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A Study of Constructing of Column-Orthogonal Experimental Designs

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ABSTRACT

Latin hypercube design and uniform design are two kinds of most popular space-filling designs for computer experiments. The fact that the run size equals the number of factor levels in a Latin hypercube design makes it difficult to be orthogonal. While for a uniform design, it usually has good space-filling properties, but does not necessarily have small or zero correlations between factors. In this paper, we construct a class of column-orthogonal and nearly column-orthogonal designs for computer experiments by rotating groups of factors of orthogonal arrays, which supplement the designs for computer experiments in terms of various run sizes and numbers of factor levels and are flexible in accommodating various combinations of factors with different numbers of levels. The resulting column-orthogonal designs not only have uniformly spaced levels for each factor but also have uncorrelated estimates of the linear effects in first order models. Further, they are 3-orthogonal if the corresponding orthogonal arrays have strength equal to or greater than three. Along with a large factor-to-run ratio, these newly constructed designs are economical and suitable for screening factors for physical experiments.

KEYWORDS: computer experiment, Latin hypercube design, orthogonal array, rotation, uniform design

INTRODUCTION

Many physical phenomena encountered in science and engineering are governed by a set of complicated equations. These equations often have only numerical solutions that are carried out by computer programs. Latin hypercube design (LHD) and uniform design are two kinds of most popular space-filling designs for computer experiments. The fact that each factor in an LHD has as many uniformly spaced levels as its run size makes it attractive in that the design achieves the maximum stratification when projected into any univariate dimension. Efforts have been made to find orthogonal or nearly orthogonal LHDs. However, the factors in an LHD have as many levels as the run size, which makes it very difficult for an LHD to be orthogonal. Uniform designs were having received great attention in recent decades and the references therein. A uniform design seeks design points that are uniformly scattered on the design domain; it is robust against the model specification and limits the effects of aliasing to yield reasonable efficiency and robustness together. However, a uniform design does not necessarily have small or zero correlations between factors. For computer experiments, practical experiments have revealed that designs with many levels are desirable, but it is not essential that the run size equals the number of levels at which each factor is observed, as in an LHD. As we know, screening important factors and then estimating the effects accurately are the main objectives of experimental designs. Therefore, lower correlations among effect estimates are preferred, which will achieve the lowest correlation when the model matrix is orthogonal. By relaxing the condition that the number of levels for each factor must be identical to the run size, we, in this paper, propose some methods to construct column-orthogonal designs and nearly column-orthogonal designs, which not only have uniformly spaced levels for each factor but also have some other attractive properties, as will be discussed later.

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SOME NOTATIONS AND RELATED WORK ON ROTATION DESIGNS

A design with n runs and m factors, each having q1,. . . ,qm levels, respectively, is denoted by D(n, q1··· qm). A D(n, q1 ··· qm) design is an n×m matrix with entries of the jth column from a set of qj symbols, which are assumed here to be {(2i−qj −1)/2, i = 1,...,qj} for odd qj and {2i−qj −1, i = 1,...,qj} for even qj . If in each column the symbols occur equally often, the design is called a U-type design. The qj 's are not necessarily distinct, for example, a D(n, qm1 1 qm2 2) is a design that has m1 factors of q1 levels and m2 factors of q2 levels. In particular, when all the qj 's are equal, the design is said to be symmetrical, otherwise, asymmetrical. A D(n, nm) is called an LHD and denoted by LHD(n, m). A U-type design D(n, q1 ··· qm) is called a column-orthogonal design, denoted by COD(n, q1 ··· qm), if the inner product of any two columns is zero; and is called an orthogonal array of strength t, denoted by OA(n, q1 ··· qm, t), if all possible level-combinations for any t columns appear equally often. We shall call the latter orthogonality combinatorial orthogonality to distinguish it from the column-orthogonality. Clearly, the combinatorial orthogonality implies the column-orthogonality, but the inverse is not necessarily true. Furthermore, a column-orthogonal design is called 3-orthogonal (see [4]) if the sum of elementwise products of any three columns (whether they are distinct or not) is zero.

Let X denote the regression matrix for the first-order model of a column-orthogonal design with m factors, including a column of ones and the m factors in the design. Let Xint denote the n × m(m − 1)/2 matrix with all the possible bilinear interactions, and let Xquad denote the n×m matrix with all the pure quadratic terms. The alias matrices for the first-order model associated with the bilinear interactions and the pure quadratic terms are then given by (X X)−1X Xint and (X X)−1X Xquad, respectively. A good design for factor screening should maintain relatively small terms in these alias matrices. It is easy to see that if a column-orthogonal design is 3-orthogonal, then these two alias matrices are both zero matrices.

