

Effects of Using Non-Identical Quantum Memories on Quantum Repeater Performance

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Abstract

The aim of this research is to improve quantum repeater performance such as amount of key rate and the quantum channel length. To do this, a 2.5 node quantum repeater scheme utilizing non-identical quantum memories in nodes instead of using same quantum memories in all nodes. The results indicated that, performance of the quantum repeater utilizing non-identical quantum memories is improved as compared to that of QR utilizing same QMs. Also, the results show that a 2.5 node quantum repeater using non-identical quantum memories with cut-off concepts has better performance than that of single node quantum repeater with same characteristics.

Keywords: quantum repeater – non – identical quantum memory – cut off – quantum memory

1. Introduction

The current advances in quantum information field have delivered numerous interesting and novel innovations. Whereas most of these innovations still not executed physically, while some of these innovations, especially quantum key distribution QKD, been popularised [1]. Regardless of QKD being a more develop innovation contrasted with its quantum partners, despite everything it faces a threatening topic. Long distance communication not acknowledged, with current records of couple hundreds of kilometers [2]. That issue occurs due to loss experienced by photons when they move via media.

To avoid this problem, in classical communication system it's possible to exploits traditional repeater to amplify signals. Whereas, in quantum communication, this process is restricted according to no-cloning theorem [3]. Hence, to overcome the issue of exponential decay of transmitted signals in quantum communication an intermediate stations (relays) equipped between the two ends of quantum channel (Alice and Bob), who at that point independently utilize QKD to build up secret keys with it [4]. Relay could create private secret keys to both Alice and Bob. On the other hand, messages can be directed through relay, that encrypts and decrypts messages utilizing two secret keys as suitable. This idea can be quickly reached out of numerous intermediate stations extending over a subjective separation. The relays or intermediate stations are described as quantum repeaters QR [5], which first depicted in [6], full repeater plan might include utilization of numerous stations, containing different qbits [7-11].

These stations referred to as nodes, at the end of each node there two two quantum memories QMs. Entanglement swapping operation will be utilized on every node of QR in a nested way to produce a long-distance entanglement. QMs are essential part in most proposed schemes of QR, it could be considered as a physical structure that can create, store, and read-out qbit [12]. There are numerous assortments of QMs, each has its special characteristics, such as rare-earth-ion doped crystals [13,14], Rydberg atoms[15], quantum dots[16], NV center[17-20], and trapped ion[21-22]. The important properties of QM are preparation time T_p , which represents the time required for QM to prepare memory-photon entangled state, preparation proficiency η_p , which is efficiency required to couple photon into fiber η_c , decoherence time T_2 of stored state, and wavelength of emitted photons.

To initiate a quantum channel, first QM decoherence time must be sufficient to successfully cover all operations times of steps mentioned above. This mean that, first QM has longer storage time than the others. The decoherence time and the efficiency of coupling photon into optical fiber which are related inversely to each other and play a big role in the performance of QR. Hence, it may be useful use non-identical types of QMs inside each node to exploit trade of T_2 and efficiencies. Two research teams—one from the University of Oxford and the other from the National Institute of Standards and Technology and the University of Washington—present entanglement between non-identical atoms [23, 24]. Two separate calcium isotopes were utilized by the University of Oxford team [23], whilst the other team used beryllium and magnesium atoms, which could maintain their states for only second and half for magnesium and around a minute for beryllium[24]. Both teams claimed that the likelihood of their two atoms being entangled is extremely high, on the order of 0.998 for the first and 0.979 for the second. Many quantum repeater schemes are proposed with different ideas, some of these are implemented practically and another just theoretical schemes. Most of the proposed quantum repeater schemes based on using optical fiber as transmission channel and an identical quantum memories in the nodes of quantum repeater [25-30] while other used non identical quantum memories [31]. Also many schemes based on a free space as transmission channel [32-35]. Also, some QR schemes use the cut-off approach [36-38] which is the minimum number of trials required to maximize key rate [39] by optimizing the decoherence time of the quantum states that are stored in QMs with cut-off. Filip. et al [40] use a cut-off QR configuration with identical QMs (e.g. NV center) in each QR node to improve the performance of QR. They also make a comparison of schemes, with and without cut-off to show the effect of cut-off on the key rate. Their scheme is a combination of single photon and single sequential quantum repeater scheme. Fig.1 shows the schematic diagram and procedure of their scheme

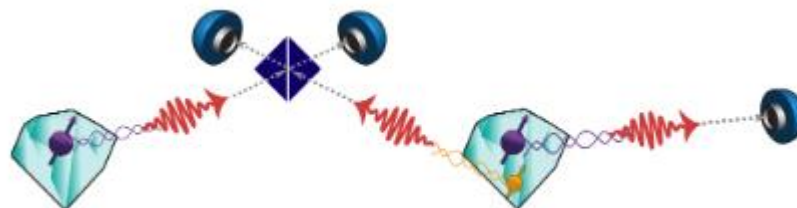


Figure 1: Schematic overview of single photon with additional detection setup scheme [40].

