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Estimation of Survival Function for type II Control Data Subject to Mixed Distribution (Topp-Leone-Exponential-Weibull) Using Least Squares Method

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Abstract
The study of the survival function of living organisms and the application of modern functions for the purpose of estimating them is one of the important matters that took the greatest part among researchers in the last decade, as the new proposed distribution (top-leon-exponential-whipple) was used to estimate the survival function for control data of the second type, as it was used The least squares method in estimating distribution parameters, including estimating the survival function, if a sample of (100) observations was taken representing patients in Nasiriyah Heart Center and the amputation rate was 30%. The difference between any two time periods of the survival function is a small difference and its variance as well, and this indicates the presence of convergence between the times of death for every two consecutive people.

Keywords :survival , type II control data , mixed distribution , least squares method

1-Introduction

The study of the survival function of living organisms is one of the important matters, as many researchers have developed several methods for generating flexible continuous distributions resulting from traditional continuous distributions. The researchers have concluded that the models resulting from the (Topp-Leone) family in many studies conducted in recent years have a clear advantage over the single distributions, that is, before they were combined. Mathematical aspect These distributions often belong to specific time-dependent families, i.e. they can be used to calculate the shed function of organisms.

The potential values of the new parameters can significantly improve the statistical capabilities of the original distribution, which positively affects the central parameters and reduces dispersion.

Because of the applications of different survival functions in daily life, the need to estimate the survival function and its applications on various organisms and knowledge of death times, to evaluate the level of work of a drug appeared. Medical or anti-vaccine for the case to be studied.

2-Topp-Leone-exponential-Weibull distribution[7][10][11][5]

To find the probability function for the proposed distribution, by replacing the cdf aggregate density function for the complex distribution (exponential-Weibull) with the probability density function (pdf) for the complex distribution (exponential-Weibull) in the distributions generating function (Topp-Leone) (pdf), we get the proposed distribution (TLEW) as follows:

$$f_{TLEW}(x) = 2\alpha\theta\eta\beta\gamma^{\beta}x^{\beta-1}e^{-(\gamma x)^{\beta}}\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta-1}\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\alpha\theta-1}\left[1-\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]\left[2-\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]^{\alpha-1}$$

حيث أن $x > 0$ ... (1)

The cumulative distribution function of the proposed distribution which is as follows::

$$F_{TLEW}(X) = \left\{\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right\}^{\alpha} \quad \dots (2)$$

prove that the (TLEW) function is a probabilistic *function*

$$\int_0^{\infty} 2\alpha\theta\eta\beta\gamma^{\beta}x^{\beta-1}e^{-(\gamma x)^{\beta}}\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta-1}\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\alpha\theta-1}\left[1-\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]\left[2-\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]^{\alpha-1}dx$$

Suppose that$(\gamma x)^{\beta}$*is equal to y from which we find that*$x = \frac{y^{\frac{1}{\beta}}}{\gamma}$*From them we find that*$dx = \frac{y^{\frac{1}{\beta}-1}}{\beta\gamma}dy$The limits of the function remain the same, by substitution:

$$\int_0^{\infty} 2\alpha\theta\eta\beta\gamma^{\beta}\left(\frac{y^{\frac{1}{\beta}}}{\gamma}\right)^{\beta-1} e^{-y}(1-e^{-y})^{\eta-1}\left[(1-e^{-y})^{\eta}\right]^{\alpha\theta-1}\left[1-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]\left[2-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]^{\alpha-1}\frac{y^{\frac{1}{\beta}-1}}{\beta\gamma}dy$$

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By simplifying, we get:

$$\int_0^\infty \frac{2\alpha\theta\eta\beta\gamma^\beta}{\beta\gamma^\beta} e^{-y}[(1-e^{-y})]^{\alpha\theta\eta-1}\left[1-\left[(1-e^{-y})^\eta\right]^\theta\right]\left[2-\left[(1-e^{-y})^\eta\right]^\theta\right]^{\alpha-1} dy$$

let$z = e^{-y}$from which we find$y = -Lnz$from which we find$dy = \frac{1}{z}dz$The limits of the function resulting from the transformation are$0 < z < 1$:

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\beta\gamma^\beta} z[(1-z)]^{\alpha\theta\eta-1}\left[1-[(1-z)]^{\eta\theta}\right]\left[2-[(1-z)]^{\eta\theta}\right]^{\alpha-1}\frac{1}{z}dz$$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\beta\gamma^\beta} [(1-z)]^{\alpha\theta\eta-1}\left[1-[(1-z)]^{\eta\theta}\right]\left[2-[(1-z)]^{\eta\theta}\right]^{\alpha-1}dz$$

suppose that$R = 1 - Z$*Of which*Z=1-R*find that*$dZ = dR$:

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\beta\gamma^\beta} [R]^{\alpha\theta\eta-1}\left[1-[R]^{\eta\theta}\right]\left[2-[R]^{\eta\theta}\right]^{\alpha-1}dR$$

let$[R]^{\eta\theta} = W$from which we find$R = W^{\frac{1}{\eta\theta}}$from which $dR = \frac{1}{\eta\theta}W^{\frac{1}{\eta\theta}-1}dW$:

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\beta\gamma^\beta}\left[\frac{w^{\left(\frac{1}{\theta\eta}\right)^{\alpha\theta\eta}}}{W^{\frac{1}{\eta\theta}}}\right][1-W][2-W]^{\alpha-1}\frac{1}{\eta\theta}W^{\frac{1}{\eta\theta}-1}dW$$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\eta\theta\beta\gamma^\beta}[W]^{\alpha-1}[1-W][2-W]^{\alpha-1}dW$$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\eta\theta\beta\gamma^\beta}[W]^{\alpha-1}[(1-W)][1+(1-W)]^{\alpha-1}dW$$

let$V = 1 - W$from which$W = 1 - V$*Where*$dW = dV$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\eta\theta\beta\gamma^\beta}[1-V]^{\alpha-1}[V][1+V]^{\alpha-1}dV$$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\eta\theta\beta\gamma^\beta}[1-V^2]^{\alpha-1}[V]dV$$

let$V^2 = S$from which$V = S^{\frac{1}{2}}$*Where*$dV = \frac{1}{2}s^{\frac{-1}{2}}dS$*By compensation*:

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{\eta\theta\beta\gamma^\beta}[1-S]^{\alpha-1}S^{\frac{1}{2}}\frac{1}{2}s^{\frac{-1}{2}}dS$$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^\beta}{2\theta\eta\beta\gamma^\beta}[1-S]^{\alpha-1}dS$$

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$$\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{2\theta\eta\beta\gamma^{\beta}}\left[-\frac{([1-S]^{\alpha}}{\alpha})\right]_{0}^{1}=\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{2\alpha\theta\eta\beta\gamma^{\beta}}=1\quad is\ pdf$$

*Now we find the moment*r$E(X^r)$(TLEW)

$$E(X^r)=\int_0^\infty 2\alpha\theta\eta\beta\gamma^{\beta}x^{r+\beta-1}e^{-(\gamma x)^{\beta}}\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta-1}\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\alpha\theta-1}\left[1-\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]\left[2-\left[\left(1-e^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]^{\alpha-1}dx$$

$$\int_0^\infty 2\alpha\theta\eta\beta\gamma^{\beta}\left(\frac{y^{\frac{1}{\beta}}}{\gamma}\right)^{r+\beta-1}e^{-y}(1-e^{-y})^{\eta-1}\left[(1-e^{-y})^{\eta}\right]^{\alpha\theta-1}\left[1-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]\left[2-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]^{\alpha-1}\frac{y^{\frac{1}{\beta}-1}}{\beta\gamma}dy$$

By simplifying, we get:

$$\int_0^\infty \frac{2\alpha\theta\eta}{\gamma^{r-1}}y^{\frac{r}{\beta}-\frac{1}{\beta}}e^{-y}[(1-e^{-y})]^{\alpha\theta\eta-1}\left[1-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]\left[2-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]^{\alpha-1}dy$$

let$z=e^{-y}$from which we find$y=-Lnz$from which we find$dy=\frac{1}{z}dz$The limits of the function resulting from the transformation are$0<z<1$:

$$\int_0^1 \frac{2\alpha\theta\eta}{\gamma^{r-1}}[Lnz]^{\frac{r}{\beta}-\frac{1}{\beta}}z[(1-z)]^{\alpha\theta\eta-1}\left[1-[(1-z)]^{\eta\theta}\right]\left[2-[(1-z)]^{\eta\theta}\right]^{\alpha-1}\frac{1}{z}dz$$