LHDs can be constructed by rotating the points in d-factor, q-level standard full factorial designs, where d is a power of 2, and defined a sequence of rotation matrices by a recursive scheme. [5] proposed the idea of independently rotating groups of factors in two-level designs. Recently, [26, 28] combined the above two ideas with the knowledge of Galois field to produce the orthogonal LHD matrix with n = qd runs, where q is a prime and d is a power of 2. This severe run size constraint is the primary limitation to their rotation methods.

Let $R_0^q = 1$, and

$$R_c^q = \begin{pmatrix} q^{2^{c-1}} R_{c-1}^q & -R_{c-1}^q \\ R_{c-1}^q & q^{2^{c-1}} R_{c-1}^q \end{pmatrix} \quad \text{for } c = 1, 2, \ldots, \tag{1}$$

Lemma 1. The matrix Rq c in (1) is a rotation of the d-factor (d = 2c), q-level standard full factorial design which yields unique and equally-spaced projections to each dimension.

Remark 1. Here, we relax the definition of rotation to be a matrix R satisfying R R = kI for some scalar k, instead of R R = I. It can be easily checked that the matrix Rq c in (1) consists of columns (and rows) of permutations of {1, q,..., q2c−1} up to sign changes, which guarantees that, for a 2c-factor q-level standard full factorial design A, ARq c yields unique and equally-spaced projections to each dimension. This paper will extend the rotation method to orthogonal arrays for accommodating various run sizes. Though the obtained designs are not always LHDs, they are column-orthogonal designs or nearly column orthogonal designs, and the factors have enough levels to be employed in computer experiments.

CONSTRUCTION OF COLUMN-ORTHOGONAL DESIGNS

In this section, we present the construction methods for column-orthogonal designs by rotating symmetrical as well as asymmetrical orthogonal arrays.

1 Construction from symmetrical orthogonal arrays

For convenience, we denote

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$$R^q_{(c_1,\ldots,c_v)} = \mathrm{diag}\{R^q_{c_1},\ldots,R^q_{c_v}\}, \tag{2}$$

and

$$R_{m,q} = \mathrm{diag}\{R^q_1,\ldots,R^q_1\}, \tag{3}$$

Theorem 1. *Suppose A is an $\mathrm{OA}(n, q^m, t)$ with $m = 2k$ and $t \geqslant 2$, $D = AR_{m,q}$, then*
(1) *D is a $\mathrm{COD}(n, (q^2)^m)$;*
(2) *if $t \geqslant 3$, D is a 3-orthogonal $\mathrm{COD}(n, (q^2)^m)$.*

The proof of Theorem 1 is given in the Appendix. Now, let us see some illustrative examples.

Example 1. Suppose A is an $\mathrm{OA}(12, 2^{11}, 2)$, A_1 consists of the first 10 columns of A and $D = A_1R_{10,2}$, then D is a $\mathrm{COD}(12, 4^{10})$. The $\mathrm{OA}(12, 2^{10}, 2)$ and $\mathrm{COD}(12, 4^{10})$ are listed in Table 1.

Example 2. Suppose A is an $\mathrm{OA}(18, 3^7, 2)$, A_1 consists of the first 6 columns of A and $D = A_1R_{6,3}$, then D is a $\mathrm{COD}(18, 9^6)$. A_1 and D are shown in Table 2.

Example 3. Suppose A is an $\mathrm{OA}(24, 2^{12}, 3)$, and $D = AR_{12,2}$, then D is a 3-orthogonal $\mathrm{COD}(24, 4^{12})$. A and D are shown in Table 3.

