JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

Iterative Image Reconstruction Algorithms on a Multi-core system based on PSNR optimization

[1]**S. Kartheeswaran**, Assistant Professor, Department of Computer Science, Ayya Nadar Janaki Ammal College(Autonomous), Sivakasi, Tamilnadu, India. *skarthiphd@gmail.com*

[2]**A. Bharathi Lakshmi**, Assistant Professor & Head of IT, V.V.Vanniaperumal College For Women, (Autonomous), Virudhunagar, Tamil Nadu, India. *bharathilakshmi@vvvcollege.org*

[3]**D. Christopher Durairaj**, Associate Professor, Research Centre in Computer Science,V.H.N.S.N College (Autonomous), Virudhunagar - 626001. *kesterkaren@gmail.com*

ABSTRACT

Iterative reconstruction methods applied to Image reconstruction in Computer Tomography is versatile and result in possibly better-quality images than FBP in limited number of projections. But these methods suffer from long reconstruction time. The main objective of this research is to reduce the time complexity by improving the performance of reconstruction algorithms. To minimize the reconstruction time, in the three algorithms - ART, MLEM and MAPEM parallel programming technique is implemented in a shared memory processing environment and pART, pMLEM and pMAPEM algorithm are introduced.

Keywords—Image Reconstruction, Iterative Image Reconstruction, Algebraic Reconstruction Technique, Statistical Iterative Methods, Macimum Likelihood Maximum Estimation, Maximum a Posteriori Expectation Maximization, Parallel Processing, OpenMP, Shared Memory Processor

I. INTRODUCTION

Image reconstruction is one type of method to arrange the images in exact position without noise. The reconstruction follows different methods such as Analytical and Iterative to produce the quality images. Analytical method such as Back Projection (BP) or Filtered Back Projection (FBP) is used for different imaging modalities such as CT and PET in clinical settings because of the speed and easy implementation. The analytical image reconstruction method assumes that the images are noise free and there is chance to find an exact mathematical representation for the known projections. For noisy projection data as well as for limited number of projections, the FBP method of image reconstruction shows very poor performance. Hence currently there is considerable interest to evaluate the use of other reconstruction methods for medical imaging techniques.

On the other hand, Iterative Methods are based on optimization strategies incorporating specific constraints about the object and the reconstruction process. The iterative reconstruction techniques perform better than the FBP method when reconstruction is attempted with limited number of projection data. Iterative method is further classified into Algebraic and Statistical Method. The linear system problems are effectively solved by Algebraic method. Algebraic Reconstruction Technique (ART), Multiplicative Algebraic Reconstruction Technique (MART), Simultaneous Iterative Reconstruction Technique (SIRT), Simultaneous Algebraic Reconstruction Technique (SART) are some of the Algebraic algorithms. Statistical image reconstruction plays a vital role in medical field. Statistical method is a kind of iterative method which is divided into weighted and likelihood [1].

Iterative process includes different methods for statistical reconstruction technique in the form of poisson process. The poisson statistical model supports the maximum posterior work, maximum likelihood, context-based Bayesian frame work. Expectation Maximization (EM) is one type of statistical method for image reconstruction process. The statistical model supports the iterative process to identify maximum posteriori parameters. EM Algorithm is an iterative algorithm that is often used for estimating parameters of Gaussian Mixture Model [2]. The Gaussian mixture method allows the expectation maximization model to reconstruct the image. Large scale information sets are not appropriate in reconstruction process, because the computational cost is high. Maximum likelihood is a technique to analyze the missing data in the multiple datasets. The best fit models can easily analyze in the multiple data sets. The analytical procedure is hard to solve maximum likelihood problem, when the value is set as zero for the partial derivatives. The expectation process can access different equations instead of the mathematical process.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

Figure 1 illustrate the steps involved in the iterative image reconstruction, in which the initial image estimate is acquired using the projection. With the initial image the unfilled values are calculated using any one of the iterative algorithms and compared with the measured data. The Discrepancies between them is applied with back projection to form the new image and the new image will act as an initial image for next iteration. The process will be repeated till the error between the data has been reduced.

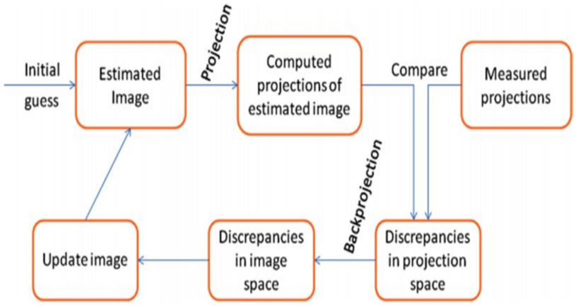

Figure 1: Steps involved iterative image reconstruction methods

The medical image acquired using CT, PET and MRI scan currently scans the object at each degree, which leads to place the object for long time. And also, existing algorithm reconstructs the image fast and in easy way. But not for limited number of angles. The limited number of angles data can be reconstructed with the help of iterative image reconstruction algorithms. The problem identified are the need for a greater number of iterations and the size of the initial image used to reconstruct an image is large.

