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$I_{G\delta S}$-CLOSED FUNCTIONS

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ABSTRACT

In this paper, we have introduced a new class of open and closed functions called $I_{g\delta s}$-closed and $I_{g\delta s}$-open functions in ideal topological spaces and also investigated some of its characterizations and properties with the existing sets.

1. Introduction

Topology has a vital role in pure mathematics and has many subfields. The topology structure is the foundation for geometry and algebra. There is no universal agreement among mathematicians as what a first course in topology should include. Topological ideas have played an important role in topology for several years. Johann Benedict (1802-1882) was the first to use the word topology. The subject of topology itself consists of several different branches, such as point set topology, algebraic topology and differential topology. The influence of topology is also important in other mathematical disciplines such as dynamical systems, algebraic geometry and several aspects of analysis and combinatorics. In 1872, Cantor introduced the concept of the first derived set or set of limit points of a set. Cantor also introduced the concept of an open set; another fundamental concept in point set topology. Topology is an indispensable object of study, with open sets as well as closed sets being the most fundamental concepts in topological spaces. The

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digital line, the digital plane and three dimensional of point set topology are used in computer graphics. Topology, as so many other branches of mathematics, evolved out of the revolutionary changes undergone by the concept of geometry during the nineteenth century. In the beginning of the century the reigning view was the classical one, according to which geometry was the mathematical theory of the real physics space that surrounds us, and its axioms were seen as self- evident elementary facts. By the end of the century mathematicians had freed themselves from this narrow approach. It had become clear that geometry was henceforth to have much wider aims, and should accordingly be made to work in abstract "spaces", such as n-dimensional manifolds, projective spaces, Riemann surfaces, function spaces and etc. Hence topology emerged. Broadly speaking, Topology is the study of space and continuity. Since topology includes the study of continuous deformations of a space, it is often popularly called rubber sheet geometry. Topology, as a branch of Mathematics, can be formally defined as the study of qualitative properties of certain objects (called topological spaces). The objects that are invariant under certain kind of transformations called continuous maps. Especially those properties that are invariant under a certain kind of equivalence known as homeomorphism. Topology is the branch of Mathematics through which we elucidate and investigate the ideas of continuity, within the frame work of Mathematics. The study of topological spaces, their continuous mappings and general properties makes up one branch of topology known as General Topology. Modern topology depends strongly on the ideas of set theory, developed by George Cantor in the later part of the 19^{th} century. The subject of ideals in topological spaces has been introduced and studied by Kuratowski [8] and Vaidyanathasamy [16]. An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (1) $A \in I$ and $B \subset A$ implies $B \in I$ and (2) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called the local function [16] of A with respect to τ and I, is defined as follows: For $A \subset X, A^*(\tau, I) = \{x \in X \mid U \cap A \notin I$ for every open neighbourhood U of x$\}$. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(\tau, I)$ called the *-topology, finer than τ is defined by $Cl^*(A) = A \cup A^*(\tau, I)$ where there is no chance of confusion, $A^*(I)$ is denoted by A^*. If I is an ideal on X, then (X, τ, I) is called an ideal topological space.

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The concept of semi-I-open sets was introduced and studied by Caksu Guler and Aslim. Our research work is an elaborate study of that set in ideal topological space.
In this paper, we have introduced a new class of open and closed functions called $\mathbf{I}_{g\delta s}$-closed and $\mathbf{I}_{g\delta s}$-open functions in ideal topological spaces and also investigated some of its characterizations and properties with the existing sets.

2. Preliminaries

The purpose of this section is to give a review of some definitions, theorems and some propertiesof sets and functions concerning our subject.

Definition 2.1. [17] *The* θ*-closure of* A, *denoted by* $Cl_\theta(A)$, *is defined to be the set of all* $x \in X$ *such that* $A \cap Cl(U) \neq \emptyset$ *for every open neighbourhood* U *of* X. *If* $A = Cl_\theta(A)$, *then* A *is called* θ *-closed. The complement of a* θ *-closed set is called a* θ *-open set. The* θ *-interior of* A *is definedby the union of all* θ *-open sets contained in* A *and is denoted by* $Int_\theta(A)$.