Table 1 OA(12, 2^{10}, 2) and COD(12, 4^{10})

$\mathrm{OA}(12, 2^{10}, 2)$										$\mathrm{COD}(12, 4^{10})$									
1	1	1	1	1	1	1	1	1	1	3	1	3	1	3	1	3	1	3	1
−1	1	−1	1	1	1	−1	−1	−1	1	−1	3	−1	3	3	1	−3	−1	−1	3
−1	−1	1	−1	1	1	1	−1	−1	−1	−3	−1	1	−3	3	1	1	−3	−3	−1
1	−1	−1	1	−1	1	1	1	−1	−1	1	−3	−1	3	−1	3	3	1	−3	−1
−1	1	−1	−1	1	−1	1	1	1	−1	−1	3	−3	−1	1	−3	3	1	1	−3
−1	−1	1	−1	−1	1	−1	1	1	1	−3	−1	1	−3	−1	3	−1	3	3	1
−1	−1	−1	1	−1	−1	1	−1	1	1	−3	−1	−1	3	−3	−1	1	−3	3	1
1	−1	−1	−1	1	−1	−1	1	−1	1	1	−3	−3	−1	1	−3	−1	3	−1	3
1	1	−1	−1	−1	1	−1	−1	1	−1	3	1	−3	−1	−1	3	−3	−1	1	−3
1	1	1	−1	−1	−1	1	−1	−1	1	3	1	1	−3	−3	−1	1	−3	−1	3
−1	1	1	1	−1	−1	−1	1	−1	−1	−1	3	3	1	−3	−1	−1	3	−3	−1
1	−1	1	1	1	−1	−1	−1	1	−1	1	−3	3	1	1	−3	−3	−1	1	−3

Table 2 OA(18, 3^6, 2) and COD(18, 9^6)

$\mathrm{OA}(18, 3^6, 2)$						$\mathrm{COD}(18, 9^6)$					
1	1	1	1	1	1	4	2	4	2	4	2
−1	−1	−1	−1	−1	−1	−4	−2	−4	−2	−4	−2
0	0	0	0	0	0	0	0	0	0	0	0
1	1	−1	0	−1	0	4	2	−3	1	−3	1
−1	−1	0	1	0	1	−4	−2	1	3	1	3
0	0	1	−1	1	−1	0	0	2	−4	2	−4
1	−1	1	0	0	−1	2	−4	3	−1	−1	−3
−1	0	−1	1	1	0	−3	1	−2	4	3	−1
0	1	0	−1	−1	1	1	3	−1	−3	−2	4
1	0	0	1	−1	−1	3	−1	1	3	−4	−2
−1	1	1	−1	0	0	−2	4	2	−4	0	0
0	−1	−1	0	1	1	−1	−3	−3	1	4	2
1	−1	0	−1	1	0	2	−4	−1	−3	3	−1
−1	0	1	0	−1	1	−3	1	3	−1	−2	4
0	1	−1	1	0	−1	1	3	−2	4	−1	−3
1	0	−1	−1	0	1	3	−1	−4	−2	1	3
−1	1	0	0	1	−1	−2	4	0	0	2	−4
0	−1	1	1	−1	0	−1	−3	4	2	−3	1

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Table 3 OA(24, 2^{12}, 3) and COD(24, 4^{12})

