JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 5255-5264
https://publishoa.com
ISSN: 1309-3452

An Application of Generalized Poisson Distribution Series Involving with Certain Subclasses of Analytic Multivalent Functions

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Received: 2022 March 15; **Revised:** 2022 April 20; **Accepted:** 2022 May 10

Abstract: The aim of the present paper is to introduce some new subclasses of analytic multivalent functions $S^*(p,\mu,\delta)$ and $C^*(p,\mu,\delta)$.In the present paper, we determine the necessary and sufficient conditions for the Generalized Poisson Distribution Series to be in the subclasses $S^*(p,\mu,\delta)$ and $C^*(p,\mu,\delta)$.Some interesting results using the Generalized Poisson distribution series on these subclasses are also derived.

Keywords: Analytic Function, Multivalent Functions, Generalized Poisson Distribution series.

1.Introduction:

Let $U = \{z: z\in C \text{ and } |z| < 1\}$ denote the unit disk in the complex plane C and let A_p denote the class of the function f(z) of the form

$$f(z) = z^p + \sum_{j=p+1}^{\infty} a_j z^j (p \in N=\{1,2,3,\dots\} \tag{1.1}$$

which are analytic and normalized by the condition $f(0) = f'(0) - 1 = 0$. Also let S_p be the subclass of A_p consisting of functions of the form (1.1) which are also multivalent in U and let T_p be the subclass of S_p consisting of functions of the form

$$f(z) = z^p - \sum_{j=p+1}^{\infty} |a_j| z^j \tag{1.2}$$

Here * denotes the convolution of two analytic multivalent functions f and g of the form

$$f(z) = z^p + \sum_{j=p+1}^{\infty} |a_j| z^j \tag{1.3}$$

$$g(z) = z^p + \sum_{j=p+1}^{\infty} |b_j| z^j \tag{1.4}$$

and is defined by

$$(f * g)(z) = z^p + \sum_{j=p+1}^{\infty} |a_j||b_j| z^j \tag{1.5}$$

The class $S_p^*(\alpha)$ of starlike functions of orderα $(0 \leq \alpha < p)$ is defined as

$$S_p^*(\alpha) = \left\{ f \in A_p : \mathrm{Re}\left\{\frac{z f'(z)}{f(z)}\right\} > \alpha,\ z \in U \right\} \tag{1.6}$$

The class $C_p^*(\alpha)$ of convex functions of ord1erα$(0 \leq \alpha < p)$ is defined as

$$C_p^*(\alpha) = \left\{ f \in A_p : \mathrm{Re}\left\{1 + \frac{z f''(z)}{f'(z)}\right\} > \alpha,\ z \in U \tag{1.7}$$

$$= \{ f \in A_p : z f' \in S_p^*(\alpha)\}$$

For p=1, the classes $S^*(\alpha)$and the class $C^*(\alpha)$of convex functions of orderα$(0 \leq \alpha < 1)$ were introduced by Robertson in [27]. We also write $S^*(0) = S^*$ where S^* denotes the class of functions

$f \in A$ such that f(U) is starlike with respect to the origin. Further, C (0) = C is the well-known standard class of convex functions.

In 2000 Altintas et al.[1] unified the classes $S^*(\alpha)$and $C^*(\alpha)$

We introduce the following new subclasses of analytic multivalent functions $S(p,\mu,\delta)$ and $C(p,\mu,\delta)$.Let $S(p,\mu,\delta)$ be the subclass of $A_p$consisting of functionsif it satisfies the condition

$$\mathrm{Re}\left\{\frac{z f'(z) + \mu z^2 f''(z)}{p(1-\mu)f(z) + \mu z f'(z)}\right\} > \delta \tag{1.8}$$

for someμ $(0 \leq \mu < p)$, $\delta(0 \leq \delta < p)$ and for all $z \in U$.

Also, let $C(p,\mu,\delta)$ be the subclass of $A_p$consisting of functions if it satisfies the condition

$$\mathrm{Re}\left\{\frac{\mu z^3 f'''(z) + (1+2\mu)z^2 f''(z) + z f'(z)}{(p(1-\mu)+\mu) z f'(z) + \mu z^2 f''(z)}\right\} > \delta \tag{1.9}$$

for some $\mu(0 \leq \mu < p)$, $\delta(0 \leq \delta < p)$ and for all $z \in U$.