The image reconstruction technique is suitable for identifying the disease based on scanned image process and the advantage of reconstruction technique is to display images in good quality. Existing techniques has some limitations such as, Higher signal to noise ratio, and increased resolution counteract and Expensive in mechanical scanning, Model Based Iterative Reconstruction (MBIR) [3] is computationally expensive and it requires high level of optimization to achieve their intended performance.

To overcome the drawback of existing methodologies, the proposed work is developed. The research is mainly used to improve the accuracy level for medical applications. So, the iterative image reconstruction techniques such as ART, MLEM and MAPEM are implemented for limited number of angles. The number of iterations is optimized for all the three algorithms and to prove the image quality the reconstructed image is compared with existing system such as FBP, SIRT and SART. ART is a linear system that reconstructs an image iteratively with best Peak to Signal Noise Ratio (PSNR) value. The Statistical data gives more accurate data than linear system. So an attempt is done using MLEM algorithm. MAPEM algorithm is relatively easy to incorporate the prior knowledge to reconstruction process. All the three systems developed suffer from high computation time. To reduce the reconstruction time a parallel computational method is proposed. It can be mainly used in medical field and material science testing process.

The main objective is to reduce the reconstruction time of an artifact free image from the data obtained at limited number of angles. A study on research papers were instrumental to focus on ART, MLEM and MAPEM iterative algorithms. As the algorithms are iteration based a work on optimizing the iteration has been done based on the PSNR value. After attaining the iteration limit shared memory parallelization has been implemented to reduce the time. To prove the parallel computing performance Amdahl’s Law has been used to analyze the multi-core platform.

The parallel processing is implemented in Fedora 11, a popular open-source Linux based Operating system. One of the main objectives of this research to check the proposed algorithm in sequential and parallel computing mode. To satisfy a parallel computing tool in MATLAB 11a is used. The multi-core environment has been created using 16 cores 64-bit AMD Opera processor.

The Shared Memory Parallelization (SMP) becomes a well-suited approach to attain the better performance with improved runtime in the applications of computer vision and image processing [4]. When compared to the distributed memory architectures, the SMP has very less drawbacks by using the inter processor communication. It is more advantageoustoutilizetheSMParchitectureforarelativelylargedatasizewithOpenMP. In recent times, a C++ code is developed by utilizing the OpenMP for content-based image retrieval for the exploitation of shared memory parallelization. The OpenMP for Multi-Processing is used through the collaborative exertion between the interested parties from the hardware and software [5].

The OpenMP supports Fortran, C and C++ that make Shared memory parallelism easier. OpenMP is based on the Unix pthreads implementation [4]. The OpenMP defines a set of compile directives, environmental variables and Runtime Library that is based on the fork/join programming model.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

The present study compares the three algorithm considered for study. Parallel computing is emerging as a principal theory in high performance computing. Recently SMP has been utilized for parallel computing. The SMP environment consists of a number of processors accessing one or more shared memory modules. For processing the large size of data, the SMP has some benefit over the distributed memory parallelization

II. METHODOLOGIES

A. ART

In the image reconstruction process, the linear system problems are effectively solved by using one of the iterative methods called ART technique. Thus, the ART technique is mainly categorized into either block or simultaneous or sequential iterative [6]. The ART method has a long history, literature and also derived as fully sequential method. This sequential characteristic is derived independently for reconstructing the image. Originally it was proposed by Kaczmarz [7], and its independent usage in image reconstruction was mentioned by Gordan, Bender and Herman [8]. The ART was first of its kind based on the iterative procedures and was first used in the EMI brain scanner developed by House Field. Currently the ART algorithm is used only in some specific application such as the case of limited view angle reconstruction. Its resulting image can compensate the drawbacks of its inefficiency of computing operations. Therefore, it is suitable enough for research and educating experiments which involve limited object matrix dimensions. The vector of unknowns is updated at each equation of the system, after which the next equation is

converges to a solution of this system. If the system is inconsistent, every sub-sequence of cycles through the system converges, but not necessarily to a least square solution [9].

ART system is considered as a linear equation Ax = b where x is a vector holding the calculated pixel values that is considered as a variable, A is described as an image process, b is a vector that contains the measured angular projections that is none other than the sinogram.