OA$(24, 2^{12}, 3)$												COD$(24, 4^{12})$											
−1	1	1	1	1	1	1	1	1	1	1	1	−1	3	3	1	3	1	3	1	3	1	3	1
−1	−1	1	−1	1	1	1	−1	−1	−1	1	−1	−3	−1	1	−3	3	1	1	−3	−3	−1	1	−3
−1	−1	−1	1	−1	1	1	1	−1	−1	−1	1	−3	−1	−1	3	−1	3	3	1	−3	−1	−1	3
−1	1	−1	−1	1	−1	1	1	1	−1	−1	−1	−1	3	−3	−1	1	−3	3	1	1	−3	−3	−1
−1	−1	1	−1	−1	1	−1	1	1	1	−1	−1	−3	−1	1	−3	−1	3	−1	3	3	1	−3	−1
−1	−1	−1	1	−1	−1	1	−1	1	1	1	−1	−3	−1	−1	3	−3	−1	1	−3	3	1	1	−3
−1	−1	−1	−1	1	−1	−1	1	−1	1	1	1	−3	−1	−3	−1	1	−3	−1	3	−1	3	3	1
−1	1	−1	−1	−1	1	−1	−1	1	−1	1	1	−1	3	−3	−1	−1	3	−3	−1	1	−3	3	1
−1	1	1	−1	−1	−1	1	−1	−1	1	−1	1	−1	3	1	−3	−3	−1	1	−3	−1	3	−1	3
−1	1	1	1	−1	−1	−1	1	−1	−1	1	−1	−1	3	3	1	−3	−1	−1	3	−3	−1	1	−3
−1	−1	1	1	1	−1	−1	−1	1	−1	−1	1	−3	−1	3	1	1	−3	−3	−1	1	−3	−1	3
−1	1	−1	1	1	1	−1	−1	−1	1	−1	−1	−1	3	−1	3	3	1	−3	−1	−1	3	−3	−1
1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	−3	−3	−1	−3	−1	−3	−1	−3	−1	−3	−1
1	1	−1	1	−1	−1	−1	1	1	1	−1	1	3	1	−1	3	−3	−1	−1	3	3	1	−1	3
1	1	1	−1	1	−1	−1	−1	1	1	1	−1	3	1	1	−3	1	−3	−3	−1	3	1	1	−3
1	−1	1	1	−1	1	−1	−1	−1	1	1	1	1	−3	3	1	−1	3	−3	−1	−1	3	3	1
1	1	−1	1	1	−1	1	−1	−1	−1	1	1	3	1	−1	3	1	−3	1	−3	−3	−1	3	1
1	1	1	−1	1	1	−1	1	−1	−1	−1	1	3	1	1	−3	3	1	−1	3	−3	−1	−1	3
1	1	1	1	−1	1	1	−1	1	−1	−1	−1	3	1	3	1	−1	3	1	−3	1	−3	−3	−1
1	−1	1	1	1	−1	1	1	−1	1	−1	−1	1	−3	3	1	1	−3	3	1	−1	3	−3	−1
1	−1	−1	1	1	1	−1	1	1	−1	1	−1	1	−3	−1	3	3	1	−1	3	1	−3	1	−3
1	−1	−1	−1	1	1	1	−1	1	1	−1	1	1	−3	−3	−1	3	1	1	−3	3	1	−1	3
1	1	−1	−1	−1	1	1	1	−1	1	1	−1	3	1	−3	−1	−1	3	3	1	−1	3	1	−3
1	−1	1	−1	−1	−1	1	1	1	−1	1	1	1	−3	1	−3	−3	−1	3	1	1	−3	3	1

Corollary 1. *Suppose $q \geqslant 2$ is a prime power, $l \geqslant 2$ and $m = (q^l - 1)/(q-1)$, then a* COD$(q^l, (q^2)^k)$ *exists, where k is the largest even integer not greater than m.*

Proof. It is seen from Theorem 3.20 of [14] that for any prime power $q \geqslant 2$, an OA$(q^l, q^m, 2)$ exists whenever $l \geqslant 2$, where $m = (q^l - 1)/(q-1)$. Thus from Theorem 1, the COD can be obtained by rotating the OA consisting of some k columns of this OA by $R_{k,q}$.

Next, we discuss the construction of asymmetrical column-orthogonal designs by rotating symmetrical orthogonal arrays.

Remark 2. In Example 1, if we take

$$R = \begin{pmatrix} R_{10,2} & 0 \\ 0 & R_0^q \end{pmatrix},$$

then AR is an asymmetrical COD(12, $4^{10}2^1$). From the OA in Example 2, a COD$(18, 9^6 3^1)$ can also be obtained similarly.

Now, we propose a general method to construct asymmetrical column-orthogonal designs. Suppose A is an OA(n, q^m, t) with $t \geqslant 2$, in which the strength of the first 2^{c_1} factors is 2^{c_1}, the strength of the next 2^{c_2} factors is $2^{c_2}, \ldots$, the strength of the last 2^{c_v} factors is 2^{c_v}, and $2^{c_1} + \cdots + 2^{c_v} = m$. Let $D = AR^q_{(c_1 \cdots c_v)}$, where $R^q_{(c_1 \cdots c_v)}$ is defined in (2). Then

Theorem 2. (1) *D is a* COD$(n, (q^{2^{c_1}})^{2^{c_1}} \cdots (q^{2^{c_v}})^{2^{c_v}})$.

(2) *If $t \geqslant 3$, then D is a 3-orthogonal* COD$(n, (q^{2^{c_1}})^{2^{c_1}} \cdots (q^{2^{c_v}})^{2^{c_v}})$.

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The proof of Theorem 2 can be easily obtained from Lemma 1 along the lines of the proof of Theorem 1. Now, let us see two examples for illustration.