Remark 2.2. [17] *The collection of* θ *-open sets in a topological space* (X, τ) *forms a topology* τ_θ *on* X.

Definition 2.3. [17] *The* δ *-closure of* A, *denoted by* $Cl_\delta(A)$, *is defined to be the set of all* $x \in X$ *such that* $A \cap Int(Cl(U)) \neq \emptyset$ *for every open neighbourhood* U *of* X. *If* $A = Cl_\delta(A)$, *then* A *is called a* δ *-closed set. The complement of a* δ *-closed set is called* δ *-open. The* δ *-interior of* A *is defined by the union of all* δ *-open sets contained in* A *and is denoted by* $Int_\delta(A)$.
A subset A of X is said to be δ*-pre open if* $A \subset int(cl_\delta(A))$. *The complement of* δ*-pre open is called as* δ*-pre closed.*

Remark 2.4. [17] *The collection of* δ*-open sets in a topological space* (X, τ) *forms a topology* τ_δ *on* X.

Definition 2.5. *A subset* A *of a topological space* (X, τ) *is said to be*

(1) *regular open* [15] *if* $A = Int(Cl(A))$,
(2) *preopen* [7] *if* $A \subset Int(Cl(A))$,
(3) *semiopen* [9] *if* $A \subset Cl(Int(A))$,
(4) α*-open* [13] *if* $A \subset Int(Cl(Int(A)))$,
(5) b*-open* [3] *if* $A \subset Int(Cl(A)) \cup Cl(Int(A))$,
(6) β*-open* [1] *if* $A \subset Cl(Int(Cl(A)))$.

The complement of a regular open (resp. preopen, semiopen, α -open, b -open, β -open) set is called a regular closed (resp. preclosed, semiclosed, α -closed, b -closed, β -closed) set. The set of all regular open (resp. preopen, semiopen, α -open, b -open, β-open, regular closed, preclosed, semiclosed, α -closed, β -closed) sets of (X, τ) is denoted by RO(X) (resp. PO(X), SO(X), αO(X), BO(X), βO(X), RC(X), PC(X), SC(X), αC(X), βC(X)) S-int(A)={∪U : U⊂A and U is semi-open sets} and scl(A)={∩G : A⊂G and G is semi-closed}.

Definition 2.6. *A subset* A *of an ideal topological space* (X, τ, I) *is said to be*
(1) *regular-*$\mathbf{I}$*-open* [15] *if* $A = \mathbf{I}nt(Cl^*(A))$,
(2) *pre*-I*-open* [7] *if* $A \subset Int(Cl^*(A))$,
(3) δ*-pre-*$\mathbf{I}$*-open* [11] *if* $A \subset \mathbf{I}nt(Cl_\delta^*(A))$.

The complement of a regular-I-open (resp. pre-I-open) set is called a regular-I-closed (resp. pre-I-closed) set. The set of all regular-I-open (resp. pre-I-open, regular-I- closed, pre-I-closed) sets of (X, τ, I) is denoted by RIO(X) (resp. PIO(X) SIO(X), RIC(X), PIC(X))

Remark 2.7. [17] *A set A⊂X is* δ*-open if and only if it is the union of regular open sets of* X.

Definition 2.8. [4] *A function* $f : (X, \tau, I) \to (Y, \sigma)$ *is called a semi-continuous function if forevery open subset* V *of* Y, $f^{-1}(V)$ *is semi-open in* X.

Definition 2.9. [4] *A subset A of X is g-closed if cl(A)⊂U whenever A⊂U and U is open.*

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Definition 2.10. [4] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be strongly continuous, if* $f^{-1}(V)$ *is semi-closed in* (X, τ) *for every closed set* V *in* (Y, σ).

Definition 2.11. [4] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be perfectly continuous, if* $f^{-1}(V)$*s clopen in* (X, τ) *for every open set* V *in* (Y, σ).