B. MLEM

Emission Computed Tomography (ECT) is a CT applied to nuclear medicine. It uses the decay of radioactive materials to image properties of the body's physiology. The projections are obtained using the detector ring around the object and are reconstructed using various reconstruction algorithm.

The image reconstruction difficulty for ECT can be observed as statistical estimation problem [10]. In statistics an EM algorithm is an iterative method to find Maximum likelihood or maximum A Posteriori estimates with limited number of projections.

In the MLEM algorithm, the collected projection data plays an important role. In a SPECT scanner, the size of the projection data depends on both the quantity of detectors and the corresponding quantity of angles [11]. If the medical imaging modalities have b number of detectors and are measured at angles, then the total number of counts in the projection data is J = a∗b. For ease of calculations, this vector is generally epitomized by way of a column vector. In PET, there is a ring of detectors around the patient which measures the annihilation event. An event is recorded only if the two events occur within a time window. If the detector ring has N detectors, the number of counts in projection data is given by J = N(N − 1)/2. The image matrix could be accessed as a column vector I with I = nx ∗ ny elements. Practitioner agreed that such emissions follow a Poisson model. Therefore, the unknown total number of emission events in the i^{th} pixel, $\hat{x}$(i) represents a Poisson random variable, with mean ¯ x(i). The system matrix characterizes the probability distribution of the projection data. Hence elements of the system matrix p(i,j) embodies the likelihood of emission i to be detected by detector j. The algorithm requires an initial estimate x0, and the maximization condition to iteratively improve the estimate. Researchers have used a diversity of initial estimates to reach the results faster. The main formula for MLEM algorithm is given in Equation 1.

$$x^{n+1}(i) = x^n(i) \sum_{j=1}^{j} \frac{n(j)p(i,j)}{x^n(x')p(i',j)} \qquad (1)$$

C. MAPEM

The image reconstruction using MAPEM will improve the resolution and reduce the noise of the images. MAPEM is introduced with a prior knowledge as a constraint that favours convergence of the expectation maximization algorithm process called as regularization. The prior is usually chosen to penalize the noisy images. The goal of the required criterion is simultaneously maximized which leads to a scheme called One Step Late (OSL) algorithm. The priori term is

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

the derivative of an energy function chosen to enforce smoothing and a value is chosen to modulate the importance of the priori.

In maximum A posteriori (MAP) estimation, one tries to maximize the posterior instead of the likelihood, so the MAP estimator can be written as Equation 2 [12].

$$\begin{aligned}\hat{\theta}_{MAP} &= \arg\max_{\theta\in\Theta} \log P(\theta|y) \\ &= \arg\max_{\theta\in\Theta}(\log P(y|\theta) + \log p(\theta)) \\ &= \arg\max_{\theta\in\Theta}(L(\theta) + \log p(\theta)) \qquad (2)\end{aligned}$$

Under the estimation theory the expectation maximization is a general optimization technique, to analyse the maximum likelihood estimation of parameters based on the stactical model. The E–Step, and the M–Step are the two steps involved in the EM algorithm. For every iteration in EM, the re-estimated parameters provides a least log likelihood value same as the previous values. The E-step, estimated parameter will calculate the maximum likelihood based on true value. In the M-step the calculated value in the E-step of maximum likelihood is used to estimate the parameters [13].

The parallel beam which determines a view or direction of the projection will be measured by an array consisting of a number of detectors, which will determine the number of sampling of the beam. Since the X-ray source can be assumed as point of the beam that pass a small fan shaped track can be assumed as a small beam itself. A small beam here is simply called as a ray. So a projection beam consists of many projection rays.

The two steps will iterate continuously until the specified convergence is occurred. Applications of the EM algorithm include estimating class–conditional densities in supervised learning settings, density estimation in unsupervised clustering and for outlier detection purposes. The Spatial EM algorithm are based on the utilizes median based location and rank based scatter estimators to replace the sample mean and covariance matrix in the M – Step of an EM algorithm. Hence it improves the stability of the finite mixture model and it is well robust to outliers. There are also many good tutorials on EM algorithms. Thus the optimal solution, the maximum likelihood estimation directly leads to the accurate quantification as well as the reliability.

E-Step Procedure: Estimates the expectation of the missing value i.e. unlabelled class information. This step corresponds to performing classification of each unlabelled document. Probability distribution is calculated using current parameter.