Initially, two NV centers run scheme of single-photon, in which, Alice measure her electron spin directly after each attempt, If success, state of middle node swap to carbon spin. Middle node attempt to send entangled pairs of electron-photon, encoded in the time-bin degree of freedom, to Bob. The attempt continue until Bob measure photon successfully or until cut-off is reached. If the cut-off is reached, then, scheme restart process again, else, middle node executes entanglement swapping on the two QMs and links classical result to both Alice and Bob, who can correct their outcomes to get bit of raw key [40].

In this work, an analyze of the effect of utilizing non-identical QMs instead of identical QMs on the performance of two-half nodes QR scheme with the concept of the cut-off is presented.

2. Methodology

The scheme of a two-half nodes QR exploiting non-identical QMs with the concept of cut-off is shown in (Fig.2). The role of decoherence time and coupling efficiency on the performance of the QM1 & QM2 will be utilized in this scheme. The decoherence of the quantum state stored in a QMs is the main difficulty in the implementation of a QR, which mean, in the case Bob needs large number of trials so as to receive photon successfully, the state in QM1 will significantly be decohered. This may reduce the generation of a secret key rate R . To avoid such an issue a cut-off concept is introduced, It reduce the number of trials Bob can make to receive photon.

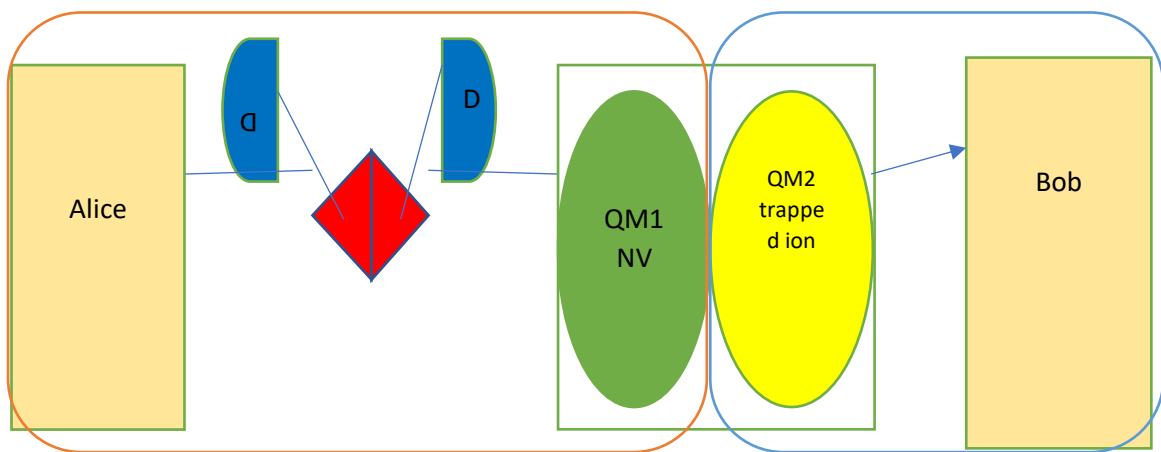


Figure 2: Schematic of non-identical QMs with cut-off QR. the orange represents the QM1 NV while yellow represents the QM2 trapped ion, the blue represents the detectors, the red represents beam splitters, the orange border represents the single-photon scheme and the blue border represents single sequential quantum repeater scheme.

In this scheme, the middle node (QM1 & QM2) is positioned at two-thirds from Alice. To satisfy the probability criterion mentioned above, QM1 is chosen as NV center and QM2 as trapped ions with their parameters given by [20, 41-42]. Several tunable parameters, such as cut-off n^* (the number of trail attempts by Bob until restart) and the time for preparing the emission of entangled photons are used, to optimize key rate.