By simplifying, we get:

$$\int_0^1 \frac{2\alpha\theta\eta}{\gamma^{r-1}}[Lnz]^{\frac{r}{\beta}-\frac{1}{\beta}}[(1-z)]^{\alpha\theta\eta-1}\left[1-[(1-z)]^{\eta\theta}\right]\left[2-[(1-z)]^{\eta\theta}\right]^{\alpha-1}dz$$

suppose that$R=1-Z$*Of which*$Z=1-R$*find that*$dZ=dR$:

$$E(X^r)=\int_0^1 \frac{2\alpha\theta\eta}{\gamma^{r-1}}[Ln(1-R)]^{\frac{r}{\beta}-\frac{1}{\beta}}[R]^{\alpha\theta\eta-1}\left[1-[R]^{\eta\theta}\right]\left[2-[R]^{\eta\theta}\right]^{\alpha-1}dR$$

Using the previous assumptions in the transformation of complex functions and the integration rule of partial functions in(Mathematical program)

$$\mu_t^{'}=\alpha^{r}\sum_{k=0}^{\infty}\gamma_k[(\alpha+k)\theta]^{\frac{r}{\beta}}\Gamma\left(1-\frac{r}{\beta}\right)\qquad \ldots(3)$$

3-Survival function[8][9][11]

It is defined as a function during which a particular device can continue to work correctly (without failure) during a specified period (0, t), and the survival function can also be known as a measure of the ability to work a certain system or part of that system continuously without stopping during A specific period of time (0, t), and the survival function can be expressed mathematically as:

$$\boldsymbol{S}(t) = pr\ (T > t\ ) \qquad \dots (4)$$

t: the operating time of the device, which is greater or equal to zero.

T: The accumulated lifetime of the device during and its value is limited (0,t .).

The formula for the dependency function in continuous distributions is

$$:\boldsymbol{S}(t) = \int_t^{Maxt} f(u)du \qquad \dots (5)$$

The survival function has such properties as:

- *Its value is between zero and one.*

It is a probability function: $0 \quad \leq \boldsymbol{S}(t) \leq 1)$

- It is inversely proportional to time,

that is, as the time of operation of the machine progresses, its value decreases:

$$\boldsymbol{S}(t_1\ ) > \boldsymbol{S}(t_2) > \boldsymbol{S}(t_3) > \cdots > \boldsymbol{S}(t_\infty)$$

The survival function of the proposed distribution(TLEW):

$$\boldsymbol{S}(\boldsymbol{x}) = \mathbf{1} - \mathrm{F}_{\mathrm{TLEW}}\ (X)\ = \mathbf{1} - \left\{\left(1 - \mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2 - \left(1 - \mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right\}^{\alpha} \quad \dots (6)$$

4-Data classification [4] [3]

4-1complete data

Complete data means all the data (sample units) that have been put to a life test. The test stops after the failure of all units, and the failure time for each sample unit is known and observed.

The maximum possibility function for this type of data

$$L = \prod_{i=1}^{n} f(t, \theta) \dots \qquad (7)$$

since:

$f(t, \theta)$:The probability density function of the distribution

The disadvantages of using complete data is to monitor all the items of the sample subject to the life test, and that this results in (loss of time, cost, effort, and sometimes destructive testing) so it can be replaced by complete data with control data.

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4- 2censored Data

Two types of monitoring data will be discussed, although there are several types, and it is possible to clarify the monitoring data as follows:

:

- **type- I -censord Data**

This type is called Time censored data. In this type of data, the observation time is fixed (t_0) and predetermined and varies from one experiment to another for all sample data (sample units) subject to the test.

When testing the life of n of units at zero time, we will watch (watch) the work of the sample units until the predetermined fixed time expires, i.e. the life experience (the test) stops.

And that the units that failed the test are m units, and that m is a random variable that we cannot know or determineexcept after the expiry of time (t_0), and (n-m) is the number of units left after time (t_0)

$0< t_1 < t_2 < t_3 < \cdots \dots \dots \dots . < t_m < t_0$

since:

is the failure time of unit i before time t_0

The greatest possibility function for observation data of the first type is:

$$L = \frac{n!}{(n-m)!} \prod_{i=1}^{m} f(t,\theta)[S(t_0)]^{n-m} \qquad \dots . (8)$$

since:

$S(t_0)$Survival function over timet_0

$f(t,\theta)$failure density function

$(n-m)$Number of units left after timet_0

And that this type of samples is concerned with the experiences of life tests in which the cost is increasing.

- **type – II-censored Data**

This type of data is called failure censored data.

