personally own may sometimes be up for sale. Short selling describes this kind of transaction. The procedure is roughly as follows. Let's say you've decided to short (or sell short) XXX stock. To get started, you might ask your broker whether XXX is already included in your stock portfolio. You may ask to sell as many shares of stock XXX as the brokerage business really owns, if any. Your available balance is reduced by an amount equal to the debt for the number of stocks XXX they sell on your behalf. That is to say, rather than a dollar amount, the value of your loan is denoted by the number of shares of XXX stock that you have borrowed against. On the asset sheet for your account, the short sale will appear as a deducted amount next to the shorted asset. Do not confuse this negative number with actual currency; it represents shorted stocks or assets. You are x0 dollars richer thanks to the sale of stock XXX. You'll need to ask your broker to purchase back shares of stock XXX in an amount equal to what you sold, so that the shares may be added back to the brokerage's customer-owned stock. You inform your broker that on the day you plan to return stock XXX, you want to sell it back to the brokerage at the market price of x1 dollars. In this case, you gained a profit if x1 < x0; otherwise, you lost money. This investment's yield and rate of return are calculated as

$$R = \frac{-x_1}{-x_0} = \frac{x_1}{x_0} \quad \text{and} \quad r = \frac{(-x_1) - (-x_0)}{-x_0} = \frac{x_1 - x_0}{x_0}$$

respectively. Due to the high degree of risk involved, several brokerages prohibit short selling. Nonetheless, there is potential for gain.

So, let's imagine we're putting up a portfolio of n assets. These assets will initially be funded at a level of x_{0i} dollars. For each asset i= 1, 2,..., n), If asset i has a weight of wi, then its value is x0i = wix0. A short position in the related asset is represented by negative weights. In order to stay within our financial means, the weights must add up to 1.

$$\sum_{i=1}^{n} w_i = 1$$

That is,

$$\text{the sum of the investments} = \sum_{i=1}^{n} w_i x_0 = x_0 \sum_{i=1}^{n} w_i = x_0$$

When we short a stock, we get the dollar value of that stock right now, which we may then use to buy other assets or put toward other investments. Our portfolio's total return, symbolized by the symbol Ri, where i is the number of assets in the portfolio, is

$$x_1 = \sum_{i=1}^{n} R_i w_i x_0 = x_0 \sum_{i=1}^{n} R_i w_i$$

Hence, the portfolio's total return is

$$R = \sum_{i=1}^{n} R_i w_i$$

For each asset i, the rate of return is written as ri = Ri-1, where i may be any positive integer from 1 to n. The portfolio's return rate is thus

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$$r = R - 1 = (\sum_{i=1}^{n} R_i w_i) - (\sum_{i=1}^{n} w_i) = \sum_{i=1}^{n} (R_i - 1) w_i = \sum_{i=1}^{n} r_i w_i$$

Making Markowitz's Portfolio Optimization Theory Practically Useful

For those who have puzzled about why the Markowitz MV optimization process doesn't work, the concept of the high-dimensional random matrix provides an answer. To make the optimization process more applicable in real life, we will be using a novel strategy in the next part.

Optimal Solution

The returns on a portfolio of p-branch assets are assumed to have a mean of = (1,..., p)T and a covariance matrix of = (ij)T, where r = (r1,..., rp)T is the return vector. In addition, we assume the investor will allot her investable money on the assets, but realize any of the following benefits by allocating a portion of it to p-branch assets S.

- to maximize return while taking as little risk as possible, and
- to limit risk while maintaining a target rate of return.

Both of the aforementioned issues are comparable; therefore, we will just be concerned with the first one here. We suppose C ≤ 1 has an investment strategy of c = (c1,..., cp)^T without limiting ourselves to this specific case. This means that

$$\sum_{j=1}^{p} c_j = C \leq 1$$

Moreover, the expected return's mean and risk will be c ^{T}μ and c ^{T}Σc, respectively. We also assume in this work that short selling is permitted, which means that c may have negative values for any of its components. As a result, we may restate the aforementioned maximizing issue as the following optimization problem:

$$R = \max\ \mathbf{c}^T \boldsymbol{\mu}, \text{ subject to } \mathbf{c}^T \mathbf{1} \leq 1 \text{ and } \mathbf{c}^T \Sigma \mathbf{c} \leq \sigma_0^2 \quad (1)$$

Given a fixed risk threshold of 20, where l is the p-dimensional vector of zeros. The best allocation strategy, denoted by c, is reached by solving the maximizing problem, and the return R fulfilling (1) is the optimal return.