The key rate (R), that represent proportion between quantity of bits generated and number of times that channel used, has been chosen as the figure of merit for evaluating QR.

The lower-bounded of (R) for efficient BB84 protocol [43] can be calculated using the relation[40]:

$$R = \frac{1}{2} Y r \quad 1$$

Where,

r : secret-key fraction and represents total of secret keys which ay be extracted from single raw bit

Y : Yield and represents average raw bits number transmitted per channel use, and can be calculated as follows [40]:

$$Y = \frac{1}{E[N]} \quad 2$$

Where $E[N]$ is the expectation value of $[N]$, which represents the total number of the channel used.

$$E[N] = E[N_A + N_B] \quad 3$$

Where:

N_A ; denoted number of the photons must be generated to Alice, in order her detector click once.

N_B : denoted number of the photons must be generated to Bob, in order her detector click once. There are two different probabilistic events to be taken into account for any QR, probability of success in the external connection of Alice's side, denoted by P_A , and the probability of success in the external link of Bob's side, denoted by P_B [40].

$$E[N_A + N_B] = \frac{1}{P_A} + \frac{1}{P_B} \quad 4$$

Average number of using channel increased as the cut-off (n^*) is utilized because any time Bob reaches n^* trials, then Alice and Bob abort operation then start new round.

For our 2.5 non-identical model equations (4) become:

$$E[N_A + N_B] = \frac{1}{P_{AQM1}} + \frac{1}{P_{BQM2}} \quad 5$$

The probability on Alice's side P_{AQM1} is:

$$P_{AQM1} = P_{QM1}(1 - (1 - P_{QM2})^{n^*}) \quad 6$$

The probability which single attempt of QM1 on Alice's (P_{QM1}) can be calculated from the following equation:

$$P_{QM1} = 2(1 - P_D)[\delta_a \cos^2 \theta \left(1 - \frac{\delta_a}{2} \cos^2 \theta\right) + (1 - \delta_a \cos^2 \theta)^2 P_D] \quad 7$$

While The probability (P_{BQM2}) of QM2 on Bob's side respectively succeeds is calculated by:

$$P_{B_{QM2}} = 1 - (1 - P_{eff_{QM2}} \delta_B)(1 - P_D)^2 \quad 8$$

Where, $P_{eff_{QM2}}$ is the total apparatus efficiency and can be calculated as follows:

$$P_{eff_{QM2}} = P_{C_{QM2}} P_{em_{QM2}} P_{Det} \quad 9$$

Where:

P_C : Photon collection efficiency

$P_{em_{QM2}}$: the probability of emitting into the zero- phonon line are the two crucial parameters relying on the implementation of the optical cavity.

P_{Det} : Detector efficiency.

P_D : Probability of dark count

Secret-key fraction r is [40]:

$$r = 1 - h(e_x) - h(e_z) \quad 10$$

Where; $h(e)$ represent the binary entropy function:

$$h(x) = -x \log_2(x) - (1 - x) \log_2(1 - x) \quad 11$$

where:

$1 - h[e_x] - h[e_z]$: is privacy amplification and classical error correction protocols; e_x, e_z : QBER in (X and Z) bases [44].

$$e_x = e_y = e_{xy} = 0.5 - 0.5 F_{gm} \alpha_A \alpha_B (2 F_{prep} - 1)^2 < e^{-(a+b)n^*} > \quad 12$$

$$e_z = 0.5 - 0.5 F_{gm} \alpha_A \alpha_B < e^{-bn^*} > \quad 13$$

For non-identical model equations (2.18 and 2.19) become:

$$e_x = e_y = e_{xy} = 0.5 - 0.5 F_{gm} \alpha_A \alpha_B (2 F_{prep} - 1)^2 < e^{-(a_{QM2} + b_{QM2})n^*} > \quad 14$$

$$e_z = 0.5 - 0.5 F_{gm} \alpha_A \alpha_B < e^{-b_{QM2}n^*} > \quad 15$$

Where; $\alpha_{A,B}$: are depolarizing parameters due to the noise that are achieved by dark count at (Alice's or Bob's) detectors respectively

$$\alpha_{A,B} = \frac{P_{eff_{QM2}} \delta_B (1 - P_D)}{1 - (1 - P_{eff_{QM2}} \delta_{A,B})(1 - P_D)^2} \quad 16$$

The two parameters a_{QM2} and b_{QM2} are given by:

$$a_{QM2} = a_0 + a_1 \left(\frac{2 L_b n_{ri}}{c} + T_{prep_{QM2}} \right) \quad 17$$

$$b_{QM2} = b_0 + b_1 \left(\frac{2 L_b n_{ri}}{c} + T_{prep_{QM2}} \right) \quad 18$$

and the transmissivity of the fiber is given by:

$$\delta_{A,B} = \exp \left(-\frac{L_{a,b}}{L_{att}} \right) \quad 19$$

Where,

L_{att} : attenuation length and its about 22 km at wavelength of 1550 nm [44].