The estimate is given from the previous iteration (m),

$$Q\left(\theta|theta^{(m)}\right) = E_{x|y,\theta^{(m)}}[\log P(X|\theta)]$$

M-Step Procedure: Calculates the maximum likelihood parameters for the current estimate of the complete data.

$$Maximize\ Q\left(\theta|theta^{(m)}\right) + \log P(\theta)\ Over\ \theta \in \Theta$$

to find

$$\theta^{(m+1)} = \arg max_{\theta\in\Theta}\left(Q(\theta|theta^{(m)})\right) + \log P(\theta)$$

First the iterative reconstruction methods are applied with reduction in number of iterations. In second approach special hardware techniques were employed to do back projection on an event-by-event basis targeting at the speed of computations. Both these two approaches are not free from major problems. One significant problem was that computational speed arrived at was not remarkable but very limited. Moreover, it requires tremendous amount of computation. Hence the third approach that is parallel processing is considered promising and more reliable.

III. System Design and Implementation

A. Data Set

The input of the iterative image reconstruction uses Shepp Logan phantom data set in size 256 x 256 pixels to analyze the optimal iteration, quality of image and parallel computing performance among the image reconstruction algorithms. Figure 2 represents the 256 x 256 pixel Shepp Logan Phantom. ART, MLEM and MAPEM algorithm has been used to reconstruct the images and parallelized to reduce the reconstruction time.

Figure 2:The Shepp Logan Phantom of 256 x 256 pixels

The sinogram for the considered image set is obtained using the radon function in Matlab. Radon function produces the raw data that is sinogram of the given image in the specified angles. The projections are obtained at 18^0, 15^0, 12^0, 9^0 and 6^0 angles for the study. The radon transform at 18^0 angle produces 10, 15^0 angle produces 12, 12^0 angle produces 15, 9^0 angle produces 20, 6^0 angle produces 30 numbers of projected data. The projection data size used to reconstruct an image is very large. The large size data set is the main factor that affects the reconstruction time. Figure 3 shows the projected data of the considered image obtained using five different angles 6^0, 9^0, 12^0, 15^0 and 18^0.

6^0 9^0 12^0 15^0 18^0

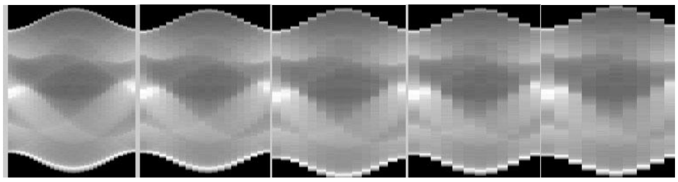

Figure 3: The projection of Shepp Logan Phantom Image

B. Design and Implementation

ART

The iterative ART algorithm executed under single core is considered as Sequential version. To standardize the number of iterations it has been optimized based on the PSNR value. After optimizing the iteration, image is reconstructed using the algorithm calling the MEX function using single core. The time taken to reconstruct the image is noted for each size of image in different angles. Under this process the projection data is directed into the MEX function, as the iteration has been optimized as the algorithm is repeatedly called to the optimized value. Projection values are calculated and correction for new estimation of the projections is carried out for all the projection values in a single iteration and the process is repeated for all iterations.

W,v,p,numIter are the input parameter used where W is the weighted matrix. The weighted matrix size is size of image x size of the projections. As the size of the matrix is large the matrix is inputted as sparse matrix. v is the vector that hold the size of the output image and initialized to zero. p represents the projection data and numIter gives the number of times the algorithm to be repeated. retV is the output parameter that is used to store the pixel values of a reconstructed image.

The Algorithm begins by initializing the output parameter to zero. As the process depends on sparse matrix the full matrix data is retrieved and normalized before entering the process. Core part of the ART manipulation as explained above is repeatedly done for the specified numIter and for all projections p.

pART

The operation is performed based on the projection data, the reading process is started and continued by calculating the corrections value. Then, sequentially the estimated correction is applied in all the projections iteratively. Calculating and applying correction for each projection is alone parallelized. The projections data are concurently read by the specified core either 2, 4 or 8 and values are calculated for the projection in each thread. Depending on the core created the worker thread are assigned.

MLEM

With the projection data obtained and the Initial guess of the estimate MLEM algorithm is passed as input to reconstruct an image. MLEM is an iterative reconstruction method based on statistical data and the required iterations

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

have been optimized based on the PSNR value. This is followed by calling a MEX function consisting of MLEM algorithm in a single core. The time taken to reconstruct an image is noted for each size at different angles of projections. As described in the methodology the forward projection is followed by comparison, Back projections, normalization and value updating for all numbers of projection data. The methodology of MLEM begins with the initial guess of the image. The forward projection stimulates the measurements from the estimate dataset. The measured and the calculated is compared. The error is corrected by applying back projection which updates image estimate. This process is repeated until convergence.

pMLEM

The iterative statistical pMLEM algorithm implemented under the multi-core is considered as parallel version. Similar steps exist as in sequential version. Optimizing the iterations are carried out in Matlab and image reconstruction algorithm MLEM for calculating each pixel value for each projection is done in parallel by calling the MEX function. In the MEX function the number of cores are set to 2, 4 or 8 using the OMP_SET_NUM_THREAD (num_thread) function. Initially the program starts with single core. #pragma omp parallel directive is used to create a master and a worker thread. The Master thread co-distributes the measured and the calculated projection data to the worker thread. After updating the value each worker updates its value and the master thread updates the final value in the vector.

