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# A Study on Prime Cordial Labelling in Graceful Graphs

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**ABSTRACT**

Graph theory is the one of the most important concepts which takes the great roll in the electronic devices $IC$'s. These components are known as chips and include complicated, layered microcircuits that can be described by lines or arcs as sets of points. By utilizing graph theory, mathematicians create integrated chips with maximum part density and minimum total interconnecting conductor length.

**Key Words:** Prime Cordial Labelling, Graceful Graphs

## Introduction

Leonhard Euler 's article on Konigsberg's Seven Bridges, written in 1736, is known as the first article in the history of graph theory. This article, as well as the one published on the knight dilemma by Vandermonde, continued with the Leibniz initiated research suits. Cauchy and L'Huilier researched and extended Euler 's formula referring to the number of sides, vertices, and faces of a convex polyhedron, which marks the beginning of the mathematics division known as topology. Cayley was led by a curiosity in basic analytical types emerging from differential calculus to research a particular class of graphs I e trees more than one hundred years after Euler 's paper on Königsberg bridges and while Listing introduced the definition of topology. For theoretical chemistry, this research has several consequences. Enumeration methods for graphs with complex properties. From the findings of Cayley and the fundamental results published by Pólya between 1935 and 1937, enumerative graph theory then proceeds. These were generalized in 1959 by De Bruijn. Via contemporary studies of chemical composition, Cayley listed his findings on trees. The proposals for contemporary chemical structure research. It has become part of the traditional vocabulary of graph theory to combine concepts from mathematics with those from chemistry.

In fact, Sylvester introduced the word “graph” in a paper published in Nature in 1878, where he drew a comparison between algebra and molecular diagrams of “quantic invariants” and “co-variants.

## Extremal Graph Theory

It is a subset of the philosophy of graphs. The theory of severe graphs studies maximal or minimal graphs that obey a certain property. We may take extremality in terms of numerous invariants of graphs, such as order, or girth. It studies a graph's global properties that affect the graph's local substructures.

For eg, “graphs on n vertices have the highest number of edges” is a basic extreme graph theory. The extreme graphs are trees on n vertices with $n-1$ edges. We want to find the minimum value of m because of graph $P$, an invariant u, and a set of graphs $H$, such that any graph in H that has u greater than m has the property $P$. In the example above, $H$ was the $n-vertex$ graph collection, $P$ was the cyclic property, and $u$ was the number of edges in the graph. There must be a loop for each graph on n vertices with more than $n-1$ edges.

The above-mentioned type is concerned with many basic outcomes in severe graph theory. For eg, Turán 's theorem asks the question of how many edges an $n-$vertex graph may have until it must have a clique of size k as a sub graph. Instead of cliques, if total multi-party graphs are asked the same question, the Erdős-Stone theorem offers the answer.

An example of a severe graph is the Turán graph T $(n, r)$. For a graph on n vertices without $(r+1)$, the four colour issue remained unresolved for more than a century. It has the highest possible number of edges. A procedure for solving the problem using computers was published in 1969 by Heinrich Heesch. "A computer-aided proof developed by Kenneth Appel and Wolfgang Haken in 1976 allows fundamental use of Heesch's notion of" discharging.

With the theory of Ringel (1964) and a paper by Rosa (1967), much curiosity in graph marking started in the middle of 1960. The famous conjecture of Ringel-Kotzig (1964, 1965) that all trees are graceful remains unsettled. Rosa (1967) implemented $\beta$- valuation, alph-valuation and other marking in his classic paper as a method for decomposing complete graphs. The $\beta-$ valuation was later called Golomb's graceful labelling (1972) and now this is the most commonly used term. In conjunction with their studies on the issue of additive bases resulting from error-correcting protocols, Graham and Sloane (1980) adopted harmonious marking.