L_a : distance between Alice (A) and first quantum memory.

L_b : distance between Bob (B) and last quantum memory .

c : speed of light in vacuum = 3×10^8 m/sec.

$T_{prep_{QM2}}$: time trapped ion needs to emit entangled photons = 210 μ s [26].

a_0 and b_0 : the noise that comes from single attempt to generate entangled state where a_0 (NV) = 1/2000 per attempt,

b_0 (NV) = 1/5000 per attempt [19, 45].

a_1 and b_1 : quantify the noise during storage per second where a_1 (NV) = 1/3 per second, b_1 (NV) = 1/3 per second [42, 44].

$P_{em_{QM1}}$: the probability of generating photon from NV into the optical fiber = 0.49 [26, 46]

$P_{em_{QM2}}$: the probability of generating photon from trapped ion into the optical fiber = 0.9 [47]

$P_{c_{QM1}}, P_{c_{QM2}}$: probability of coupling quantum memory with fiber for Alice and Bob side respectively = 0.46 for NV center [48], = 0.4 for trapped-ion for Bob side [49]

T: NV emission characteristic time = 6.48 ns [48, 50]

n_{ri} : Refractive index of the fiber 1.44 [51].

F_{gm} : depolarizing parameter = 0.9

F_{prep} : dephasing parameter for preparing memory-photon state = 0.99 [46].

P_{det} : (detector efficiency) = 0.9 the probability of photons producing a click in the detector [46].

The aim of this study can be illustrated through study the following:

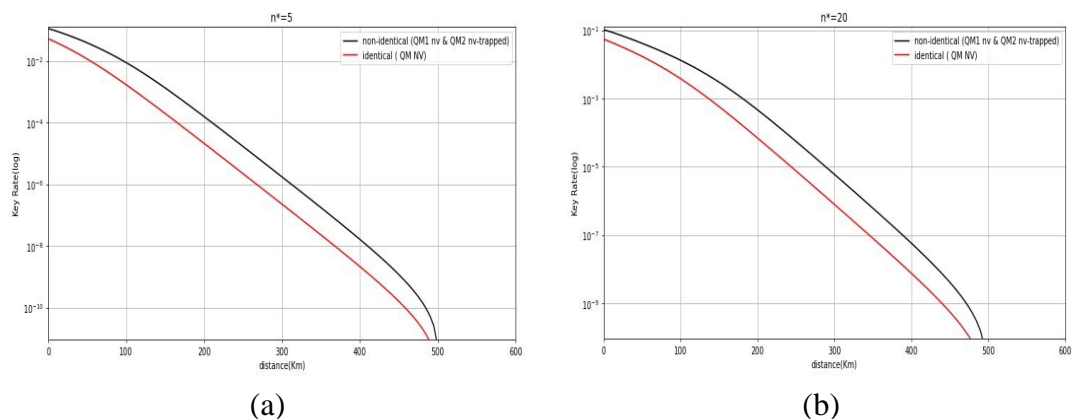
1. Effect of using identical and non-identical QMs on the performance of 2.5 node QR.
2. Comparison based on channel length between 2.5 node and single node QRs utilizing non-identical QMs.
3. Effect of detection efficiency on 2.5 node QR performance.

3. Results and Discussion

In this section of study, the results and its explanation is presented as follows:

1. Effect of using identical and non-identical QMs on the performance of 2.5 node QR.

In this step a comparison between the performance of 2.5 node QR schemes based on identical QMs and non-identical QMs with different no. of trails (n^*) (5, 20, 50, 75, and 100). The results obtained are presented in Fig. 3.