Also denote $S^*(p,\mu,\delta) = S(p,\mu,\delta) \cap T_p$and $C^*(p,\mu,\delta) = C(p,\mu,\delta) \cap T_p$.

Note that $f \in C^*(p,\mu,\delta)$ if and only if $z f' \in S^*(p,\mu,\delta)$.

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Volume 13, No. 3, 2022, p. 5255-5264
https://publishoa.com
ISSN: 1309-3452

Remark 1. For univalent function (p=1), the classes P_λ (α) and D_λ (α) were introduced by Porwal and Kumar [24].It is of interest to note that for p=1 andμ = 0 the above classes reduced to ordinary classes of univalent starlike and convex functions [29]. Recently the Generalized Poisson distribution series [25, 26] in terms of a power series whose coefficients are probabilities of Poisson distribution is defined as

$$K(\lambda, z, p) = z^p + \sum_{j=p+1}^{\infty} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!} z^j \qquad (1.10)$$

Here by ratio test, the radius of convergence of the above series is infinity. Now, we introduce the series

$$T(\lambda, z, p) = 2z^p - K(\lambda, z, p) = z^p - \sum_{j=p+1}^{\infty} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!} z^j \qquad (1.11)$$

Eminent authors have obtained several importantresults on connections between various subclasses ofanalytic univalent and multivalent functions by using various distribution series. They used efficiently many distribution series such as Hypergeometric distribution series [6, 7, 10, 12, 14,16, 28, 30], Generalized Bessel functions [2, 3, 4, 11, 13, 17, 22, 23, 31], Poisson distribution series [8,9,15,18,21,24, 25, 26] and Generalized distribution series [5,20], Binomial distribution [19], Beta-Binomial distribution, Zeta distribution, Geometric distribution and Bernoulli distribution,Yule-Simon distribution, Logarithmic distribution in their work. Motivated by several earlier results, by using Poisson distribution series and by recent investigations of Purohit et.al [25, 26] in the present paper we determine the necessary and sufficient condition for functions T (λ, z, p) in $S^*(p, \mu, \delta)$ and$C^*(p, \mu, \delta)$.Finally, wegive conditions for the integral operator

$G(\lambda, z, p) = p\int_0^z \frac{T(\lambda, t, p)}{t} dt$ belonging to the above classes.

To characterize our main results, we will require the following Coefficients inequalities:

2. The Necessary and Sufficient Conditions

Theorem 2.1. A function f (z) of the form (1.2) is in the class$S^*(p, \mu, \delta)$ if and only if

$$\sum_{j=p+1}^{\infty} [j\{(j-1)\mu + (1-\mu\delta)\} - p\delta(1-\mu)]|a_j| \le p[(p-1)\mu + (1-\delta)] \qquad (2.1)$$

Proof. Suppose that f(z) $\in S^*(p, \mu, \delta)$. Then we have from (1.8) that

$$\mathrm{Re}\left\{\frac{z f'(z) + \mu z^2 f''(z)}{p(1-\mu)f(z) + \mu z f'(z)}\right\} > \delta$$

$$\mathrm{Re}\left\{\frac{[p+\mu p(p-1)]-\sum_{j=p+1}^{\infty}[j+\mu j(j-1)]|a_j|z^{j-p}}{[p(1-\mu)+\mu p]-\sum_{j=p+1}^{\infty}[p(1-\mu)-\mu j]|a_j|z^{j-p}}\right\}>\delta \tag{2.2}$$

If we choose z real and let $z\to 1^-$, we get

$$\mathrm{Re}\left\{\frac{[p+\mu p(p-1)]-\sum_{j=p+1}^{\infty}[j+\mu j(j-1)]|a_j|}{[p]-\sum_{j=p+1}^{\infty}[p(1-\mu)+\mu j]|a_j|}\right\}\geq\delta \tag{2.3}$$

which is equivalent to desired result (2.1).