PROPOSITION 1: For the optimization issue in (1), we derive the optimum return, R, and the accompanying investment strategy, c, in the following way:

1. If

$$\frac{\mathbf{1}^T \Sigma^{-1} \boldsymbol{\mu} \sigma_0}{\sqrt{\boldsymbol{\mu}^{\mathbf{T}} \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}} < 1$$

Then the best possible rate of return, R, and the best possible strategy for investing, c, are

$$R = \sigma_0 \sqrt{\boldsymbol{\mu}^T \Sigma^{-1} \boldsymbol{\mu}}$$

And

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$$\boldsymbol{c} = \frac{\sigma_0}{\sqrt{\boldsymbol{\mu}^T\Sigma^{-1}\boldsymbol{\mu}}}\Sigma^{-1}\boldsymbol{\mu}.$$

2. If

$$\frac{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{\mu}\sigma_0}{\sqrt{\boldsymbol{\mu}^{\mathbf{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}} > 1,$$

Then the best possible rate of return, R, and the best possible strategy for investing, c, are

$$R = \frac{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{\mu}}{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{1}} + b\left(\boldsymbol{\mu}^T\Sigma^{-1}\boldsymbol{\mu} - \frac{(\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{\mu})^2}{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{1}}\right)$$

And

$$\boldsymbol{c} = \frac{\Sigma^{-1}\boldsymbol{1}}{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{1}} + b\left(\Sigma^{-1}\boldsymbol{\mu} - \frac{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{\mu}}{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{1}}\Sigma^{-1}\boldsymbol{1}\right)$$

Where

$$b = \sqrt{\frac{\boldsymbol{1}^T\sum^{-1}\boldsymbol{1}\sigma_0^2 - 1}{\boldsymbol{\mu}^T\Sigma^{-1}\boldsymbol{\mu}\,\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{1} - (\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{\mu})^2}}.$$

REMARK 1: The following exemplifies the reasoning behind the inequalities used to categorize Proposition 1's two sorts of solutions: The intersection of the ellipsoid cTc 0 and the half space cT l 1 is where the greatest value is found. If the ellipsoid fits fully within the half space, then the solution is the same as the maximizing problem without the half space restriction, in which case it does not cross with the hyperplane $c^T l = 1$. Hence, the answer is provided by the first scenario. Unless the goal function $c^T\mu$ is a linear function in c, the maximizer should be located on the hyperplane $c^T l = 1$, which passes through the origin and the ellipse $c^T\Sigma c = \sigma_0$. This disparity

$$\frac{\boldsymbol{1}^T\Sigma^{-1}\boldsymbol{\mu}\sigma_0}{\sqrt{\boldsymbol{\mu}^{\mathbf{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}} < 1$$

might then be used to determine whether the ellipsoid contains the maximizer of $c^T\mu$.

$$\mathbf{c} = \frac{\Sigma^{-1}\boldsymbol{\mu}\sigma_0}{\sqrt{\boldsymbol{\mu}^{\mathbf{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}}$$

is situated inside the center of the halo.

The MV efficient frontier is the set of optimally balanced portfolios that may be constructed for every degree of portfolio risk.

Conclusion

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Markowitz's approach was a groundbreaking development in the field of portfolio theory. Portfolios provide a helpful framework for mitigating risk by suggesting that investors spread their money out over a variety of unrelated investments. Based on his theory of portfolio optimization, Harry Markowitz argues that investors care about both return and risk and that both should be quantified together. Markowitz means-variance optimization has been shown produce provide ideal returns, but conventional estimated returns have been shown to significantly deviate from these returns. According to current portfolio theory, the risk and return of individual investments should be assessed in light of their impact on the portfolio as a whole. When it comes to optimum portfolio creation, asset allocation, and investment diversification, the mean-variance (MV) portfolio optimization process stands as the benchmark of contemporary financial theory.

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