Definition 2.12. [4] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called completely continuous, if the inverse image of every open set in* Y *is regular open in* (X, τ).

Definition 2.13. [4] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called totally semi-continuous, if the inverse image of every semi-open set in* Y *is clopen in* (X, τ).

Definition 2.14. [4] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be contra-continuous if* $f^{-1}(V)$ *is closed in* (X, τ) *for each open set* V *in* (Y, σ).

Definition 2.15. [4] *A space X is said to be* $I_{g\delta s}$*-space if for any distinct points x and y there exists two disjoint open sets U and V such that x∈U and y∈V respectively.*

Definition 2.16. [7] *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be weakly continuous if for each* $x \in X$ *and an open set* V *in* Y *containing* $f(x)$, *there exists an open set* U *of* X *containing* x *such that* $f(U) \subset Cl(V)$.

Definition 2.17. [5] *A subset* S *of an ideal topological space* (X, τ, I) *is called semi-I-open if* $S \subset Cl^*(Int(S))$. *The complement of a semi-I-open set is called a semi-I-closed set. The family of all semi-I-open (resp. semi-I-closed) sets of* (X, τ, I) *is denoted by* $SIO(X)$(*resp.* $SIC(X)$).*We set* $SIO(X, x) = \{U : U \in SIO(X)$ *and* $x \in U\}$ *and* $SIC(X, x) = \{U : U \in SIC(X)$ *and* $x \in U\}$.

Definition 2.18. [4] *A subset* B_x *of an ideal topological space* (X, τ, I) *is said to be a semi-I- neighbourhood of a point* $x \in X$ *if there exists a semi-I-open set* U *such that* $x \in U \subset B_x$.

Definition 2.19. [4] *The intersection of all semi-I-closed sets containing* A *is called the semi-* I*-closure of* A *and is denoted by* $I_{scl}(A)$. *A subset* A *is semi-*I*-closed if, and only if* $I_{scl}(A) = A$. *The union of all semi-I-open subsets of* (X, τ) *contained in* $A \subset X$ *is called the semi-I-interior of* A *and is denoted by* $I_{sint}(A)$.

Definition 2.20. [4] *A function* $f : (X, \tau, I) \to (Y, \sigma)$ *is called a semi-I-continuous function if for every open subset* V *of* Y, $f^{-1}(V)$ *is semi-I-open in* X.

We set $SIC(X, Y) = \{f : f : (X, \tau, I) \to (Y, \sigma)$ is a semi-I-continuous function

Theorem 2.21. [4] *For a function* $f : (X, \tau, I) \to (Y, \sigma)$, *the following statements are equivalent*

(1) f *is semi-I-continuous;*

(2) *For each open subset* V *in* Y *containing* $f(x)$, *there exists* $U \in SIO(X, x)$ *such that* $(U) \subset V$

(3) *For each* $x \in X$ *the inverse image of every neighbourhood of* $f(x)$ *is a semi-I- neighbour-hood of* x;

(4) *For every closed subset* V *of* Y, $f^{-1}(V)$ *is semi-I-closed in* X.

(5) *For every subset* A *of* X, $f(sICl(A)) \subset Cl(f(A))$;

(6) *For every subset* B *of* Y, $sICl(f^{-1}(B)) \subset f^{-1}(Cl(B))$.

Lemma 2.22. *The following statements are true*

(1) *Let* A *be a subset of a topological space* (X, τ). *Then* $A \in PO(X)$ *if and only if* $sCl(A) = Int(Cl(A))$ [14].

(2) *A subset A of a topological space* (X, τ) *is* β*-open if and only if* $Cl(A)$ *is regular closed*[2].

Theorem 2.23. [5] *If* $\{A_\alpha\}_{\alpha\in\Delta}$ *is a collection of semi-I-open subsets of an ideal topological space* (X, τ, I), *then* $\cup_{\alpha\in\Delta} A_\alpha$ *is also a semi-I-open set in* (X, τ, I).

