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Estimation of the Reliability Function for type II Control Data Subject to Mixed Distribution (Topp-Leone-Exponential-Weibull) by Using the Moment Method

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Abstract: The development that science is witnessing in the current era and the advanced ages of the last century in all applied fields, the most important of which is the development of machinery technology, makes research and development required in the fields opposed to it. And drawing economic policies through it. The main idea in this research is to estimate the dependency function of the new probability distribution through the generating function of the distributions) .Topp - Leone(As we combine the exponential distribution with the Whipple distribution and put the result of them in the generated function and find the properties of the resulting distributionAs for the practical side, real data from the Ministry of Health - DhiQar Health Department about nebulizer devices were used. The distribution data and the dependency function were estimated by the method of moments. The study concluded that the proposed distribution achieved more flexible results in the applied data.

Keywords : reliability , type II control data , mixed distribution , moment method

1-Introduction

The study of the reliability of machines, equipment or living organisms is one of the important matters, as many researchers have developed several methods for generating flexible continuous distributions resulting from traditional continuous distributions. The researchers have concluded that the models resulting from the (Topp-Leone) family in many studies conducted in recent years

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have a clear advantage compared to the single distributions, i.e. before they were combined. Mathematical aspect These distributions often belong to specific time-dependent families, that is, they can be used to calculate the reliability function of machines and the survival functions of organisms.

The potential values of the new parameters can also significantly improve the statistical capabilities of the original distribution, which positively affects the central parameters and reduces dispersion. Because of the various applications of reliability in daily life, the need to estimate the reliability function and its applications to devices and machines and knowledge of failure times (malfunctions) and working times until failure appeared. , to assess the level of work of machines and equipment and to estimate the costs of maintaining and restarting machines.

2-Topp-Leone-exponential-Weibull distribution[7][10][11][5]

To find the probability function for the proposed distribution, by replacing the cdf aggregate density function for the complex distribution (exponential-Weibull) with the probability density function (pdf) for the complex distribution (exponential-Weibull) in the distributions generating function (Topp-Leone) (pdf), we get the proposed distribution (TLEW) as follows:

$$\mathrm{f_{TLEW}}\,(\mathrm{x}) = 2\alpha\theta\eta\beta\gamma^{\beta}x^{\beta-1}\mathrm{e}^{-(\gamma x)^{\beta}}\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta-1}\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\alpha\theta-1}\left[1-\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]\left[2-\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]^{\alpha-1}$$

$x > 0$ حيث أن ... (1)

The cumulative distribution function of the proposed distribution which is as follows::

$$\mathrm{F_{TLEW}}\,(X) = \left\{\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right\}^{\alpha} \quad \ldots (2)$$

prove that the (TLEW) function is a probabilistic *function*

$$\int_0^{\infty} 2\alpha\theta\eta\beta\gamma^{\beta}x^{\beta-1}\mathrm{e}^{-(\gamma x)^{\beta}}\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta-1}\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\alpha\theta-1}\left[1-\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]\left[2-\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]^{\alpha-1} dx$$

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Suppose that$(\gamma x)^{\beta}$*is equal to y from which we find that*$x=\frac{y^{\frac{1}{\beta}}}{\gamma}$*From them we find that*$dx=\frac{y^{\frac{1}{\beta}-1}}{\beta\gamma}dy$The limits of the function remain the same, by substitution:

$$\int_0^{\infty} 2\alpha\theta\eta\beta\gamma^{\beta}\left(\frac{y^{\frac{1}{\beta}}}{\gamma}\right)^{\beta-1} e^{-y}(1-e^{-y})^{\eta-1}\left[(1-e^{-y})^{\eta}\right]^{\alpha\theta-1}\left[1-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]\left[2-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]^{\alpha-1}\frac{y^{\frac{1}{\beta}-1}}{\beta\gamma}dy$$