The two basic labels that have been thoroughly researched are elegant labelling and harmonious labelling. In the field of graph labelling, varieties of graceful and harmonious labelling, including alpha-valuation, elegant labelling and cordial labelling, were implemented with various motives and domain restriction programming models. Therefore, separate labelling of graphs such as graceful labelling, prime labelling, cordial labelling, absolute cordial labelling, k-graceful labelling and unusual graceful labelling, etc., have been used in the subsequent years.

So far, multiple marking systems have been developed and many scholars are still exploring them. Within mathematics and other fields of information science and networking networks, graph marking has immense applications. In the work of Yegnanaryanan and Vaidhyanathan, different applications of graph labelling are mentioned.

More than six hundred articles have published on this issue over the course of four decades. This illustrates the field's accelerated development. The fundamental understanding, however, that the characterization of graceful and other labelled graphs appears to be one of graph theory's most difficult and difficult problems. Recognizing the simplest labelled graph, namely the cordial graph, is currently an NP-complete problem, see Kirchherr (1993). Many mathematicians have shown interest in having required criteria and different appropriate criteria on labelled graphs owing to these inherent difficulties of such marking, aiming to increase the comprehension of the characteristic existence of the labelled graphs. While the area of graph labelling deals essentially with theoretical analysis, the topic of graph labelling in the applied fields has also been the topic of research for a long time. The labelled graphs serve as useful templates for a large variety of applications such as coding theory, $X$-ray crystallography, radar, physics, circuit architecture and address of the communication network (refer to Golomb (1972), Bloom and Golomb (1997) and Bermond (1979)). It is important to notice that The labelling (or measurement) of graph G is the assigning of labels f from a collection of non-negative integers to a set of vertices of graph $G$, which induces a label identified by the labels f(u) and f(v) for each edge $uv$. The two basic marks that have been researched in the field of graph labelling are graceful and harmonious labelling.

## BASIC DEFINITIONS:-

**Graph Labeling**

The marking of the graph is an attribution to the vertices or edges of the values or both using such criteria.

**Vertex Labeling (Edge Labeling):-**

If the set of vertices is the mapping(edges) area, so the labelling is called the labelling of the vertex.

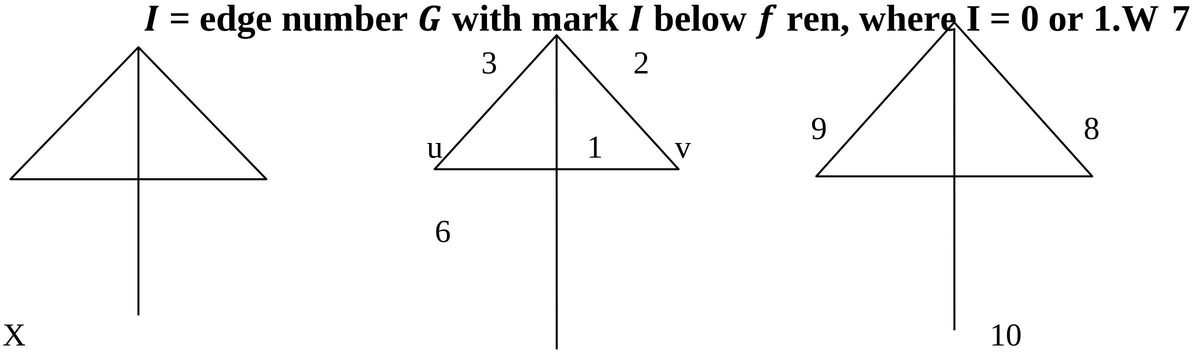
A label: $(G)$ as $\{0\ ,1\}$ is classified as a binary vertex label of $G$ and $(v)$ is classified as $G$ under $f$ vertex of $V$.

Rating 1.If an edge is $e = u\ V$, edge marking is caused.

$f$: $(G)$ to the point of $\{0\ ,1\}$ $f$ to the point of) $- f(V)$.