Example 4. Suppose A is an $\mathrm{OA}(16, 2^{15}, 2)$, columns 1–4, 5–8 and 9–12 form three full 2^4 factorial sets, respectively, A_1 consists of the first 14 columns of A, and A_2 consists of the first 12 columns of A. Then, $D = AR^2_{(2,2,2,1,0)}$, $D_1 = A_1R^2_{(2,2,2,1)}$ and $D_2 = A_2R^2_{(2,2,2)}$ are $\mathrm{COD}(16, 16^{12}4^22^1)$, $\mathrm{COD}(16, 16^{12}4^2)$ and $\mathrm{COD}(16, 16^{12})$, respectively. Moreover, the $\mathrm{COD}(16, 16^{12})$ is in fact an orthogonal $\mathrm{LHD}(16, 12)$, and each 4-column part is a 3-orthogonal column-orthogonal design. The $\mathrm{OA}(16, 2^{15},2)$ and $\mathrm{COD}(16, 16^{12}4^22^1)$ are shown in Table 4.

Methods of partitioning the saturated factorial designs to the maximal number of full factorial sets using the Galois field.

Example 5. Suppose D is a 3^{8-4}_{IV} design with the defining relation $5 = 1^2234$, $6 = 12^234$, $7 = 123^24$, $8 = 1234^2$, then $1, 2, 3, 4$ and $5, 6, 7, 8$ constitute two full factorial sets, respectively. Let $D_1 = DR^3_{(2,2)}$, then D_1 is a 3-orthogonal $\mathrm{COD}(81, 81^8)$ which is in fact an orthogonal $\mathrm{LHD}(81, 8)$ with the property that the sum of elementwise products of any three columns is zero.

Table 4 OA(16, 2^{15}, 2) and COD(16, $16^{12}4^22^1$)

OA(16, 2^{15}, 2)															COD(16, $16^{12}4^22^1$)														
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	15	5	9	3	15	5	9	3	15	5	9	3	3	1	1
1	1	1	−1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	13	1	1	−13	13	1	1	−13	1	−13	−13	−1	−3	−1	−1
1	1	−1	1	1	−1	−1	1	−1	1	1	−1	−1	−1	−1	11	7	−7	11	3	−9	−5	15	−3	9	5	−15	−3	−1	−1
1	1	−1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	9	3	−15	−5	1	−13	−13	−1	−13	−1	−1	13	3	1	1
1	−1	1	1	−1	1	−1	1	−1	−1	−1	−1	1	1	−1	7	−11	11	7	−5	15	−3	9	−15	−5	−9	−3	3	1	−1
1	−1	1	−1	−1	1	−1	−1	−1	1	1	1	−1	−1	1	5	−15	3	−9	−7	11	−11	−7	−1	13	13	1	−3	−1	1
1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	3	−9	−5	15	−9	−3	15	5	3	−9	−5	15	−3	−1	1
1	−1	−1	−1	−1	−1	1	−1	1	1	1	−1	1	1	−1	1	−13	−13	−1	−11	−7	7	−11	13	1	1	−13	3	1	−1
−1	1	1	1	−1	−1	−1	−1	1	1	−1	1	1	−1	−1	−1	13	13	1	−15	−5	−9	−3	11	7	−7	11	1	−3	−1
−1	1	1	−1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	−3	9	5	−15	−13	−1	−1	13	5	−15	3	−9	−1	3	1
−1	1	−1	1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	−5	15	−3	9	−3	9	5	−15	−7	11	−11	−7	−1	3	1
−1	1	−1	−1	−1	1	1	1	−1	−1	1	1	1	−1	−1	−7	11	−11	−7	−1	13	13	1	−9	−3	15	5	1	−3	−1
−1	−1	1	1	1	−1	1	−1	−1	−1	1	−1	1	−1	1	−9	−3	15	5	5	−15	3	−9	−11	−7	7	−11	1	−3	1
−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1	−11	−7	7	−11	7	−11	11	7	−5	15	−3	9	−1	3	−1
−1	−1	−1	1	1	1	−1	−1	1	−1	1	1	−1	1	−1	−13	−1	−1	13	9	3	−15	−5	7	−11	11	7	−1	3	−1
−1	−1	−1	−1	1	1	−1	1	1	1	−1	−1	1	−1	1	−15	−5	−9	−3	11	7	−7	11	9	3	−15	−5	1	−3	1

Table 5 OA(36, $2^83^46^2$, 2) and COD(36, $4^89^436^2$)