Conversely, suppose that (2.1) holds true. Then, adding

$$p[(p-1)\mu+(1-\delta)][(p-1)-\sum_{j=p+1}^{\infty}\{p(1-\mu)+\mu j\}]|a_j|$$

in both sides of (2.1), we obtain

$p(p-1)[(p-1)\mu+(1-\delta)]+\sum_{j=p+1}^{\infty}[j\{(j-1)\mu+(1-\mu\delta)\}-p\delta(1-\mu)-p\{(p-1)\mu+$

1−δ} {p1−μ+ μ j}aj≤ p p−1μ+(1−δ)[{□}−j=p+1∞ {p1−μ+ μ j}]aj (2.4)

On the other hand, we see that

$$\left|[\frac{z f'(z)+\mu z^2 f''(z)}{p(1-\mu)f(z)+\mu z f'(z)}]-[1+(p-1)(2\mu-p\mu-1+\delta)]\right|$$

$$=\left|[\frac{p+\mu p(p-1)-\sum_{j=p+1}^{\infty}\{j+\mu j(j-1)\}|a_j|}{[p]-\sum_{j=p+1}^{\infty}[p(1-\mu)+\mu j]|a_j|}]-[1+(p-1)(2\mu-p\mu-1+\delta)]\right|$$

$$\leq\frac{p(p-1)[(p-1)\mu+(1-\delta)]+\sum_{j=p+1}^{\infty}[j\{(j-1)\mu+(1-\mu\delta)\}-p\delta(1-\mu)-p\{(p-1)\mu+(1-\delta)\}\{p(1-\mu)+\mu j\}]|a_j|}{[p]-\sum_{j=p+1}^{\infty}[p(1-\mu)+\mu j]|a_j|} \tag{2.5}$$

it follows from (2.4) that last expression in (2,5) is bounded above by $p[(p-1)\mu+(1-\delta)]$

Theorem 2.2 A function f (z) of the form (1.2) is in the class $C^*(p,\mu,\delta)$ if and only if

$$\sum_{j=p+1}^{\infty}j[j\{(j-1)\mu+(1-\mu\delta)\}-p\delta(1-\mu)]|a_j|\leq p^2[(p-1)\mu+(1-\delta)] \tag{2.6}$$

Proof. Note that $f(z)\in C^*(p,\mu,\delta)$ if and only if $zf'(z)\in S^*(p,\mu,\delta)$

Hence replacing a_j by $\frac{j}{p}a_j$ in Theorem 2.1, we have the inequality (2.6).

Corollary 2.1 Taking p =1, the Theorem 2.1 and Theorem 2.2 get reduced to the results due to Altintas et al.[1].

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 5255-5264
https://publishoa.com
ISSN: 1309-3452

Corollary 2.2 By specializing the parameter p=1, $\mu = 0$ and $\mu = 1$ the Theorem 2.1 and Theorem 2.2 get reduced to the results due to Silverman [24].

3. Main Results

Theorem 3.1 If $\lambda > 0$, then T (λ, z, p) is in $S^*(p, \mu, \delta)$ if and only if

$$e^{\lambda}\,[\,\lambda^2\mu + (2\mu p + 1 - \mu\delta))\lambda] \le p^2\mu + p(1-\mu) - p\delta \qquad (3.1)$$

Proof. Since

$$T(\lambda, z, p) = z^p - \sum_{j=p+1}^{\infty} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!} z^j$$

According to the Theorem 2.1, it is sufficient to show that

$$\sum_{j=p+1}^{\infty} [j\{(j-1)\mu + (1-\mu\delta)\} - p\delta(1-\mu)] \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!} \le [p + \mu p(p-1) - \delta]$$

Let $M_1(\lambda, \mu, \delta) = \sum_{j=p+1}^{\infty} \left[j^2\mu + j(1-\mu-\mu\delta) - p\delta(1-\mu)\right] \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!}$

Writing $j^2 = (j-p)(j-p-1) + (2p+1)(j-p) + p^2$ and

$j = (j-p) + p$ and by simple computation we get

$M_1(\lambda, \mu, \delta) =$

$$\sum_{j=p+1}^{\infty} [\mu\{(j-p)(j-p-1) + (2p+1)(j-p) + p^2\} + \{(j-p) + p\}\{(1-\mu-\mu\delta)\} - p\delta(1 - \mu)] \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!}$$

$$= \mu \sum_{j=p+1}^{\infty} \{(j-p)(j-p-1)\} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!} + (2p\mu + 1 - \mu\delta) \sum_{j=p+1}^{\infty} \{j-p\} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!} + \{p^2\mu + p(1-\mu) -$$

pδ□ = □ +1∞λ□ − □ □ −λ j−p !