By simplifying, we get:

$$\int_0^{\infty}\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\beta\gamma^{\beta}}e^{-y}[(1-e^{-y})]^{\alpha\theta\eta-1}\left[1-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]\left[2-\left[(1-e^{-y})^{\eta}\right]^{\theta}\right]^{\alpha-1}dy$$

let$z=e^{-y}$from which we find$y=-Lnz$from which we find$dy=\frac{1}{z}dz$The limits of the function resulting from the transformation are$0<z<1$:

$$\int_0^{1}\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\beta\gamma^{\beta}}z[(1-z)]^{\alpha\theta\eta-1}\left[1-[(1-z)]^{\eta\theta}\right]\left[2-[(1-z)]^{\eta\theta}\right]^{\alpha-1}\frac{1}{z}dz$$

$$\int_0^{1}\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\beta\gamma^{\beta}}[(1-z)]^{\alpha\theta\eta-1}\left[1-[(1-z)]^{\eta\theta}\right]\left[2-[(1-z)]^{\eta\theta}\right]^{\alpha-1}dz$$

suppose that$R=1-Z$*Of which*Z=1-R*find that*$dZ=dR$:

$$\int_0^{1}\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\beta\gamma^{\beta}}[R]^{\alpha\theta\eta-1}\left[1-[R]^{\eta\theta}\right]\left[2-[R]^{\eta\theta}\right]^{\alpha-1}dR$$

let$[R]^{\eta\theta}=W$from which we find$R=W^{\frac{1}{\eta\theta}}$from which $dR=\frac{1}{\eta\theta}W^{\frac{1}{\eta\theta}-1}dW$:

$$\int_0^{1}\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\beta\gamma^{\beta}}\left[\frac{w^{\left(\frac{1}{\theta\eta}\right)^{\alpha\theta\eta}}}{W^{\frac{1}{\eta\theta}}}\right][1-W][2-W]^{\alpha-1}\frac{1}{\eta\theta}W^{\frac{1}{\eta\theta}-1}dW$$

$$\int_0^{1}\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\eta\theta\beta\gamma^{\beta}}[W]^{\alpha-1}[1-W][2-W]^{\alpha-1}dW$$

$$\int_0^{1}\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\eta\theta\beta\gamma^{\beta}}[W]^{\alpha-1}[(1-W)][1+(1-W)]^{\alpha-1}dW$$

let$V=1-W$from which$W=1-V$ *Where* $dW=dV$

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$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\eta\theta\beta\gamma^{\beta}}[1-V]^{\alpha-1}[\mathrm{V}][1+\mathrm{V}]^{\alpha-1}dV$$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\eta\theta\beta\gamma^{\beta}}[1-V^2]^{\alpha-1}[\mathrm{V}]dV$$

let$V^2 = S$from which$V = S^{\frac{1}{2}}WheredV = \frac{1}{2}s^{\frac{-1}{2}}dSBy\ compensation$:

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^{\beta}}{\eta\theta\beta\gamma^{\beta}}[1-S]^{\alpha-1}S^{\frac{1}{2}}\frac{1}{2}s^{\frac{-1}{2}}dS$$

$$\int_0^1 \frac{2\alpha\theta\eta\beta\gamma^{\beta}}{2\theta\eta\beta\gamma^{\beta}}[1-S]^{\alpha-1}dS$$

$$\frac{2\alpha\theta\eta\beta\gamma^{\beta}}{2\theta\eta\beta\gamma^{\beta}}\left[-\frac{([1-S]^{\alpha}}{\alpha})\right]\begin{matrix}1\\0\end{matrix} = \frac{2\alpha\theta\eta\beta\gamma^{\beta}}{2\alpha\theta\eta\beta\gamma^{\beta}} = 1 \quad is\ pdf$$

*Now we find the moment*r$\mathrm{E}(X^r)$(TLEW)

$$E(X^r) = \int_0^{\infty} 2\alpha\theta\eta\beta\gamma^{\beta}x^{\mathrm{r}+\beta-1}\mathrm{e}^{-(\gamma x)^{\beta}}\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta-1}\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\alpha\theta-1}\left[1 - \left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]\left[2-\left[\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\eta}\right]^{\theta}\right]^{\alpha-1}dx$$

$$\int_0^{\infty} 2\alpha\theta\eta\beta\gamma^{\beta}\left(\frac{y^{\frac{1}{\beta}}}{\gamma}\right)^{\mathrm{r}+\beta-1}\mathrm{e}^{-\mathrm{y}}(1-\mathrm{e}^{-\mathrm{y}})^{\eta-1}\left[(1-\mathrm{e}^{-\mathrm{y}})^{\eta}\right]^{\alpha\theta-1}\left[1 - \left[(1-\mathrm{e}^{-\mathrm{y}})^{\eta}\right]^{\theta}\right]\left[2-\left[(1-\mathrm{e}^{-\mathrm{y}})^{\eta}\right]^{\theta}\right]^{\alpha-1}\frac{y^{\frac{1}{\beta}-1}}{\beta\gamma}dy$$