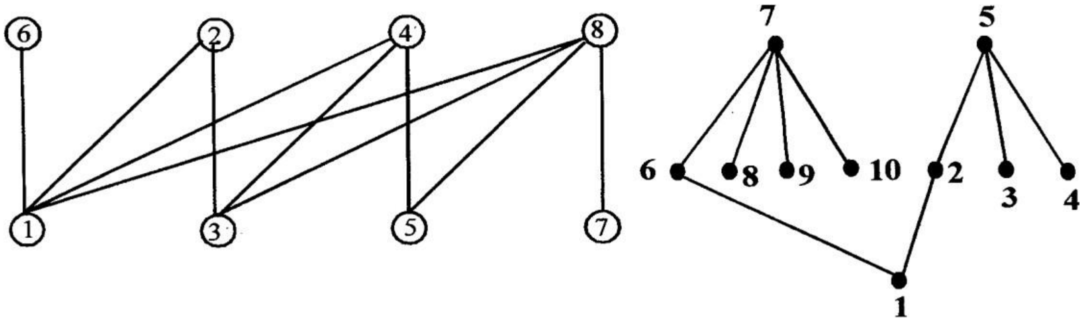
$V(i)$ = number of vertices of $G$ with the mark $I$ below $f$

**$I$ = edge number $G$ with mark $I$ below $f$ ren, where I = 0 or 1.W 7**




**Unlabelled graph** **edge-labeled graph** **vertex-labeled graph**

**Prime Labeling**




An injective function is the primary marking of a graph $\boldsymbol{G}$: $(\boldsymbol{G})$ da $\{\mathbf{1}\ ,\mathbf{2},.\,|\boldsymbol{V}(\boldsymbol{G})|\}$ , for all adjacent pairs $\boldsymbol{u}$ and $\boldsymbol{v}$, $\boldsymbol{gcd}$ of $(\boldsymbol{f}(\boldsymbol{u}), \boldsymbol{f}(\boldsymbol{v})) = \mathbf{1}$. And the prime number graph is known as the main labelling graph.

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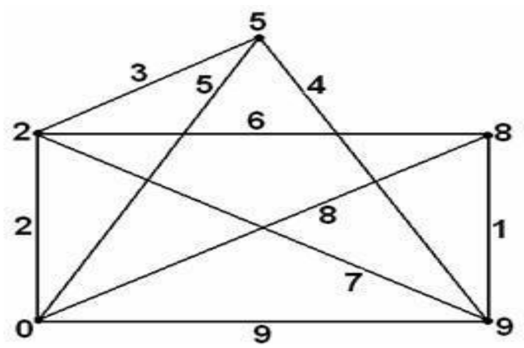
**Prime Cordial Labeling**

A prime cordial labeling of a graph $G$ where vertex set $(G)$ is a bijection$f'$: $(G) \to$ {1, 2, 3, 4,. . . , $|V(G)|$} and if the induced function $f$: $E(G) \to$ {0, 1} is defined by $f'(e = u\mathrm{V}) = 1$, if $gcd(f\ (u)\ , f\ (\mathrm{V})) = 1 = 0$, otherwise, the number of edges labeled with 0 and edges labeled with 1 differ by at most 1. A graph which labeled with prime cordial labeling is called a prime cordial graph.

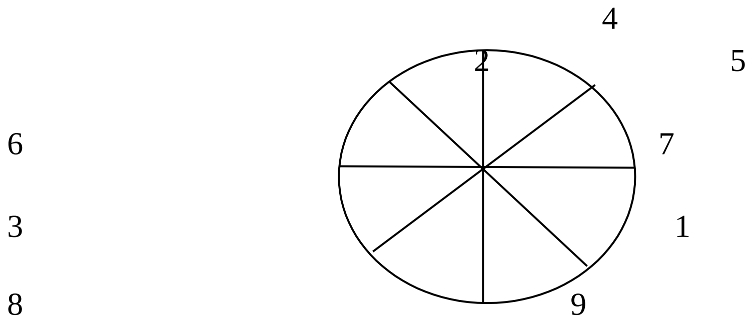
The values $f\ (u),\ f'\ (u, v)$ are called graceful labels of the vertex $U$ and the edge ($U$, $V$) respectively.