Theorem 2.24. [5] *If* $f : (X, \tau, I) \to (Y, \sigma)$ *is semi-I -continuous and if* $g : (Y, \sigma) \to (Z, \eta)$ *is continuous, then* $g \circ f$ *is semi-I-continuous.*

Lemma 2.25. [6] *For any function* $f : (X, \tau, I) \to (Y, \sigma)$, $f(I)$ *is an ideal on* Y. *If* f *is an injection and* $\mathbf{J}$ *is any ideal on* Y, *then* $f^{-1}(\mathbf{J}) = \{f^{-1}(J) : J \in \mathbf{J}\}$ *is an ideal on* X.

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Theorem 2.26. [10] *Let* (X, τ) *and* (Y, σ) *be topological spaces. Then the following are equivalent*

(1) $f : (X, \tau) \to (Y, \sigma)$ *is weakly continuous.*

(2) *For every open set* V *in* Y*, there exists an open set* G *in* Y *such that* $G \subset V$ *and* $f^{-1}(G) \subset Int(f^{-1}(Cl(V)))$

Definition 2.27. *A subset A of X is called ideal generalized δ semi-closed (briefly* $I_{g\delta s}$*-closed) set if* $I_{scl}(A) \subset U$ *whenever* $A \subset U$ *and* U *is δ-open in* (X, τ).
A subset A of X is called ideal generalized δ semi-open (briefly $I_{g\delta s}$*-open) set if* A^c *is* $I_{g\delta s}$*-closed.*
The family of all $I_{g\delta s}$*-closed subsets of the space X is denoted by* $I_{g\delta s}$*-C(X) and* $I_{g\delta s}$*-open subsets of the space X is denoted by* $I_{g\delta s}$*-O(X).*

3. $I_{g\delta s}$-closed and $I_{g\delta s}$-open functions

Definition 3.1. *A function* $f : (X, \tau) \to (Y, \sigma, I)$ *is said to be* $I_{g\delta s}$*-closed if* $f(V)$ *is* $I_{g\delta s}$*-closed in* Y *for every closed set* V *in* X.

Definition 3.2. (1) *A function* $f : (X, \tau, I) \to (Y, \sigma, J)$ *is* $I_{g\delta s}$*-irresolute if* $f^{-1}(V)$ *is* $I_{g\delta s}$*- closed in X for every* $I_{g\delta s}$*-closed set V of Y.*
(2) *A function* $f : (X, \tau, I) \to (Y, \sigma)$ *is* $I_{g\delta s}$*-continuous if* $f^{-1}(V)$ *is* $I_{g\delta s}$*-closed in X for every closed set V of Y.*

Theorem 3.3. *A function* $f : (X, \tau) \to (Y, \sigma, I)$ *is* $I_{g\delta s}$*-closed if and only if* $f(V)$ *is* $I_{g\delta s}$*-open in* Y *for every open set* V *in* X.

Proof : Suppose $f : (X, \tau) \to (Y, \sigma, I)$ is $I_{g\delta s}$-closed function and V is an open set in X. Then $X - V$ is closed in X. By hypothesis $f(X - V) = Y - f(V)$ is a $I_{g\delta s}$-closed set in Y and hence $f(V)$ is $I_{g\delta s}$-open set in Y.
On the other hand, if F is closed set in X, then $X - F$ is an open set in X. By hypothesis $f(X - F) = Y - f(F)$ is $I_{g\delta s}$-open set in Y, implies $f(F)$ is $I_{g\delta s}$-closed set in Y. Therefore f is $I_{g\delta s}$-closed function.