In this type of data, a predetermined number of sample units that is being monitored is determined (r) fixed units. Therefore, the time of these units t_r is a random variable that cannot be determined. When the life test begins at zero time, we will monitor (watch) the work of the units. r We stop the

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experiment after obtaining r of the failed units that were previously specified, while the remaining units after time t_r are (n-r*)*

The maximum possibility function for this type of data is*:*

$$L = \frac{n!}{(n-r)!} \prod_{i=1}^{m} f(t,\theta)[S(t_r)]^{n-r} \qquad \dots (9)$$

since:

$0 < t_1 < t_2 < \ldots < t_r$

$f(t,\theta)$: failure density function.

$S(t_r)$: Survival function at time t_r.

(n-r): Number of units left (non-failed) after the test stops on failure Unit No. r.

These samples are often concerned with examining expensive units or those in which the examination is destructive.

5-(Least square method Method)[2] [12] [1]

The least squares method is one of the important methods in estimating the parameters of the probability distribution, and the estimators in this way are more accurate in medium and small samples compared to large samples, and the basic principle of it is to work on finding estimators by minimizing the sum of the squares of errors (the difference) between the cumulative distribution function CDF of the distribution The studied and one of the nonparametric capabilities of the cumulative probability function, and mathematically it can be written as follows:

$$LS = \sum_{i=1}^{r} \left(F(t_i) - \frac{i}{r+1} \right)^2 \qquad \dots (9)$$

or

$$LS = \sum_{i=1}^{r} \left(F(t_i) - \frac{i-0.5}{r+2} \right)^2 \dots (10)$$

We will apply the first formula, which is

$$LS = \sum_{i=1}^{r} \left(F(t_i) - \frac{i}{r+1} \right)^2$$

$$LS=\sum_{i=1}^{r}\left(\left\{\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right\}^{\alpha}-\frac{i}{r+1}\right)^{2}\ \dots(11)$$

Derive for the parameters to be estimated, we get:

$$\frac{\partial\log LS}{\partial\alpha}=\sum_{i=1}^{r}\left(\left(\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right)^{\alpha}-\frac{i}{r+1}\right)\left(\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right)^{\alpha}\ln\left(\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right)\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]=0\qquad\dots(12)$$

$$\frac{\partial\log LS}{\partial\theta}=\sum_{i=1}^{r}\left(\left(\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right)^{\alpha}-\frac{i}{r+1}\right)\left(\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right)=0\qquad\dots(13)$$

$$\frac{\partial\log LS}{\partial\beta}=\sum_{i=1}^{r}\left(\left(\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right)^{\alpha}-\frac{i}{r+1}\right)\frac{1}{\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]}\left(\left(\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}})^{\theta\eta}\right]\right)^{\alpha}\alpha\left(\frac{\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\eta\theta\gamma x^{\beta}\ln(\gamma x)\mathrm{e}^{-(\gamma x)^{\beta}}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]}{1-\mathrm{e}^{-(\gamma x)^{\beta}}}-\frac{\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{2\theta\eta}\eta\theta\gamma x^{\beta}\ln(\gamma x)\mathrm{e}^{-(\gamma x)^{\beta}}}{1-\mathrm{e}^{-(\gamma x)^{\beta}}}\right)\right)=0\qquad\dots(14)$$

$$\frac{\partial \log LS}{\partial \eta}=\sum_{i=1}^{r}\left(\left(\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right)^{\alpha}-\frac{i}{r+1}\right)\left(\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}-\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right)=0 \quad \ldots(15)$$

$$\frac{\partial \log LS}{\partial \gamma}=\sum_{i=1}^{r}\left(\left(\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right)^{\alpha}-\frac{i}{r+1}\right)\frac{1}{\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]}\left(\left(\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right)^{\alpha}\alpha\left(\frac{\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\eta\theta(\gamma x)^{\beta}\beta x e^{-(\gamma x)^{\beta}}}{\gamma x\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}}-\frac{\left(\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right)^{2}\eta\theta(\gamma x)^{\beta}\beta x e^{-(\gamma x)^{\beta}}}{\gamma x\left(1-e^{-(\gamma x)^{\beta}}\right)^{\theta\eta}}\right)\right)=0 \quad \ldots(16)$$