A cyclic decomposition of $K_{2q}{+}1$ is obtained as follows

1) Choose any graph $G$ with graceful labeling.
2) Identify the edges of $G$ with $q$ suitable edges of $K_{2q}{+}1$ , where$q = |E(G)|$
3) Each vertex and each edge of $G$ is rotated 2q times from the original position.




**Wheel Labeling**




**Graceful graph**

- May the graph $\boldsymbol{G = (V, E)}$ be a $\boldsymbol{q}$ edge graph. Graceful $\boldsymbol{G}$ labelling is an $\boldsymbol{f}$-injective function: $\boldsymbol{v}$ by the way,$\{\boldsymbol{0, 1, 2, \cdots, q}\}$ so that the induced labelling of the edge, $\boldsymbol{f(uv)} =$ is a bijection of E into the set$\{\boldsymbol{1, 2, \cdots, q}\}$.
- The conjecture given is the problem of definition of all graceful graphs and the graceful tree. The explanation why the diagrams are named differently. Few variants of successful labelling have also recently been added, for example, edge graceful labelling, graceful labelling by Fibonacci unusual graceful labelling.
- **Steps for Graceful Labeling of a graph**

  Use distinct nonnegative integers like $\{0,1,2,3, \ldots .,9\}$ to label vertices no need to use all
  Each edge value is the absolute difference of the values on adjacent vertices The adjacent 3

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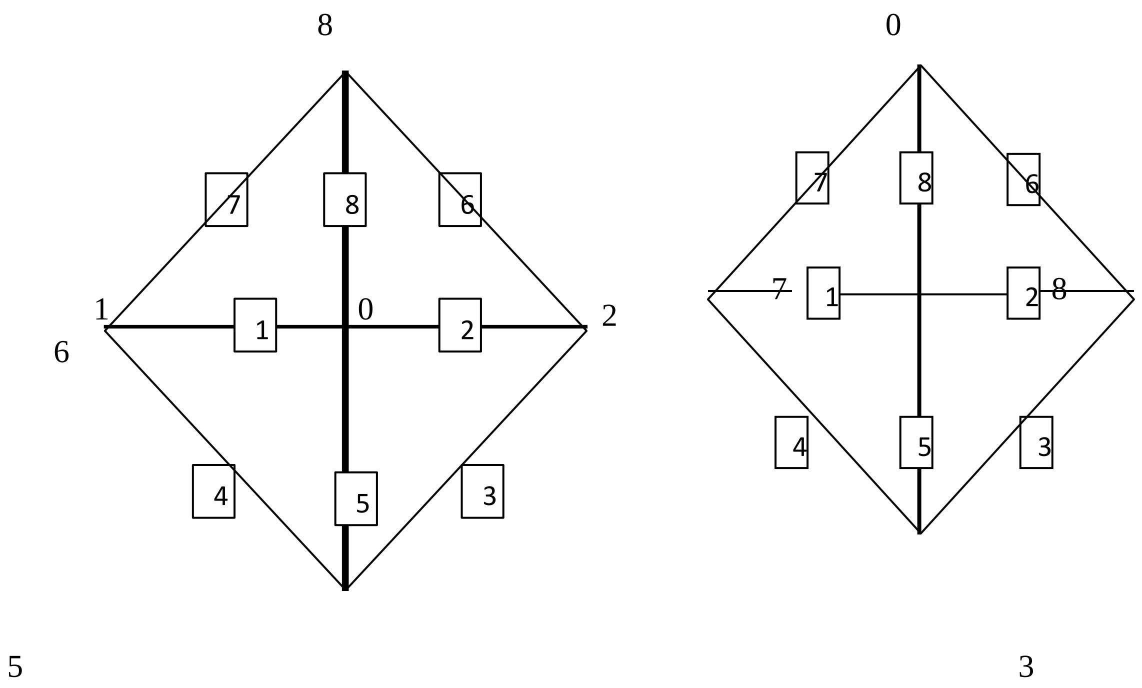
vertices should not be labeled with 0,1,2.