Definition 3.4. *An ideal topological space X is said to be* $TI_{g\delta s}$*-space if every* $I_{g\delta s}$*-closed set is closed in X.*

Definition 3.5. *An ideal topological space X is called* δ-$T_{1/2}$ *space if every* $I_{g\delta s}$*-closed set is semi-*I-*closed.*

Theorem 3.6. *If* $f : (X, \tau) \to (Y, \sigma, I)$ *is* $I_{g\delta s}$*-closed function and* Y *is* $TI_{g\delta s}$*-space, then* f *is a closed function.*

Proof: Let V be a closed set in X. Since f is a $I_{g\delta s}$-closed function, implies $f(V)$ is $I_{g\delta s}$-closed in Y. Now Y is $TI_{g\delta s}$-space, implies $f(V)$ is a closed set in Y. Therefore f is a closed function.

Theorem 3.7. *If* $f : (X, \tau) \to (Y, \sigma, I)$ *is* $I_{g\delta s}$*-closed function and* Y *is* δ-$T_{1/2}$ *space, then* f *is semi-closed function.*

Proof : Let V be a closed set in X. Since f is a $I_{g\delta s}$-closed function, $f(V)$ is $I_{g\delta s}$-closed set in Y. Now Y is δ-$T_{1/2}$ space, implies f(V) is a semi-I-closed set in Y and since every semi-I-closed set is semi closed. Therefore f is a semi-closed function.

Theorem 3.8. *For the function* $f : (X, \tau) \to (Y, \sigma, I)$*, the following statements are equivalent.*

(1) f *is a* $I_{g\delta s}$*-open function.*

(2) *For each subset* A *of* X*,* $f(int(A)) \subset I_{g\delta s}$-$int(f(A))$

(3) *For each* $x \in X$*, the image of every nhd of* x *is* $I_{g\delta s}$*-nhd of* $f(x)$.

Proof : (1) ⇒ (2) Suppose (i) holds and $A \subset X$. Then int(A) is an open set in X. By (1), $f(int(A))$ is a $I_{g\delta s}$-open set in Y. Therefore $I_{g\delta s}$-$int(f(int(A))) = f(int(A))$. Since $f(int(A)) \subset f(A)$, implies $I_{g\delta s}$-$int(f(int(A))) \subset I_{g\delta s}$-$int(f(A))$. That is $f(int(A)) \subset I_{g\delta s}$-$int(f(A))$.

(2) ⇒ (3) Suppose (2) holds. Let $x \in X$ and A be an arbitrary nhd of x in X. Then there exists an open set G in X such that $x \in G \subset A$. By (2), $f(G) = f(int(G)) \subset I_{g\delta s}$-$int(f(G))$. But $I_{g\delta s}$-$int(f(G)) \subset f(G)$ is always true. Therefore, $f(G) = I_{g\delta s}$-$int(f(G))$ and hence $f(G)$ is $I_{g\delta s}$-open set in Y. Further $f(x) \in f(G) \subset f(A)$, this implies, $f(A)$ is a $I_{g\delta s}$-nhd of $f(x)$ in Y. Hence (3) holds.

(3) ⇒ (1) Suppose (3) holds. Let V be any open set in X and $x \in V$. Then $y = f(x) \in f(V)$. By (3), for

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each $y \in f(V)$, there exists a $I_{g\delta s}$-nhd Z_y of y in Y. Since Z_y is a $I_{g\delta s}$-nhd of y, there exists a $I_{g\delta s}$-open set V_y in V such that $y \in V \subset Z_y$.Therefore $f(V) = \cup\{V_y : y \in f(V)\}$, which is union of $I_{g\delta s}$-open sets and hence $I_{g\delta s}$-open set in Y. Therefore f is $I_{g\delta s}$-open function.

Theorem 3.9. *A function $f : (X, \tau) \to (Y, \sigma, I)$ is $I_{g\delta s}$-closed if and only if for each subset S of Y and for each open set U in X containing $f^{-1}(S)$, there exists a $I_{g\delta s}$-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.*