$$= \mu \sum_{j=p+2}^{\infty} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p-2)!} + (2p\mu + 1 - \mu\delta) \sum_{j=p+1}^{\infty} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p-1)!} +$$

$$\{p^2\mu + p(1-\mu) - p\,\delta\} \sum_{j=p+1}^{\infty} \frac{\lambda^{j-p} e^{-\lambda}}{(j-p)!}$$

$$= e^{-\lambda}[\,\mu\lambda^2 e^{\lambda} + (2p\mu + 1 - \mu\delta)\lambda e^{\lambda} + \{p^2\mu + p(1-\mu) - p\delta\}\,\{e^{\lambda} - 1\}]$$

$$= \mu\lambda^2 + (2p\mu + 1 - \mu\delta)\lambda + \{p^2\mu + p(1-\mu) - p\delta\}\,\{1 - e^{-\lambda}\}$$

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https://publishoa.com
ISSN: 1309-3452

But this last expression is bounded above by $p^2\mu + p(1-\mu) - p\delta$ if and only if (3.1) is satisfied.

Theorem 3.2 If $\lambda > 0$, then $T(\lambda, z, p)$ is in $C^*(p, \mu, \delta)$ if and only if

$$e^{\lambda}\left[\lambda^3\mu + (3\mu p + 2\mu + 1 - \mu\delta)\lambda^2 + \{(3p^2 + p)\mu + (2p+1)(1-\mu\delta) - p\delta\}\lambda\right]$$
$$\le p^3\mu + p^2(1-\mu-\mu\delta) - p^2\delta \qquad (3.2)$$

Since

$$T(\lambda, z, p) = z^p - \sum_{j=p+1}^{\infty}\frac{\lambda^{j-p}e^{-\lambda}}{(j-p)!}z^j$$

According to the Theorem 2.2, it is sufficient to show that

$$\sum_{j=p+1}^{\infty} j[j\{(j-1)\mu + (1-\mu\delta)\} - p\delta(1-\mu)]\frac{\lambda^{j-p}e^{-\lambda}}{(j-p)!} \le p[p + \mu p(p-1) - \delta]$$

Let $M_2(\lambda, \mu, \delta) = \sum_{j=p+1}^{\infty}\left[j^3\mu + j^2(1-\mu-\mu\delta) - jp\delta(1-\mu)\right]\frac{\lambda^{j-p}e^{-\lambda}}{(j-p)!}$

Writing

$j^3 = (j-p)(j-p-1)(j-p-2) + (3p+3)(j-p)(j-p-1) + (3p^2+3p+1)(j-p) +$
$+ p^3$ and

$j^2 = (j-p)(j-p-1) + (2p+1)(j-p) + p^2$ and

$j = (j-p) + p$ and by simplifying we get

$M_2(\lambda, \mu, \delta)) = \sum_{j=p+1}^{\infty}[\mu\{(j-p)(j-p-1)(j-p-2) + (3p+3)(j-p)(j-p-1) + (3p^2 + 3p +$

1)j−p+ p3 }+ 1−μ−μδ {j−pj−p−1+2p+1j−p+p2 }−pδ {j−p+pλj−pe− λ j−p !

$= \mu\sum_{j=p+1}^{\infty}\{(j-p)(j-p-1)(j-p-2)\}\frac{\lambda^{j-p}e^{-\lambda}}{(j-p)!} + (3p\mu + 2\mu + 1 - \mu\delta)\sum_{j=p+1}^{\infty}\{(j-p)(j-p-$

1)λj−pe− λ j−p !+{3 p2+pμ+(2p+1)(1−μδ)− pδ)}j=p+1∞j−pλj−pe− λ j−p !+p3μ+ p21−μ−μδ−p2δj=p+1∞λj−pe− λ j−p !