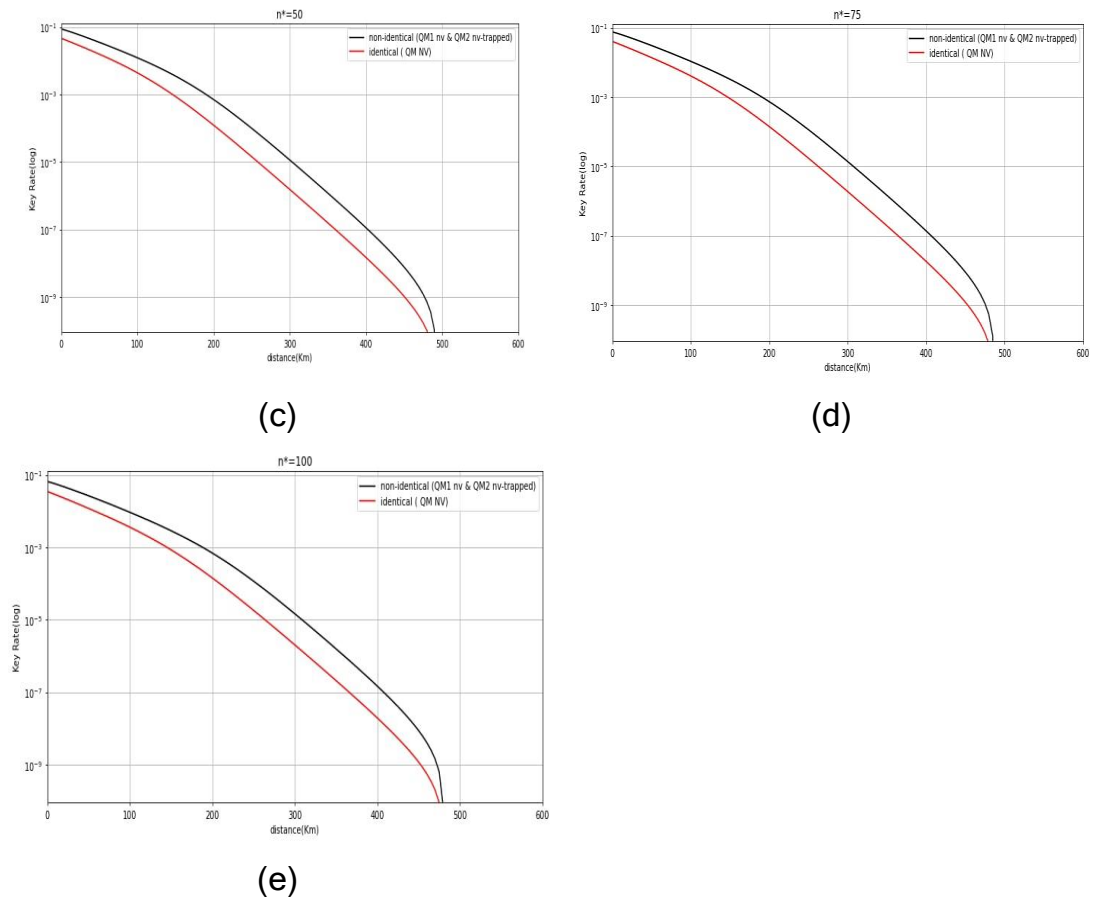


Figure 3: A comparison between the performance of 2.5 node QR scheme based on identical QM and non-identical QMs with different no. of cut-off .

From the results shown in (Fig. 3) it is evident that, for any number of trials, the length of the quantum channel and the amount of key rate per channel use based on using non-identical quantum memories are better than those based on using identical quantum memories. This result is caused by the interaction between decoherence time and quantum efficiency, as using different quantum memories in the same quantum repeater allows us to use QM with long decoherence time and low efficiency in locations where long time is required. While using QMs with short decoherence time and high efficiency in places where long time isn't required. Therefore, utilizing both a lengthy decoherence period and high efficiency at the same time has advantages over using identical QMs.

2. Comparison between 2.5 node and single node quantum repeater (both are non-identical).

Herein, a comparison between the quantum channel length of 2.5 – node and single node quantum repeater for different number of trials (5, 10, 20, 50, 75, 100) is presented, both repeaters using non identical QMs. The results are presented in (Fig. 4. a, b, c, d, e, f).