Values of edge must be distinct integers $\{1,2,3,\ldots,9\}$.

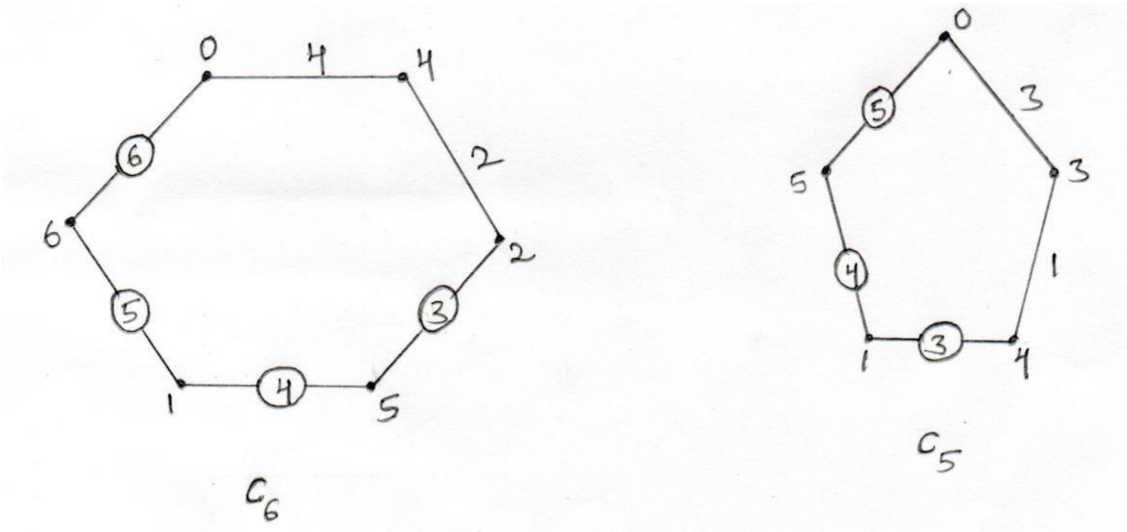
The labeling of the vertices should be in the manner, where we get unique values to each edge.

Example: Graph with 5 vertices and 8 edges.




**Graceful Labeling of cyclic graph $C_n$**

According to S. P. Rao Hebbare, Graceful Cycles, Utilitas Mathematica, (1976) states that steps for graceful labeling of Cycle graph $c_n$ is Graceful if $n = 4k \; or \; n = 4k + 3$ for some integer $k \geq 0$, And non-graceful if $n = 4k + 1 \; or \; n = 4k + 2$. Non-graceful graphs: - If $K = 1$ $n = 4K + 1$ then $n = 5 \; Or \; n = 4K + 2 = 6$. Therefore $C_6$ and $C_5$ are non graceful graph.



Graceful graphs: - C4

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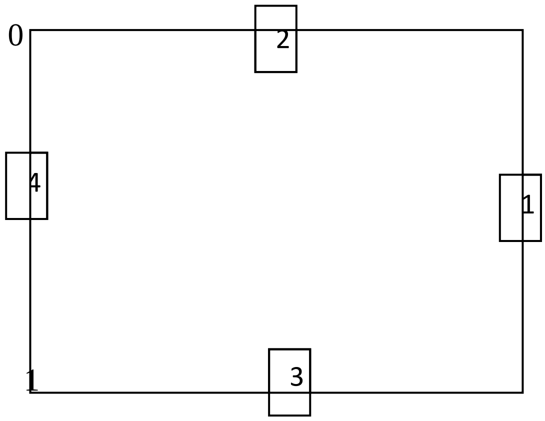