OA(36, $2^83^46^2$, 2)														COD(36, $4^89^436^2$)													
−5	−5	0	0	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−35	−15	0	0	4	2	−3	−1	−3	−1	−3	−1	−3	−1
−3	−3	0	0	1	1	1	−1	−1	−1	1	−1	1	−1	−11	−15	0	0	4	2	1	−3	−3	−1	1	−3	1	−3
−5	−3	1	1	0	0	−1	1	1	1	1	−1	−1	−1	−33	−13	4	2	0	0	−1	3	3	1	1	−3	−3	−1
−3	−5	1	1	0	0	1	1	1	−1	1	−1	−1	1	−13	−17	4	2	0	0	3	1	1	−3	1	−3	−1	3
−1	3	0	1	0	1	−1	1	−1	−1	1	1	1	1	−3	19	1	3	1	3	−1	3	−3	−1	3	1	3	1
1	5	0	1	0	1	−1	−1	1	−1	−1	1	−1	1	11	19	1	3	1	3	−3	−1	1	−3	−1	3	−1	3
−1	5	1	0	1	0	1	−1	−1	1	1	1	−1	1	−1	31	3	−1	3	−1	1	−3	−1	3	3	1	−1	3
1	3	1	0	1	0	1	1	−1	1	−1	1	−1	−1	9	17	3	−1	3	−1	3	1	−1	3	−1	3	−3	−1
3	−1	0	1	1	0	1	−1	1	1	−1	−1	1	1	17	−9	1	3	3	−1	1	−3	3	1	−3	−1	3	1
5	1	0	1	1	0	−1	1	−1	1	−1	−1	1	1	31	1	1	3	3	−1	−1	3	−1	3	−3	−1	3	1
3	1	1	0	0	1	1	1	1	−1	−1	1	1	−1	19	3	3	−1	1	3	3	1	1	−3	−1	3	1	−3
5	−1	1	0	0	1	−1	−1	1	1	1	1	1	−1	19	−11	3	−1	1	3	−3	−1	3	1	3	1	1	−3
−1	−1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−7	−5	4	2	−4	−2	−3	−1	−3	−1	−3	−1	−3	−1
1	1	1	1	−1	−1	1	−1	−1	−1	1	−1	1	−1	7	5	4	2	−4	−2	1	−3	−3	−1	1	−3	1	−3
−1	1	−1	−1	1	1	−1	1	1	1	1	−1	−1	−1	−5	7	−4	−2	4	2	−1	3	3	1	1	−3	−3	−1
1	−1	−1	−1	1	1	1	1	1	−1	1	−1	−1	1	5	−7	−4	−2	4	2	3	1	1	−3	1	−3	−1	3
3	−5	1	−1	1	−1	−1	1	−1	−1	1	1	1	1	13	−33	2	−4	2	−4	−1	3	−3	−1	3	1	3	1
5	−3	1	−1	1	−1	−1	−1	1	−1	−1	1	−1	1	17	−13	2	−4	2	−4	−3	−1	1	−3	−1	3	−1	3
3	−3	−1	1	−1	1	1	−1	−1	1	1	1	−1	1	15	−11	−2	4	−2	4	1	−3	−1	3	3	1	−1	3
5	−5	−1	1	−1	1	1	1	−1	1	−1	1	−1	−1	15	−35	−2	4	−2	4	3	1	−1	3	−1	3	−3	−1
−5	3	1	−1	−1	1	1	−1	1	1	−1	−1	1	1	−17	13	2	−4	−2	4	1	−3	3	1	−3	−1	3	1
−3	5	1	−1	−1	1	−1	1	−1	1	−1	−1	1	1	−13	33	2	−4	−2	4	−1	3	−1	3	−3	−1	3	1
−5	5	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−15	35	−2	4	2	−4	3	1	1	−3	−1	3	1	−3
−3	3	−1	1	1	−1	−1	−1	1	1	1	1	1	−1	−15	11	−2	4	2	−4	−3	−1	3	1	3	1	1	−3
3	3	−1	−1	0	0	−1	−1	−1	−1	−1	−1	−1	−1	11	15	−4	−2	0	0	−3	−1	−3	−1	−3	−1	−3	−1
5	5	−1	−1	0	0	1	−1	−1	−1	1	−1	1	−1	35	15	−4	−2	0	0	1	−3	−3	−1	1	−3	1	−3
3	5	0	0	−1	−1	−1	1	1	1	1	−1	−1	−1	13	17	0	0	−4	−2	−1	3	3	1	1	−3	−3	−1
5	3	0	0	−1	−1	1	1	1	−1	1	−1	−1	1	33	13	0	0	−4	−2	3	1	1	−3	1	−3	−1	3
−5	−1	−1	0	−1	0	−1	1	−1	−1	1	1	1	1	−31	−1	−3	1	−3	1	−1	3	−3	−1	3	1	3	1
−3	1	−1	0	−1	0	−1	−1	1	−1	−1	1	−1	1	−17	9	−3	1	−3	1	−3	−1	1	−3	−1	3	−1	3
−5	1	0	−1	0	−1	1	−1	−1	1	1	1	−1	1	−19	11	−1	−3	−1	−3	1	−3	−1	3	3	1	−1	3
−3	−1	0	−1	0	−1	1	1	−1	1	−1	1	−1	−1	−19	−3	−1	−3	−1	−3	3	1	−1	3	−1	3	−3	−1
−1	−5	−1	0	0	−1	1	−1	1	1	−1	−1	1	1	−11	−19	−3	1	−1	−3	1	−3	3	1	−3	−1	3	1
1	−3	−1	0	0	−1	−1	1	−1	1	−1	−1	1	1	3	−19	−3	1	−1	−3	−1	3	−1	3	−3	−1	3	1
−1	−3	0	−1	−1	0	1	1	1	−1	−1	1	1	−1	−9	−17	−1	−3	−3	1	3	1	1	−3	−1	3	1	−3
1	−5	0	−1	−1	0	−1	−1	1	1	1	1	1	−1	1	−31	−1	−3	−3	1	−3	−1	3	1	3	1	1	−3