Proof : Assume that f is $I_{g\delta s}$-closed function. Let $S \subset Y$ and U be an open set of X containing $f^{-1}(S)$. Since f is a $I_{g\delta s}$-closed function and $X - U$ is -closed in X, implies $f(X - U)$ is $I_{g\delta s}$-closed set in Y. Then $V = Y - f(X - U)$ is $I_{g\delta s}$-open set in Y such that $S \subset V$ and $f^{-1}(V) \subset U$.
Conversely, let F be a closed set in X, then $X-F$ is an open set in X and $f^{-1}(Y - f(F)) \subset X-F$. By hypothesis, there is a $I_{g\delta s}$-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore, $Y - V \subset f(F) \subset f(X - F^{-1}(V)) \subset Y - V$, this implies $f(F) = Y - V$. Since V is a $I_{g\delta s}$-open set in Y and so $f(F)$ is $I_{g\delta s}$-closed in Y. Hence f is $I_{g\delta s}$-closed function.

Theorem 3.10. *If $f : (X, \tau) \to (Y, \sigma, I)$ is $I_{g\delta s}$-closed, then for each $I_{g\delta s}$-closed set K of Y and each open set G of X containing $f^{-1}(K)$, there exists $I_{g\delta s}$-open set V in Y containing K such that $f^{-1}(V) \subset G$.*

Proof : Suppose $f : (X, \tau) \to (Y, \sigma, I)$ is $I_{g\delta s}$-closed function. Let K be any $I_{g\delta s}$-closed set of Y and U is an open set in X containing $f^{-1}(K)$, by theorem 3.9, there exists a $I_{g\delta s}$-open set G of Y such that $K \subset G$ and $f^{-1}(G) \subset U$. Since K is $I_{g\delta s}$-closed set and G is $I_{g\delta s}$-open set containing K implies $K \subset I_{g\delta s}-int(G)$. Put $V = I_{g\delta s}-int(G)$, then $K \subset V$ and V is $I_{g\delta s}$-open set in Y and $f^{-1}(V) \subset U$.

Theorem 3.11. *A function $f : (X, \tau) \to (Y, \sigma, I)$ is $I_{g\delta s}$-closed, if and only if $I_{g\delta s}-cl(f(A)) \subset f(cl(A))$, for every subset A of X.*

Proof : Suppose $f : (X, \tau) \to (Y, \sigma, I)$ is a $I_{g\delta s}$-closed and $A \subset X$. Then $f(cl(A))$ is $I_{g\delta s}$-closed in Y. Since $f(A) \subset f(cl(A))$, implies $I_{g\delta s}-cl(f(A)) \subset I_{g\delta s}-cl(f(cl(A))) = f(cl(A))$. Hence $I_{g\delta s}-cl(f(A)) \subset f(cl(A))$.
Conversely, let A is any closed set in X. Then $cl(A) = A$. Therefore, $f(A) = f(cl(A))$. By hypothesis, $I_{g\delta s}-cl(f(A)) \subset f(cl(A)) = f(A)$. Implies $I_{g\delta s}-cl(f(A)) \subset f(A)$. But $f(A) \subset I_{g\delta s}-cl(f(A))$ is always true. This shows, $f(A) = I_{g\delta s}-cl(f(A))$. Therefore $f(A)$ is $I_{g\delta s}$-closed set in Y and hence f is $I_{g\delta s}$-closed.

Theorem 3.12. *Let $f : (X, \tau) \to (Y, \sigma, I)$ and $g : (Y, \sigma, I) \to (Z, \mu, J)$ be any two functions. Then $(g \circ f) : (X, \tau) \to (Z, \mu, J)$ is $I_{g\delta s}$-closed function if f and g satisfy one of the following conditions*

(1) f, g *are $I_{g\delta s}$-closed functions if Y is $TI_{g\delta s}$-space.*
(2) f *is closed and* g *is $I_{g\delta s}$-closed function.*

Proof :

(1) Suppose F is closed set in X. Since f is $I_{g\delta s}$-closed function $f(F)$ is $I_{g\delta s}$-closed set in Y. Now Y is $TI_{g\delta s}$-space, implies $f(F)$ is closed set in Y. Also g is $I_{g\delta s}$-closed function, implies $g(f(F)) = (g \circ f)(F)$ is $I_{g\delta s}$-closed set in Z. Hence $(g \circ f)$ is $I_{g\delta s}$-closed function.