Equations (12) to (16) One of the numerical iterative methods and the fsolve function in Matlab program will be used to find the estimator of the survival function for the distribution as follows:

$$\hat{R}_{\text{TLEW}\_OLS}=1-\left\{\left(1-e^{-(\gamma_{OLS}x)^{\beta_{OLS}}}\right)^{\theta_{OLS}\eta_{OLS}}\left[2-\left(1-e^{-(\gamma_{OLS}x)^{\beta_{OLS}}}\right)^{\theta_{OLS}\eta_{OLS}}\right]\right\}^{\alpha_{OLS}} \quad \ldots(17)$$

$$S(t)_{RSS}=1-\exp(-\left(\frac{\hat{\lambda}_{ls}}{t}\right)^{\hat{\theta}_{ls}} \quad \ldots(18)$$

5-Applied

The data were taken by the DhiQar Health Department and with a large sample size of (100n =) observations of patients in Nasiriyah Heart Center - Ministry of Health / DhiQar Health Department, and the sample observations represent observational data until death in weeks for each

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patient, as for the time period that These observations were calculated for her, and they were 12 months for the year 2021, which are, in order, a month (1, 2, 3….., 12))

5- 1Real data analysis)(

To estimate the survival function, real control data of the second type at a size of 100 and an amputation rate of 30% using the least squares method, and after using the mathematical equations numbered from (14) to (18) that represent the estimations of the parameters of the proposed distribution TLEW by the method of least squares OLS, and in light of that, the The survival function was estimated using the least squares method. The results of the estimation were as shown in the table below:

Table (1) True survival function values estimated according to the squares method

	t_i	S_Real	S_ OLS	t_i	S_Real	S_ OLS
n=100 r=40	10.22	0.98182	0.97457	19.32	0.31982	0.35423
2.8=$\hat{\alpha}$	11.22	0.97001	0.96026	20.14	0.31616	0.35092
3.3=$\hat{\theta}$	11.29	0.95818	0.94658	20.44	0.23317	0.27475
3.4=$\hat{\beta}$	12.11	0.93305	0.91885	20.86	0.21626	0.25885
$\hat{\gamma}$=1.4	12.12	0.91292	0.89751	21.29	0.19791	0.24139
$\hat{\eta}$=5.1	12.14	0.89306	0.87702	21.43	0.19555	0.23913
	12.22	0.86213	0.84596	21.57	0.11464	0.15829
	12.35	0.81786	0.80283	22.14	0.10100	0.14376
	18.14	0.47215	0.48976	23.71	0.06060	0.09806
	18.33	0.46586	0.48421	24.19	0.06034	0.09775
	18.57	0.46434	0.48286	24.34	0.05442	0.09056
	18.57	0.38356	0.41130	25.14	0.01565	0.03673
	19.12	0.37406	0.40285	25.22	0.01390	0.03376

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	19.23	0.33762	0.37025	25.71	0.01170	0.02989
	19.26	0.32336	0.35742	26.57	0.01086	0.02835

Table (1) at the sample size of 100 and the percentage of cut-off 30% and the values of the parameters estimated by the least squares method

The values of the survival function have clearly decreased with time, and this is what matches the behavior of this function as it is decreasing with time.

The survival function curve estimated according to the least squares method is closer to the survival curve curve for the real data and this indicates the fit of the real data to the studied distribution TLEW.

The average time between the survival function estimated by the method of least squares is equal to (0.024546), while the variance is equal to (0.001175), meaning that the difference between any two periods of survival is very little difference and its variance as well, and this indicates a convergence between the continuation of the work of the devices.

Also, the average time between the real survival function is (0.024364), while the variance is equal to (0.001094), meaning that the difference between any two time periods of the survival function is a small difference and its variance as well, and this indicates a convergence between the times of death for each two successive people.

Conclusions6-

1-The study showed that the proposed distribution (TLEW) achieved an advantage in estimating the survival function compared to the distributions (exponential, Whipple, exponential - Whipple) at all comparison criteria between distributions when applied to real second type control data.

2- The study showed that the distribution-generating function (Topp Leone) is a flexible function in generating more suitable distributions for applied phenomena compared to other functions.

3-The study concluded that the average failure times for all devices taken in the study sample amounted to 02450. The variance amounted to 0.0012, meaning that the difference between any two successive periods of failure times is very small and its variance is close to zero.

Recommendations7-

Based on the conclusions reached, the researchers recommend the following:

1-Using the moment method at a small and medium sample size to estimate the survival of control data of the second type in the case of using the distributions generating function (Topp Leone(

2-Adoption of the mean of integral error squares (IMSE) criterion, as it measures the mean squares of error at the total time (Ti.(

3-Using Bayesian estimation methods (EM algorithm, Downhill Simplex algorithm(

To estimate the survival of the proposed distribution (TLEW) and measure the survival function for type I control and complete data.

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