MAPEM

The algorithm initially reads the projection data, apply E-step and M-step and update the vector. It then checks if error exists. If error exists, the process continues else the projection image will be displayed or else the process will be continued. The EM technique is separated into two sections as E-step and M-step. The E-step, calculates the maximum likelihood based on true value. The M-step update the values using the value obtained from the E-step to estimate the parameters. Once the error value is not found it will be moved for the smoothness process for improving the resolution and to reduce the noise of the projection images. This operation will happen sequentially resulting in time complexity.

pMAPEM

The parallel version of MAPEM implemented on a multi-core environment is termed as parallel Maximum A Posteriori Expectation Maximization (pMAPEM). pMAPEM is same as the sequential operation, but with the help of OpenMP the operation will be functioned in multiple processes. For all the projection the E- step followed by the M-step will be carried out. In this method the operation will be done in parallel to reduce the time complexity. The master thread controls the worker thread by assigning the job to them. At the end the master thread collects all the values from worker thread and updates the value.

IV. RESULTS AND DISCUSSION

Iterative reconstruction algorithms ART, MLEM and MAPEM executed sequentially and in parallel as pART, pMLEM and pMAPEM is given in Figure 4. The input of the reconstructed Shepp Logan phantom is the projected data provided in Figure 3.

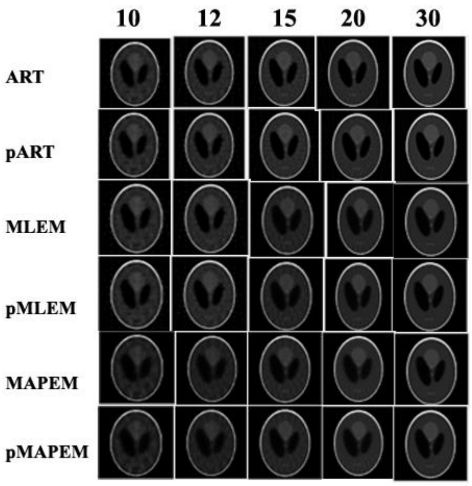

Figure 4: Image reconstructed using ART, pART, MLEM, pMLEM, MAPEM, pMAPEM

In this figure the image reconstructed using ART is at first row and its corresponding parallel version pART is at next row. Similarly, other two MLEM and MAPEM reconstructed images are placed at third and fifth row and its parallel version is next to them. Each column represents input sinogram obtained using number of projections considered for study.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

A. Optimized Iteration

The iteration for all three algorithms has been optimized to get the better-quality image. The sequential and parallel version algorithms give the same quality of image.

TABLE I. OPTIMIZED ITERATION TO RECONSTRUCT PHANTOM WITH HIGH PERCEPTUAL FIDELITY

Algorithm	10	12	15	20	30
ART	158	142	124	46	24
MLEM	59	40	51	40	69
MAPEM	61	39	45	29	61

The optimized number of iterations has been tabulated in Table I. The Table clearly shows that MLEM algorithm reconstructs enhanced image in minimum number of iterations compared to ART and MAPEM.

B. Peak to Signal Noise Ratio

Peak to Signal Noise Ratio proves the higher perceptual fidelity of the reconstructed image using ART, MLEM and MAPEM. The PSNR calculated for the reconstructed image using the above said algorithm both in sequential and parallel version is tabulated in Table II. The table proves that MLEM with minimum number of iterations reconstructs image in same quality as other two algorithms reconstruct.

TABLE II. MEASURED PSNR VALUES WITH 10, 12, 15, 20 AND 30 NUMBER OF PROJECTIONS

Algorithm	PSNR				
	10	***12***	***15***	***20***	***30***
ART	69.3787	69.5936	70.676	70.3131	71.0605
pART	69.3787	69.5936	70.676	70.3131	71.0605
MLEM	69.5807	69.7042	70.5335	70.4742	71.0236
pMLEM	69.5807	69.7042	70.5335	70.4742	71.0236
MAPEM	69.5812	69.7039	70.251	70.3148	71.0151
pMAPEM	69.5812	69.7039	70.251	70.3148	71.0151

As image size increases, also PSNR value increases. The PSNR value obtained for the reconstructed image is above 60db even with limited number of projections. The image reconstructed using sequential and parallel versions produces same PSNR value.