(2) Suppose F is closed set in X. Since f is closed function f(F) is closed set in Y. Now g is $I_{g\delta s}$-closed function, implies $g(f(F)) = (g \circ f)(F)$ is $I_{g\delta s}$-closed set in Z. Hence $(g \circ f)$ is $I_{g\delta s}$-closed function.

Theorem 3.13. *If f is δ-continuous and $I_{g\delta s}$-closed then $f(H)$ is $I_{g\delta s}$-closed in Y for each $I_{g\delta s}$-closed H in X whrere X is $TI_{g\delta s}$- space.*

Proof : Suppose H is an $I_{g\delta s}$-closed set in X and V is an δ-open set in Y containing $f(H)$, this implies $H \subset f^{-1}(V)$. Since f is δ-continuous, $f^{-1}(V)$ is an δ-open set containing H, therefore $I_{g\delta s}$-$cl(H) \subset f^{-1}(V)$ and hence $f(I_{g\delta s}$-$cl(H)) \subset V$. Since f is $I_{g\delta s}$-closed and $I_{g\delta s}$-$cl(H)$ is closed in X then $f(I_{g\delta s}$-$cl(H))$ is $I_{g\delta s}$-closed in Y. Therefore $I_{g\delta s}$-$cl(f(I_{g\delta s}$-$cl(H))) \subset V$. Thus, $I_{g\delta s}$- $cl(H) \subset I_{g\delta s}$-$cl(f(I_{g\delta s}$-$cl(H)) \subset V$. This shows that $f(H)$ is $I_{g\delta s}$-closed in Y.

Theorem 3.14. *Let $f : (X, \tau) \to (Y, \sigma, I)$ and $g : (Y, \sigma, I) \to (Z, \mu, J)$ be any two functionssuch that $(g \circ f) : X \to Z$ be $I_{g\delta s}$-closed function. Then following results hold*

(1) *If f is -continuous surjection, then g is $I_{g\delta s}$-closed function.*

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(2) *If* g *is* $\mathbf{I}_{g\delta s}$*-irresolute and injective, then* $\mathbf{f}$ *is* $\mathbf{I}_{g\delta s}$*-closed function.*

Proof : (1) Suppose A is a closed set in Y. Since f is continuous and surjective, $f^{-1}(A)$is a closed set in $\mathbf{X}$. Therefore, $(g \circ \mathbf{f})(\mathbf{f}^{-1}(A)) = g(A)$ is $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{Z}$ and hence g is $\mathbf{I}_{g\delta s}$-closed function.
(2) Suppose H is closed set in $\mathbf{X}$. Then $(g \circ \mathbf{f})(H)$ is $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{Z}$. Since g is $\mathbf{I}_{g\delta s}$- irresolute, $g^{-1}((g \circ \mathbf{f})(H)) = \mathbf{f}(H)$ is $\mathbf{I}_{g\delta s}$-closed set in Y. Hence $\mathbf{f}$ is $\mathbf{I}_{g\delta s}$-closed function.

Theorem 3.15. *For any bijection* $f : (X, \tau) \to (Y, \sigma, I)$, *the following statements are equivalent:*

(1) $\mathbf{f}^{-1}$ *is* $\mathbf{I}_{g\delta s}$*-continuous.*
(2) $\mathbf{f}$ *is a* $\mathbf{I}_{g\delta s}$*-open function.*
(3) $\mathbf{f}$ *is a* $\mathbf{I}_{g\delta s}$*-closed function.*