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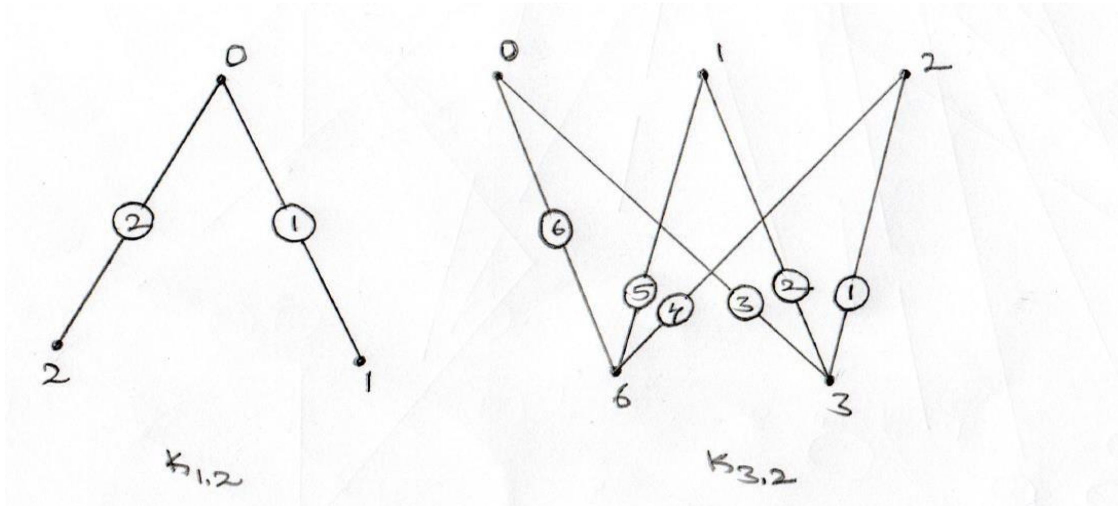
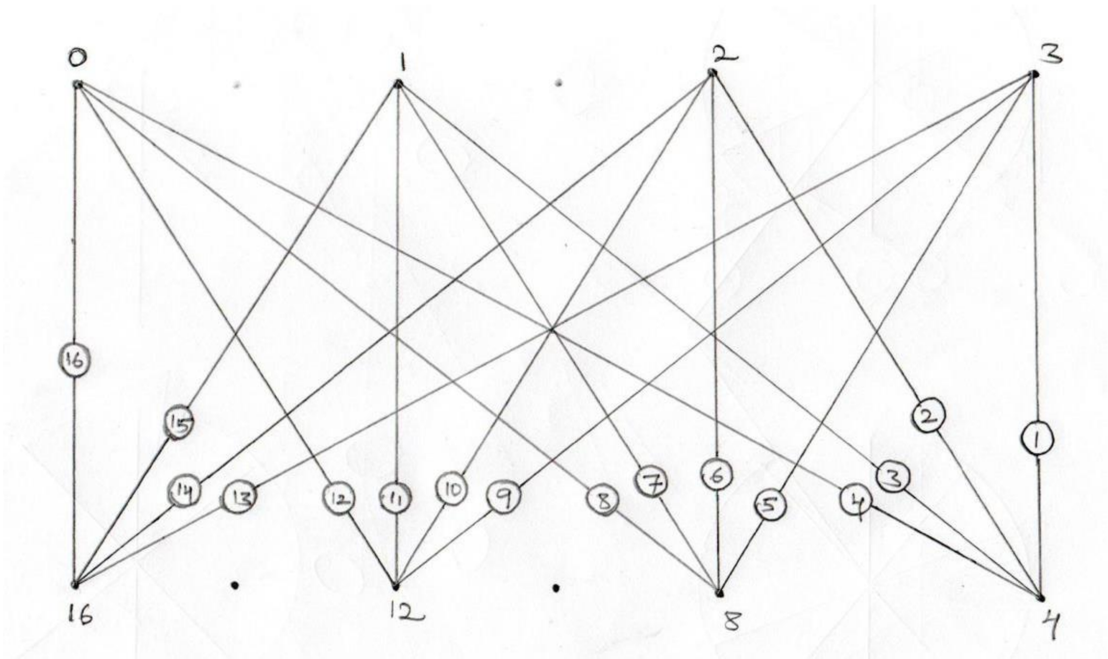
**Graceful graph of $K_{m,n}$**