C. Time Complexity

Time complexity is measured as the time taken by ART, MLEM and MAPEM for 256 x 256 image size to reconstruct in sequential and parallel using 2, 4 and 8 cores in AMD processor. This time complexity of the reconstructed images under 1, 2, 4 and 8 cores in regard to the number of projections is tabulated in Table III for all three algorithms.

TABLE III. TIME TAKEN BY ART, PART, MLEM, PMLEM, MAPEM, PMAPEM TO RECONSTRUCT SHEPP LOGAN PHANTOM IMAGE USING VARIOUS NUMBER OF PROJECTIONS

Algorithm	Cores	10	12	15	20	30
ART	1	1609.1	1699.73	1889.8	918.3131	723.983
pART	2	1118.1	972.613	1365.52	658.447	524.681
	4	905.076	866.0894	943.866	444.67	314.839
	8	683.986	470.454	557.619	273.9013	195.646
MLEM	1	522.894	462.973	750.215	709.861	2134.4
pMLEM	2	508.943	341.971	743.438	766.535	1987.55
	4	375.436	266.157	435.619	550.758	1304.25
	8	200.919	179.447	264.467	485.687	797.848
MAPEM	1	726.522	532.309	727.098	532.317	771.465
pMAPEM	2	502.087	332.341	502.65	332.347	463.192
	4	398.953	297.146	399.495	297.143	447.483
	8	198.488	145.926	199.045	145.934	259.513

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

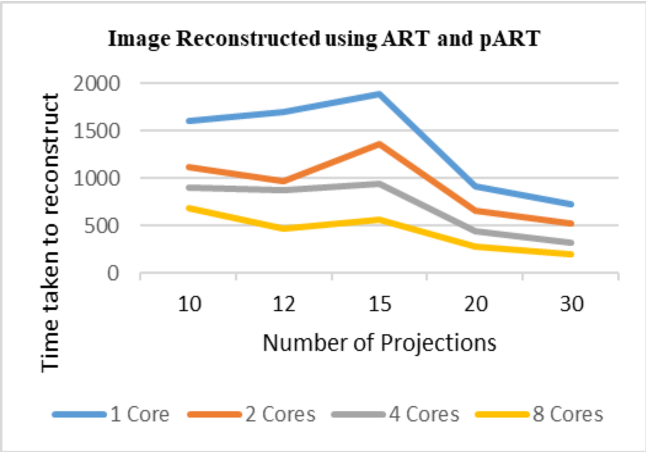

Figure 5: The reconstruction time obtained by ART and pART

A graph has been plotted for the given values in Table III and shown in the Figure 5, Figure 6 and Figure 7 for ART and pART, MLEM and pMLEM, MAPEM and pMAPEM respectively.

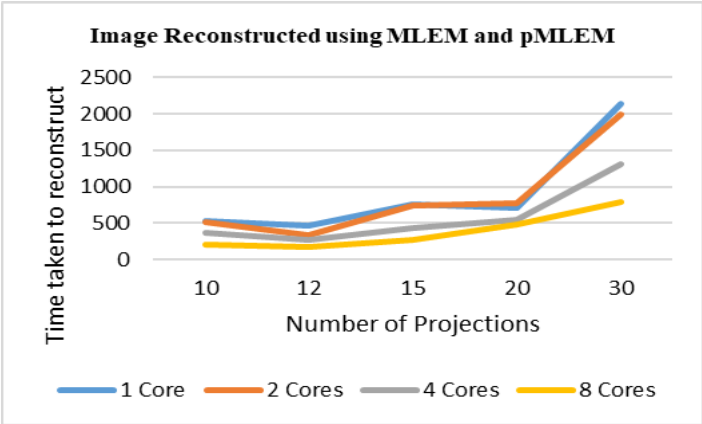

Figure 6: Time achieved to reconstruct 256 x 256 image with projection data obtained at 18^0, 15^0, 12^0, 9^0 and 6^0 angles using MLEM and pMLEM.

From the observation evidently it proves that the time gradually reduces as the number of cores increases, for a given set of projections. The time complexity of the system implementing parallel processor has got a considerable reduction of time consumptions which is certainly a high degree of utility to the user. The specific value of this finding is that the maximum number of core reconstructing the image is faster even for minimum number of projections.