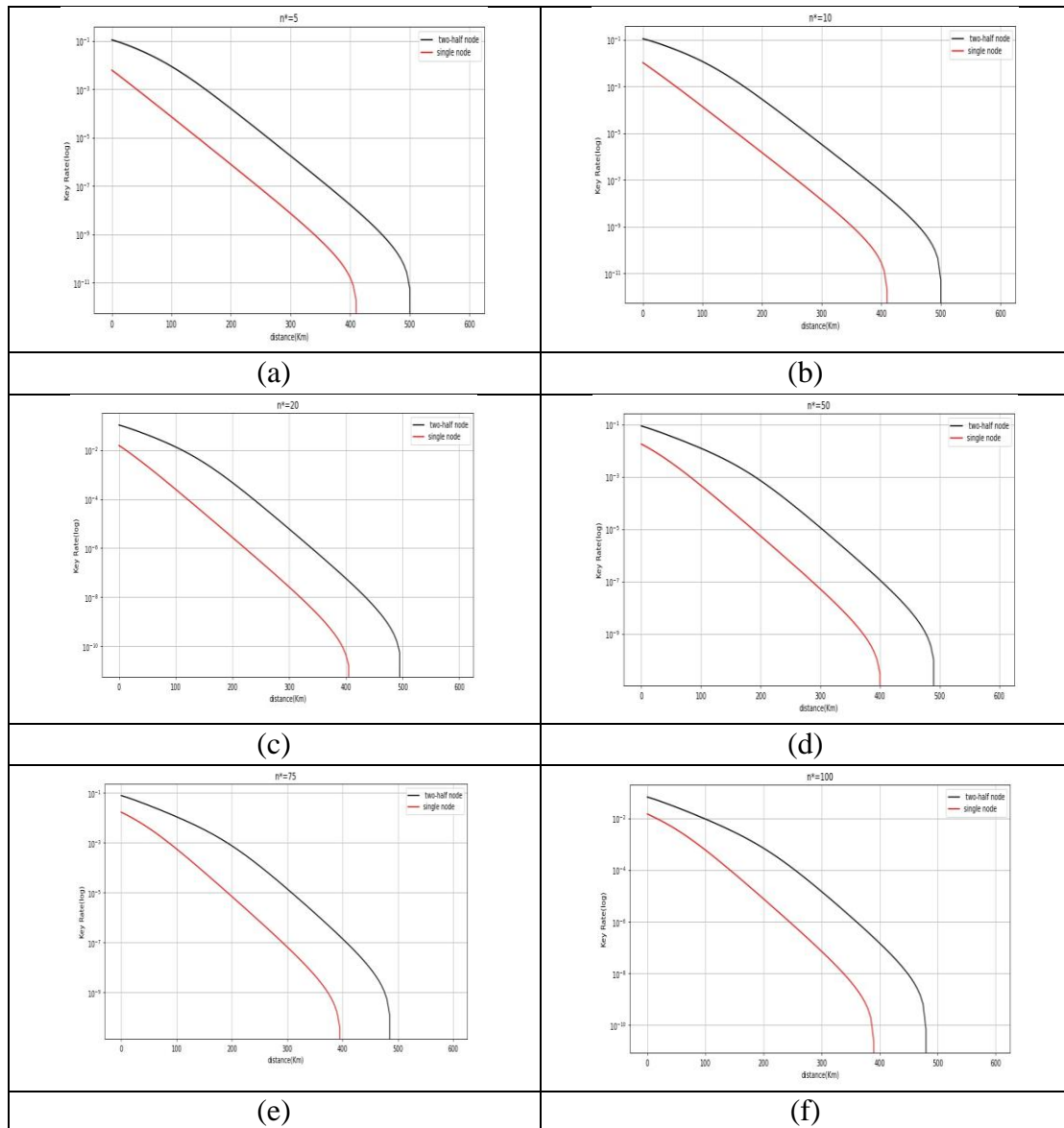


Figure 4: A comparison of the channel length between 2.5 node and single node quantum repeater for different number of trails (5, 10, 20, 50, 75, 100).

The results of (Fig. 4) indicates clearly that, the length of the quantum channel is improved by about (20%) if a 2.5 – node quantum repeater is used to distribute the entanglement between Alice and Bob instead of using single node quantum repeater., as well as the key rate.

3. Effect of using different (P_{det}) in identical and non-identical QMs of 2.5 node QR on the quantum channel length.

In this step, the effect of using different detectors with different detection efficiency P_{det} (0.7, 0.75, 0.8, and 0.9) on the performance of 2.5 node QR schemes based on identical QM and non-identical QMs. The results obtained are presented in Fig. 5.