By simplifying, we get:

$$\int_0^{\infty} \frac{2\alpha\theta\eta}{\gamma^{r-1}}y^{\frac{r}{\beta}-\frac{1}{\beta}}\mathrm{e}^{-\mathrm{y}}[(1-\mathrm{e}^{-\mathrm{y}})]^{\alpha\theta\eta-1}\left[1-\left[(1-\mathrm{e}^{-\mathrm{y}})^{\eta}\right]^{\theta}\right]\left[2-\left[(1-\mathrm{e}^{-\mathrm{y}})^{\eta}\right]^{\theta}\right]^{\alpha-1}dy$$

let$z = e^{-y}$from which we find$y = -Lnz$from which we find$dy = \frac{1}{z}dz$The limits of the function resulting from the transformation are$0 < z < 1$:

$$\int_0^1 \frac{2\alpha\theta\eta}{\gamma^{r-1}}[Lnz]^{\frac{r}{\beta}-\frac{1}{\beta}}\mathrm{z}[(1-z)]^{\alpha\theta\eta-1}\left[1-[(1-z)]^{\eta\theta}\right]\left[2-[(1-z)]^{\eta\theta}\right]^{\alpha-1}\frac{1}{z}dz$$

By simplifying, we get:

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$$\int_0^1 \frac{2\alpha\theta\eta}{\gamma^{r-1}}[Lnz]^{\frac{r}{\beta}-\frac{1}{\beta}}[(1-z)]^{\alpha\theta\eta-1}\Big[1-[(1-z)]^{\eta\theta}\Big]\Big[2-[(1-z)]^{\eta\theta}\Big]^{\alpha-1}dz$$

suppose that $R = 1 - Z$ *Of which Z=1-R find that* $dZ = dR$:

$$E(X^r) = \int_0^1 \frac{2\alpha\theta\eta}{\gamma^{r-1}}[Ln(1-R)]^{\frac{r}{\beta}-\frac{1}{\beta}}[R]^{\alpha\theta\eta-1}\Big[1-[R]^{\eta\theta}\Big]\Big[2-[R]^{\eta\theta}\Big]^{\alpha-1}dR$$

Using the previous assumptions in the transformation of complex functions and the integration rule of partial functions in(Mathematical program)

$$\mu_t' = \alpha^r \sum_{k=0}^{\infty} \gamma_k [(\alpha+k)\theta]^{\frac{r}{\beta}} \Gamma\left(1-\frac{r}{\beta}\right) \qquad \dots (3)$$

3-Reliability function[8][9][11]

It is defined as a function during which a particular device can continue to work correctly (without failure) during a specified period (0, t), and the reliability function can also be known as a measure of the ability to work a certain system or part of that system continuously without stopping during A specific period of time (0, t), and the reliability function can be expressed mathematically as:

$$R(t) = pr\ (T > t\ ) \qquad \dots (4)$$

t: the operating time of the device, which is greater or equal to zero.

T: The accumulated lifetime of the device during and its value is limited (0,t .).

The formula for the dependency function in continuous distributions is

$$:R(t) = \int_t^{Maxt} f(u)du \qquad \dots (5)$$

The reliability function has such properties as:

- *Its value is between zero and one. It is a probability function:* $0 \quad \leq R(t) \leq 1)$
- It is inversely proportional to time, that is, as the time of operation of the machine progresses, its value decreases:

$$R(t_1\,) > R(t_2) > R(t_3) > \cdots > R(t_\infty)$$

The reliability function of the proposed distribution(TLEW):

$$\boldsymbol{R}(\boldsymbol{x}) = \mathbf{1} - \mathrm{F}_{\mathrm{TLEW}}\,(X)\ = \mathbf{1} - \left\{\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\left[2-\left(1-\mathrm{e}^{-(\gamma x)^{\beta}}\right)^{\theta\eta}\right]\right\}^{\alpha} \quad \dots (6)$$

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4-Data classification [4] [3]

4-1complete data

Complete data means all the data (sample units) that have been put to a life test. The test stops after the failure of all units, and the failure time for each sample unit is known and observed.