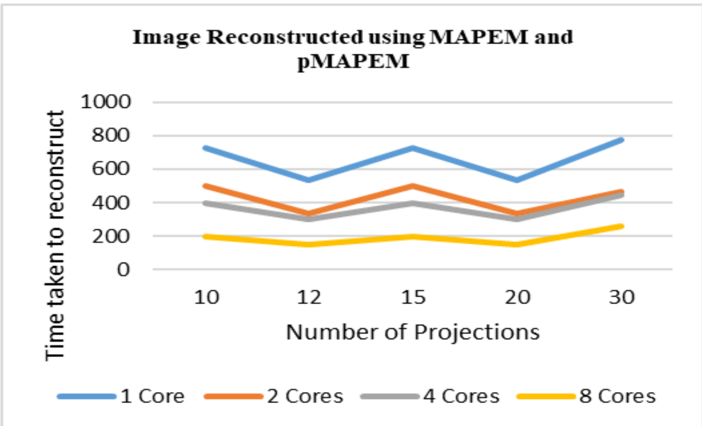

Figure 7: A plot of reconstruction time achieved using MAPEM and pMAPEM.

As a result, for a limited number of projections, it takes high time complexity in single core execution. On the whole, for each number of projections for 256 x 256 size considered for study, the time has been reduced.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

D. Speed up

The analysis of speedup calculation for pART, pMLEM and pMAPEM is obtained by applying the Amdahl's law for the serial and parallel time. Table IV lists the speed up calculation for the image reconstructed.

TABLE IV. SPEED UP CALCULATION FOR 256 X 256 SIZE RECONSTRUCTED SHEPP LOGAN PHANTOM IMAGE

Algorithm	Cores	10	12	15	20	30
ART	1	1	1	1	1	1
	2	1.19581	1.54942	1.54023	1.75870	1.78459
	4	1.22511	2.8198	3.15109	2.93728	3.39847
	8	1.57189	6.13920	6.11620	7.18971	7.21240
MLEM	1	1	1	1	1	1
	2	1.20794	1.73988	1.70430	1.63692	1.61799
	4	1.21839	2.38677	2.26052	2.16825	2.17675
	8	1.2495	3.14354	3.08499	3.15482	2.89362
MAPEM	1	1	1	1	1	1
	2	1.27736	1.97445	1.99581	1.98687	1.99471
	4	1.51692	3.91267	3.95574	3.8835	3.91221
	8	1.6212	7.50019	7.74031	7.91742	7.71174

The reconstructed image speedup is calculated for the projection data measured at 18^0 angle producing 10 number of projections, 15^0 angle producing 12 number of projections, 12^0 angle producing 15 number of projections, 9^0 angle producing 20 number of projections and 6^0 angle producing 30 number of projections. Also this speedup calculation is plotted against various projection angles considered for study and different number cores 1, 2, 4 and 8.

The speed up graph for all the sizes specified at different angles shows a good efficiency. This survey as the evidence of parallel programming applied at any field gives a better performance in reduction of time complexity.

V. SUMMARY

The performance measure of MAPEM compared to first two algorithms MLEM & ART shows that the iterations required to reconstruct an image is lesser than MLEM and higher than ART for small size of images. The MAPEM and MLEM algorithms results in minimum iteration than ART for large size of images. MLEM algorithm results better PSNR values compared to MAPEM and ART.

The reconstruction process shows a constant speedup and increase in efficiency when all the implemented algorithms are executed in the parallel environment with the OpenMP architecture.

The applications designed using pART, pMLEM and pMAPEM in this research study shows more promising result. Thus, it is proven that this research has brought out a remarkable advancement in the field of image reconstruction in terms of reduction of time consumption, performance of image quality with reduced number of iterations in a multicore environment.

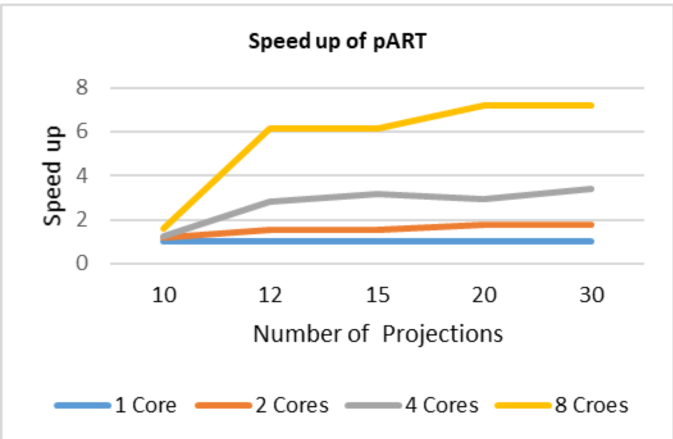

Figure 8: A Graph showing the performance Analysis of the multi-core environment using pART.