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# Prime Labeling Of $K_{4,4}$

## Graphs Introduction

As the analysis of prime numbers is of considerable interest since prime numbers are distributed and there are arbitrarily wide holes in the series of prime numbers, the notion of prime marking has drawn many scholars. Many observations on prime marking have been investigated by Vaidya and Prajapati. Prime marking in the sense of replication of graph elements was addressed by the same authors. Motivated by the principles of prime labelling and cordial labelling, Sundaram launched a new idea called a prime cordial labelling that includes a combination of both labels. In certain fields of computer science, network representations play an important role, ranging from data structures and graph algorithms to concurrent and hierarchical computation and communication networks. Typical representations of networks are typically global in nature. That is, one must enter a global data framework covering the whole network in order to obtain valuable details. From social and networking networks to the World Wide Web, huge diagrams are everywhere. A strong aid to visualizing and interpreting the data is the geometric depiction of the graph form imposed on these data sets.

Roger Entringer proposed the notion of prime marking and Tout [7] addressed it in a report. If their greatest common divisor is 1., two integers a and b are said to be comparatively prime. In both analytic and algebraic number theory, comparatively prime numbers play an significant part. Most physicists have researched the prime graph. The direction $Pn$ on n vertices has been shown to be a prime graph by $Fu.$ H[3]. Deretsky et al [2] have prove that the cycle $Cn$ on n vertices is a prime graph. Roger Etringer conjectured around 1980 that all trees have prime labelling that was not resolved until today.

Sundaram et al[6], Lee. S.et.al., are studying the Prime planar grid marking. [4] has shown that, if and only if n is even, the wheel is a prime graph. A node replication of a graph creates a new graph by connecting a node with it). Duplicating the graph obtained by

## Prime Labelling of Tree

Entringer conjectured around 1980 that all trees have a prime marking. Paths, stars, caterpillars, complete binary trees, spiders and all trees with fewer than 16 orders are among the groups with trees considered to have prime marking. For a tree to have a prime marking that would specify many families of prime trees, we give a necessary requirement. We are still studying the prime marking of banana trees in special groups. All cycles Cn and the disjoint union of $C2k$ contain other graphs with prime naming. The full graph Kndoes does not have a primary labelling for n4 and the wheel Wn with n spokes is primary if and only if we review the primary labelling of such join graphs and product graphs, namely, n is also studied.$C_{n+mk1}$, $P_{n+mk1}$, $P_n \times P_2$, $P_n \times P_3$ $and$ $K_{1n} \times P_2$ .

## Results Proved on Labeling Of Tree

Every tree is prime, Entringer conjectured. We give a necessary requirement in the theorems below for a tree to have prime marking. To the list of prime named plants, we even introduce a few unique groups of banana trees.

**Like above applications some other applications listed below**

- Graph Labelling in Communication Relevant to Adhoc Networks

- Secure Communication in Graph
- By Using Key Graphs
- Identification of Routing Algorithm with Short Label Names
- Automatic Routing with labeling
- Security with reducing the packet size using labeling schema
- Fast Communication in sensor networks Using Radio Labelling.

## Conclusion

Some findings corresponding to prime labelling and some results corresponding to elegant Fibonacci labelling are investigated and some results corresponding to tree graph prime labelling are investigated. For other graph families and in the sense of deferred graph labelling problems with the specified condition, related findings may be derived. We applied the conditions given in the theorem of graph prime labelling and tree prime labelling to the separate graphs here, and we succeeded in drawing the graph with the conditions given. Graph Labelling is a valuable tool that, as described above, allows it simple in different areas of networking.

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