The maximum possibility function for this type of data

$$L = \prod_{i=1}^{n} f(t,\theta) \ldots \qquad (7)$$

since:

$f(t,\theta)$:The probability density function of the distribution

The disadvantages of using complete data is to monitor all the items of the sample subject to the life test, and that this results in (loss of time, cost, effort, and sometimes destructive testing) so it can be replaced by complete data with control data.

4-2censored Data

Two types of monitoring data will be discussed, although there are several types, and it is possible to clarify the monitoring data as follows:

:

- **type- I -censord Data**

This type is called Time censored data. In this type of data, the observation time is fixed (t_0) and predetermined and varies from one experiment to another for all sample data (sample units) subject to the test.

When testing the life of n of units at zero time, we will watch (watch) the work of the sample units until the predetermined fixed time expires, i.e. the life experience (the test) stops.

And that the units that failed the test are m units, and that m is a random variable that we cannot know or determineexcept after the expiry of time (t_0), and (n-m) is the number of units left after time (t_0)

$0 < t_1 < t_2 < t_3 < \cdots \ldots \ldots \ldots . < t_m < t_0$

since:

is the failure time of unit i before time t_0

The greatest possibility function for observation data of the first type is:

$$L = \frac{n!}{(n-m)!} \prod_{i=1}^{m} f(t,\theta)[S(t_0)]^{n-m} \qquad \ldots . (8)$$

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since:

$S(t_0)$Reliability function over timet_0

$f(t,\theta)$failure density function

$(\text{n}-\text{m})$Number of units left after timet_0

And that this type of samples is concerned with the experiences of life tests in which the cost is increasing.

- **type – II-censored Data**

This type of data is called failure censored data.

In this type of data, a predetermined number of sample units that is being monitored is determined (r) fixed units. Therefore, the time of these units t_r is a random variable that cannot be determined. When the life test begins at zero time, we will monitor (watch) the work of the units. r We stop the experiment after obtaining r of the failed units that were previously specified, while the remaining units after time t_r are (n-r)

The maximum possibility function for this type of data is:

$$L = \frac{n!}{(n-r)!}\prod_{i=1}^{m} f(t,\theta)[S(t_r)]^{n-r} \qquad \dots (9)$$

since:

$0 < t_1 < t_2 < \ldots < t_r$

$f(t,\theta)$: failure density function.

$S(t_r)$: Reliability function at time t_r.

(n-r): Number of units left (non-failed) after the test stops on failure Unit No. r.

These samples are often concerned with examining expensive units or those in which the examination is destructive.

5-(Moments Method)[1] [2] [5] [6]

The moment method is one of the most famous methods used in estimating the parameters of the statistical distributions. This method depends mainly on finding k of the moments of the community in terms of k of parameters, then equating the moments of the society with the corresponding moments of the sample, thus we get k equations (the number of parameters) and by solving the resulting equations we get on the required capabilities.

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It is known that the k-order of the Topp-Leon-Exponential-Weibull distribution is:

$$EX^k = \mu_t' = \alpha^r \sum_{k=0}^{\infty} \gamma_k [(\alpha + k)\theta]^{\frac{r}{\beta}} \Gamma(1 - \frac{r}{\beta})$$

since:

$$EX = \alpha \sum_{k=0}^{\infty} \gamma_k [(\alpha + k)\theta]^{\frac{1}{\beta}} \Gamma\left(1 - \frac{1}{\beta}\right) \qquad \dots (10)$$

$$EX^2 = \alpha^2 \sum_{k=0}^{\infty} \gamma_k [(\alpha + k)\theta]^{\frac{2}{\beta}} \Gamma\left(1 - \frac{2}{\beta}\right) \qquad \dots (11)$$

$$EX^3 = \alpha^3 \sum_{k=0}^{\infty} \gamma_k [(\alpha + k)\theta]^{\frac{3}{\beta}} \Gamma\left(1 - \frac{3}{\beta}\right) \qquad \dots (12)$$

$$EX^4 = \alpha^4 \sum_{k=0}^{\infty} \gamma_k [(\alpha + k)\theta]^{\frac{4}{\beta}} \Gamma\left(1 - \frac{4}{\beta}\right) \qquad \dots (13)$$

$$EX^5 = \alpha^5 \sum_{k=0}^{\infty} \gamma_k [(\alpha + k)\theta]^{\frac{5}{\beta}} \Gamma\left(1 - \frac{5}{\beta}\right) \qquad \dots (14)$$