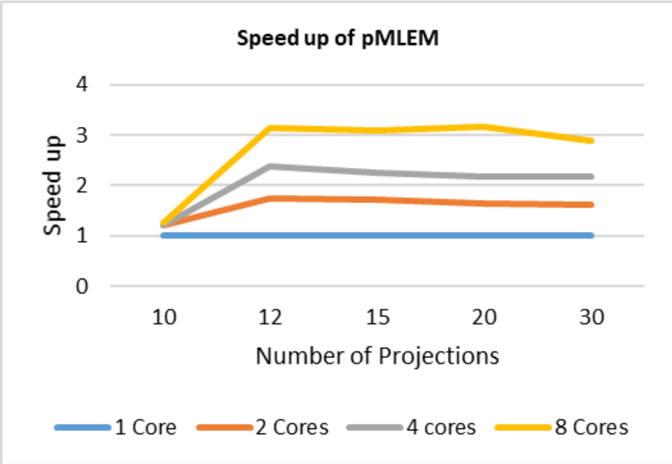

Figure 9: A Graph showing the performance Analysis of the multi-core environment using pMLEM

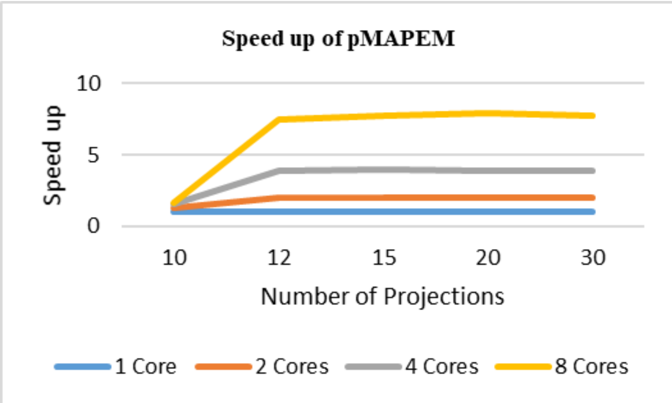

Figure 10: A Graph showing the performance Analysis of the multi-core environment using pMAPEM

References

[1] M. Beister, D. Kolditz, and W. A. Kalender, "Iterative reconstruction methods in X-ray CT," Physica Medica: European Journal of Medical Physics, vol. 28, pp. 94-108, 2012.

[2] G. Wang and J. Qi, "PET image reconstruction using kernel method," IEEE transactions on medical imaging, vol. 34, pp. 61-71, 2015.

[3] T. Würfl, F. C. Ghesu, V. Christlein, and A. Maier, "Deep learning computed tomography," in International Conference on Medical Image Computing and Computer-Assisted Intervention, 2016, pp. 432-440.

[4] Cristobal A. Navarro, Nancy Hitschfeld-Kahler and Luis Mateu, "A Survey on Parallel Computing and its Applications in Data-Parallel Problems Using GPU Architectures", Communications in Computational Physics, vol 15, Issue 2, pp 285-329, February 2014.

[5] M. Sabbagh, M. Uecker, A. J. Powell, M. Leeser, and M. H. Moghari, "Cardiac MRI compressed sensing image reconstruction with a graphics processing unit," in Medical Information and Communication Technology (ISMICT), 2016 10th International Symposium on, 2016, pp. 1-5.

[6] Y. Censor and S. A. Zenios, "Parallel Optimization: Theory, Algorithms and Applications", Oxford University Press, New York, NY, USA, 1997.

[7] S. Kaczmarz, Angenaherte auflosung von systemen linearer gleichungen, Bulletin de l'Academic Polonaise des Sciences et Lettres, A35 (1937) 355 -357.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4224-4234
https://publishoa.com
ISSN: 1309-3452

[8] G. T. Herman, Image Reconstruction from Projections: The Fundamentals of Computerized Tomography, Academic Press, New York, NY, USA, 1980.

[9] P. P. B. Eggermont, G. T. Herman and A. Lent, Iterative algorithms for large partitioned linear systems, with applications to image reconstruction, Linear Algebra and Its Applications, 40 (1981), pp. 37–67.

[10] G. T. Herman, Image Reconstruction from Projections: The Fundamentals of Computerized Tomography, Academic Press, New York, NY, USA, 1980.

[11] M. M. Khalil, J. L. Tremoleda, T. B. Bayomy, and W. Gsell, "Molecular SPECT imaging: an overview," *International journal of molecular imaging,* vol. 2011.

[12] Mark D Hill and Michael R Marty. Amdahl's Law in Multicore era. Computer Science Department Technical Report #1593, April 2007.

[13] J. Ma, H. Zhang, Y. Gao, J. Huang, Z. Liang, Q. Feng, et al., "Iterative image reconstruction for cerebral perfusion CT using a pre-contrast scan induced edgepreserving prior," *Physics in Medicine & Biology*, vol. 57, p. 7519, 2012.