$= \mu\sum_{j=p+3}^{\infty}\frac{\lambda^{j-p}e^{-\lambda}}{(j-p-3)!} + (3p\mu + 2\mu + 1 - \mu\delta)\sum_{j=p+2}^{\infty}\frac{\lambda^{j-p}e^{-\lambda}}{(j-p-2)!} + \{(3p^2 + p)\mu + (2p+1)(1-$

μδ− pδ□=□+*1*∞λ□−□□− λ j−p−1!+□*3*μ+ □*2*1−μ−μδ−p2δ□=□+*1*∞λ□−□□− λ j−p !λ

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https://publishoa.com
ISSN: 1309-3452

$= e^{-\lambda}[\mu\lambda^3 e^m + (3p\mu + 2\mu + 1 - \mu\delta)\lambda^2 e^\lambda + \{(3p^2 + p)\mu + (2p+1)(1-\mu\delta) - p\delta\}\lambda e^\lambda +$
$\{p^3\mu + p^2(1-\mu-\mu\delta) - p^2\delta\}\{e^\lambda - 1\}]$

$= \quad \mu\lambda^3 + (3p\mu + 2\mu + 1 - \mu\delta)\lambda^2 + \{(3p^2 + p)\mu + (2p+1)(1-\mu\delta) - p\delta\}\lambda + \{p^3\mu +$
□21−μ−μδ−p2δ {1−□−λ)

But this last expression is bounded above by $p^3\mu + p^2(1-\mu-\mu\delta) - p^2\delta$ if and only if (3.2) is satisfied.

4. An Integral Operator

Here we introduce, an integral operator G (λ, z , p)as follows:

$$G(\lambda, z, p) = p\int_0^z \frac{T(\lambda, t, p)}{t}\,dt \qquad (4.1)$$

where T(λ, z, p) is defined by (1.11) and we get a necessary and sufficient condition for belonging to the class$C^*(p, \mu, \delta)$.

Theorem 4.1 Let λ> 0, then

$$G(\lambda, z, p) = p\int_0^z \frac{T(\lambda, t, p)}{t}\,dt$$

is in the class$C^*(p, \mu, \delta)$ if and only if

$$e^\lambda [\lambda^2\mu + (2\mu p + 1 - \mu\delta))\lambda] \le p^2\mu + p(1 - \mu) - p\delta \qquad (4.2)$$

Proof. Since

$$G(\lambda, z, p) = z^p - \sum_{j=p+1}^{\infty} \frac{p\,\lambda^{j-p} e^{-\lambda}}{j\,(j-p)!} z^j$$

By Theorem (3.1), we need only to show that

$$\sum_{j=p+1}^{\infty} j[j\{(j-1)\mu + (1-\mu\delta)\} - p\delta(1-\mu)]\frac{p\,\lambda^{j-p} e^{-\lambda}}{j\,(j-p)!} \le p^2[(p-1)\mu + (1-\delta)]$$

i.e.

$$\sum_{j=p+1}^{\infty}[j\{(j-1)\mu+(1-\mu\delta)\}-p\delta(1-\mu)]\frac{\lambda^{j-p}e^{-\lambda}}{(j-p)!}\leq p[(p-1)\mu+(1-\delta)]$$

Now let

$M_3(\lambda, \mu, \delta) = \sum_{j=p+1}^{\infty}[j\{(j-1)\mu+(1-\mu\delta)\}-p\delta(1-\mu)]\frac{\lambda^{j-p}e^{-\lambda}}{(j-p)!}$

Writing $j^2 = (j-p)(j-p-1)+(2p+1)(j-p)+p^2$

$j = (j-p) + p$ and by using the similar arguments as in the proof of Theorem 3.1, we have

$M_3(\lambda, \mu, \delta) = \mu\lambda^2 + (2p\mu + 1 - \mu\delta)\lambda + \{p^2\mu + p(1-\mu) - p\delta\}\{1-e^{-\lambda}\}$

But this last expression is bounded above by $p^2\mu + p(1-\mu) - p\delta$ if and only if (4.2) is satisfied.

Corollary 4.1 Taking p=1, Theorems 3.1, 3.2 and 4.1 get reduced to the results due to Porwal and Kumar [24].

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