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2 Construction from asymmetrical orthogonal arrays

We can also obtain column-orthogonal designs by rotating asymmetrical orthogonal arrays. Suppose A is an $\mathrm{OA}(n, q_1^{m_1} \cdots q_v^{m_v}, t)$ with $t \geqslant 2$, and $R(q_1^{m_1} \cdots q_v^{m_v}) = \mathrm{diag}(R_{m_1,q_1}, \ldots, R_{m_v,q_v})$, where R_{m_i,q_i} is defined in (3) and m_i is even, $i = 1, \ldots, v$. Let $D = AR(q_1^{m_1} \cdots q_v^{m_v})$, then we have the following theorem, which can be proved easily along the lines of the proof of Theorem 1.

Theorem 3. (1) *D is a* $\mathrm{COD}(n, (q_1^2)^{m_1} \cdots (q_v^2)^{m_v})$.
(2) *If $t \geqslant 3$, D is a 3-orthogonal* $\mathrm{COD}(n, (q_1^2)^{m_1} \cdots (q_v^2)^{m_v})$.

Remark 3. Let $A = (A_1, \ldots, A_v)$, where A_i is an $\mathrm{OA}(n, q_i^{m_i}, t)$ with $t \geqslant 2$, a more flexible partition of A_i can be obtained according to the discussion that precedes Theorem 2 when we rotate A.

Example 6. Suppose A is an $\mathrm{OA}(36, 2^8 3^4 6^2, 2)$, then from Theorem 3 we can construct a COD(36, $4^8 9^4 36^2$). The two designs are shown in Table 5.

CONCLUSION

In this paper, we propose some methods to construct column-orthogonal designs and nearly column orthogonal designs by rotating orthogonal arrays. The methods are easy to implement, and the resulting column-orthogonal designs keep the estimates of the linear effects of all factors uncorrelated with each other, sometimes even uncorrelated with the estimates of all quadratic effects and bilinear interactions, along with flexible and economical run sizes. In addition, in each rotation part, the resulting designs also preserve the geometric configuration of orthogonal arrays, thus have good space-filling properties. It is seen from our methods in the previous sections that if the orthogonal arrays have no repeated runs, so do the constructed designs. Therefore, such designs can be used for computer experiments. In addition, our asymmetrical column-orthogonal designs and nearly column-orthogonal designs are useful if one feels the need of studying some factors in more detail than others. Note that the column-orthogonality and uniformity do not necessarily agree with each other, i.e., the uniformity does not guarantee that the design possesses low correlations among its effects, and vice versa. The proposed designs can guarantee the nice column-orthogonality properties, and thus are optimal in terms of the column-orthogonality criteria as we have discussed, but they may be not optimal under the uniformity criteria.

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