Equations (10) to (14) represent the moments of the community with the number of parameters

$$m_1 = \frac{1}{n} \sum_{i=1}^{r} X_i$$

$$m_2 = \frac{1}{n} \sum_{i=1}^{r} X_i^2$$

$$m_3 = \frac{1}{n} \sum_{i=1}^{r} X_i^3$$

$$m_4 = \frac{1}{n} \sum_{i=1}^{r} X_i^4$$

$$m_5 = \frac{1}{n} \sum_{i=1}^{r} X_i^5$$

The moments of the sample are represented by the number of parameters

By equating the moments of the population with the moments of the sample, we get the following equations

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$$:\alpha \sum_{k=0}^{\infty} \gamma_k [(\alpha+k)\theta]^{\frac{1}{\beta}} \Gamma\left(1-\frac{1}{\beta}\right) = \frac{1}{n}\sum_{i=1}^{r} X_i \; \ldots (15)$$

$$\alpha^2 \sum_{k=0}^{\infty} \gamma_k [(\alpha+k)\theta]^{\frac{2}{\beta}} \Gamma\left(1-\frac{2}{\beta}\right) = \frac{1}{n}\sum_{i=1}^{r} X_i^2 \; \ldots (16)$$

$$\alpha^3 \sum_{k=0}^{\infty} \gamma_k [(\alpha+k)\theta]^{\frac{3}{\beta}} \Gamma\left(1-\frac{3}{\beta}\right) = \frac{1}{n}\sum_{i=1}^{r} X_i^3 \quad \ldots (17)$$

$$\alpha^4 \sum_{k=0}^{\infty} \gamma_k [(\alpha+k)\theta]^{\frac{4}{\beta}} \Gamma\left(1-\frac{4}{\beta}\right) = \frac{1}{n}\sum_{i=1}^{r} X_i^4 \; \ldots (18)$$

$$\alpha^5 \sum_{k=0}^{\infty} \gamma_k [(\alpha+k)\theta]^{\frac{5}{\beta}} \Gamma\left(1-\frac{5}{\beta}\right) = \frac{1}{n}\sum_{i=1}^{r} X_i^5 \quad \ldots (19)$$

And equations (15) to (19) cannot be solved by ordinary analytical methods, so one of the numerical iterative methods will be used and using the fsolve function in Matlab program to find the estimator of the reliability function of the Tobelion exponential-Whipple distribution as follows:

$$\hat{R}_{\text{TLEW}\_MOM} = 1-\left\{\left(1-\mathrm{e}^{-(\gamma_{MOM}\,x)^{\beta_{MOM}}}\right)^{\theta_{MOM}\,\eta_{MOM}}\left[2-\left(1-\mathrm{e}^{-(\gamma_{MOM}\,x)^{\beta_{MOM}}}\right)^{\theta_{MOM}\,\eta_{MOM}}\right]\right\}^{\alpha_{MOM}} \quad \ldots (20)$$

6-Practical application

The data was taken by the Medical Devices Maintenance Department in the Technical Affairs Department of the DhiQar Health Department, with a large sample size of (n=100) Views of the nebulizer devices, the Ministry of Health / DhiQar Health Department, and the sample observations represent the time of operation of the devices until failure In weeks for each device, as for the time period for which these observations were calculated, it was 12 months for the year 2021, which are, respectively, a month (12, 11, 10, 9, 8, 7.6, 5, 4, 3, 2, 1)

6-1Real data analysis

To estimate the reliability of real control data of the second type at a size of 100 and an amputation rate of 30% using the moment method, and after using the mathematical equations numbered from (16) to (19) that represent the estimations of the parameters of the proposed distribution TLEW by the moment method MOM, and in light of that, the reliability function was estimated By the moment method, the results of the estimation were as in the table below:

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Table (1) The values of the real and estimated reliability by the moment method

	t_i	R_Real	R_ MOM	t_i	R_Real	R_ MOM
n=100 r=40	10.22	0.98182	0.97457	19.32	0.31982	0.35423
$2.8=\widehat{\alpha}$	11.22	0.97001	0.96026	20.14	0.31616	0.35092
$3.3=\widehat{\theta}$	11.29	0.95818	0.94658	20.44	0.23317	0.27475
$3.4=\widehat{\beta}$	12.11	0.93305	0.91885	20.86	0.21626	0.25885
$\widehat{\gamma}=1.4$	12.12	0.91292	0.89751	21.29	0.19791	0.24139
$\widehat{\eta}=5.1$	12.14	0.89306	0.87702	21.43	0.19555	0.23913
	12.22	0.86213	0.84596	21.57	0.11464	0.15829
	12.35	0.81786	0.80283	22.14	0.10100	0.14376
	18.14	0.47215	0.48976	23.71	0.06060	0.09806
	18.33	0.46586	0.48421	24.19	0.06034	0.09775
	18.57	0.46434	0.48286	24.34	0.05442	0.09056
	18.57	0.38356	0.41130	25.14	0.01565	0.03673
	19.12	0.37406	0.40285	25.22	0.01390	0.03376
	19.23	0.33762	0.37025	25.71	0.01170	0.02989
	19.26	0.32336	0.35742	26.57	0.01086	0.02835

From Table (1) at the sample size of 100 and the cut -off rate of 30% and the values of the parameters estimated by the moment method were $\hat{\alpha} = 2.8$, $\theta = 3.3$, $\beta = 3.4$, $\hat{\gamma}=1.4$ and $\hat{\eta} = 5.1$ the true reliability value was when (t_1=10.22)and(t_2=11.22) ranging between (0.98182) and (0.97001), respectively, while the estimated reliability value by the torque method at the same time ranged between (0.97457) and (0.96026). The real value starts decreasing gradually until it reaches the lowest reliability before stopping at (i) time, i.e. when (t_{29}=25.71) and (t_{30}=26.57)if the value of

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the real reliability R_Real ranges between (0.01170) and (0.01086) As for the reliability estimated by the torque method At the same time, it ranged between (0.02989) and (0.02835), respectively

- Decreased values of the reliability function with time in a clear manner and this is what matches the behavior of this function as it is decreasing with time
- The average time between the reliability function estimated by the torque method is equal to (0.024546), while the variance is equal to (0.001175), meaning that the difference between any two reliability time periods is very little difference and its variance as well, and this indicates a convergence between the continued work of the devices.
- The average time between the reliability function estimated by the torque method is equal to (0.024546), while the variance is equal to (0.001175), meaning that the difference between any two reliability time periods is very little difference and its variance as well, and this indicates a convergence between the continued work of the devices.
- • The average time between the real dependency function is equal to (0.024364), while the variance is equal to (0.001094), meaning that the difference between any two time periods of the dependency function is a small difference and its variance as well, and this indicates a convergence between the continuation of the work of the devices.

7-Conclusions:

1-The study showed that the distribution-generating function (Topp Leone) is a flexible function in generating more suitable distributions for applied phenomena compared to other functions.

2-The study concluded that the average failure times for all devices taken in the study sample (nebulizer) amounted to 02450. The discrepancy amounted to 0.0012, meaning that the difference between any two successive periods of failure times is very small and its variance is close to zero, and therefore the reliability of all types of devices is close.

3-The use of the mean integral error squares (IMSE) criterion gives more accuracy than the other criteria because it measures the mean squares of error at the total time Ti

8-Recommendations:

From the conclusions reached, the researcher recommends the following:

1-Using the moment method at a small and medium sample size in order to estimate the reliability of control data of the second type in the case of using the distribution-generating function (Topp Leone(

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2-Using nebulizer devices that are less expensive and more abundant than other types of devices, because the risk ratio between two time periods is very low, meaning that the reliability is close between the different types of devices.
3-Adoption of the mean of integral error squares (IMSE) criterion, as it measures the mean squares of error at the total time (Ti.(
4-Using Bayesian estimation methods (EM algorithm, Downhill Simplex algorithm(
To estimate the reliability of the proposed distribution (TLEW) and to measure the reliability function of the first type of control data and complete data.

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