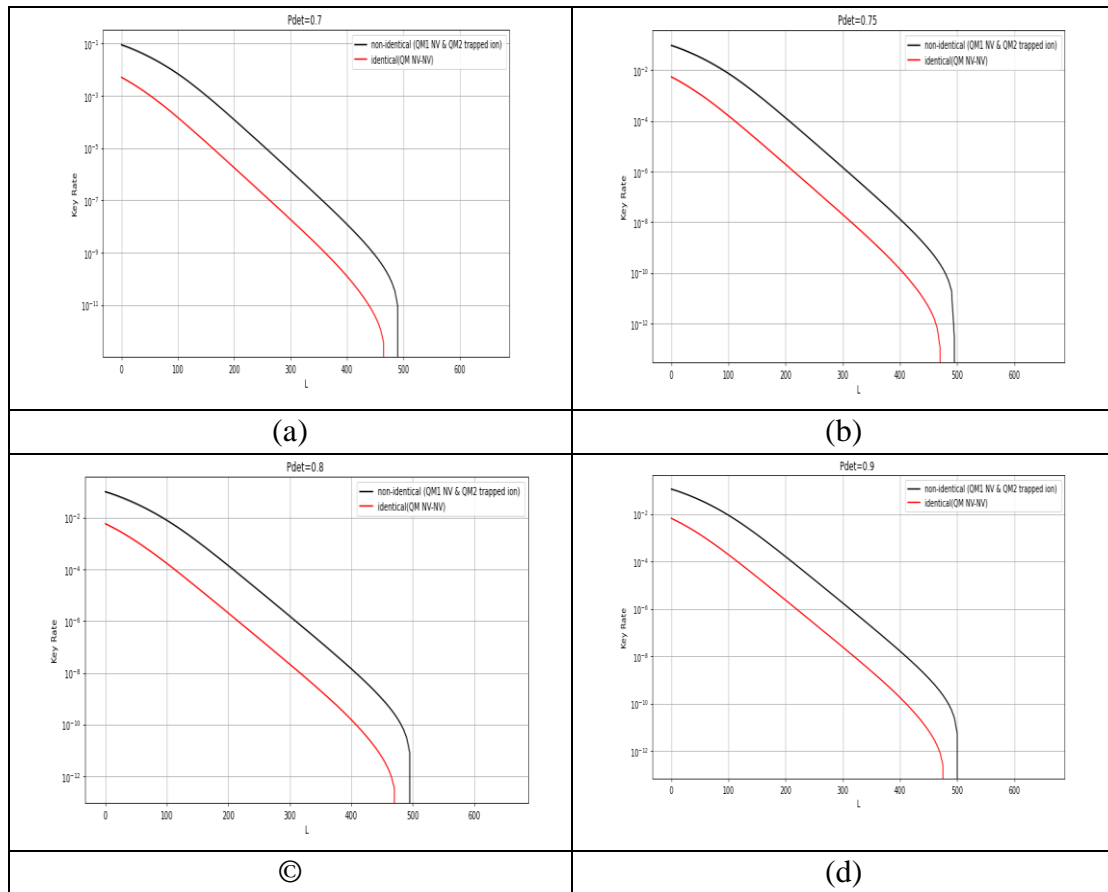


Figure 5: Effect of $P_{\text{det}}(0.7, 0.75, 0.8, 0.9)$ on the performance of 2.5 node QR scheme based on identical QM and non-identical QMs.

The results of (Fig. 5) indicate that, as the detection efficiency become high the performance of quantum repeater become better and hence the length of the quantum channel extended to more distances. The length obtained was about (490, 495, 498, and 500) Km for P_{det} of (0.7, 0.75, 0.8, and 0.9) respectively. The reason for such results is that with high detection efficiency detectors the time required for success detection is less and hence number of trials become less which enhance the QR key rate and extend the channel length.

4. Conclusions

Based on results of the study, one can conclude the following:

1. Utilizing non-identical QMs in 2.5 – node QR improves its performance.
2. Channel length that can be obtained by utilizing 2.5 – node QR with non-identical QMs is better than that of single node QR with same specifications.
3. Interplay between decoherence time and coupling efficiency of QMs enhance the performance of QR.

5. Future Work

1. Implementation of 2.5-node QR analyzed this study practically to discover the effect of any other parameters on quantum repeater performance.

2. Study the performance of quantum repeater using GHZ state instead of EPR state.

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