Proof : (1) $\Rightarrow$ (2) Suppose $\mathbf{F}$ is an open set in $\mathbf{X}$, then by (i), $(\mathbf{f}^{-1})^{-1}(\mathbf{F}) = \mathbf{f}(\mathbf{F})$ is $\mathbf{I}_{g\delta s}$-open set in Y and hence $\mathbf{f}$ is $\mathbf{I}_{g\delta s}$-open function.
(2) $\Rightarrow$ (3) Suppose F is a closed set in X, then $X - F$ is an open set in X. By (2), $f(X - F) = Y - \mathbf{f}(F)$ is $\mathbf{I}_{g\delta s}$-open in Y, implies $\mathbf{f}(F)$ is $\mathbf{I}_{g\delta s}$-closed in Y. Therefore $\mathbf{f}$ is $\mathbf{I}_{g\delta s}$-closed function.
(3) $\Rightarrow$ (1) Let F be a closed set in $\mathbf{X}$. By (3), $\mathbf{f}(F) = (\mathbf{f}^{-1})^{-1}(F)$ is $\mathbf{I}_{g\delta s}$-closed in Y. Therefore $\mathbf{f}^{-1}$ is $\mathbf{I}_{g\delta s}$ continuous function.

4. $\mathbf{I}_{pg\delta s}$-closed functions

Definition 4.1. *A function* $\mathbf{f} : (\mathbf{X}, \tau) \to (Y, \sigma, \mathbf{I})$ *is said to be* $\mathbf{I}_{pg\delta s}$ *closed if* $\mathbf{f}(V)$ *is* $\mathbf{I}_{g\delta s}$*-closed in* Y *for every semi-closed set* V *in* X.

Definition 4.2. (1) *A function* $f : (X, \tau) \to (Y, \sigma, I)$ *is said to be pre semi-closed if f(V) issemi-closed in Y for every semi-closed set V of X.*
(2) *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be pre-closed if f(V) is closed in Y for everysemi-closed set V of X.*
(3) *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be δ-continuous if* $f^{-1}(V)$ *is δ-closed in X forevery δ-closed set V of Y*

Theorem 4.3. *A function* $\mathbf{f} : (\mathbf{X}, \tau) \to (Y, \sigma, \mathbf{I})$ *is* $\mathbf{I}_{pg\delta s}$*-closed if and only if* $\mathbf{f}(V)$ *is* $\mathbf{I}_{g\delta s}$*-open in* Y *for every semi-open set* V *in* X.

Proof: Similar to the proof of theorem 3.3.

Remark 4.4. *Every pre semiclosed function is* $\mathbf{I}_{g\delta s}$*-closed function.*

The proof follows from the fact that every semiclosed set is $\mathbf{I}_{g\delta s}$-closed.

Theorem 4.5. *If* $\mathbf{f} : (\mathbf{X}, \tau) \to (Y, \sigma, \mathbf{I})$ *is* $\mathbf{I}_{pg\delta s}$*-closed function and* Y *is* δ-$T_{1/2}$ *space, then* $\mathbf{f}$ *is pre semiclosed function.*

Proof: Suppose V is a semiclosed set in $\mathbf{X}$. Since $\mathbf{f}$ is a $\mathbf{I}_{pg\delta s}$-closed function $\mathbf{f}(V)$ is $\mathbf{I}_{g\delta s}$-closed set in Y. Now Y is δ-$T_{1/2}$ space $f(V)$ is a semi-I-closed set in Y and every semi-I-closed set is semiclosed. Therefore f is a pre semiclosed function.

Theorem 4.6. *A function* $\mathbf{f} : (\mathbf{X}, \tau) \to (Y, \sigma, \mathbf{I})$ *is* $\mathbf{I}_{pg\delta s}$*-closed if and only if for each subset* S *of* Y *and for each semi-open set* U *of* $\mathbf{X}$ *containing* $\mathbf{f}^{-1}(S)$, *there exists a* $\mathbf{I}_{g\delta s}$*-open set* V *of* Y *such that* $S \subset V$ *and* $f^{-1}(V) \subset U$.

Proof: Similar to the proof of theorem 3.9.

Theorem 4.7. *If* $\mathbf{f} : (\mathbf{X}, \tau) \to (Y, \sigma, \mathbf{I})$ *is* $\mathbf{I}_{pg\delta s}$*-closed, then for each* $\mathbf{I}_{g\delta s}$*-closed set* $\mathbf{K}$ *of* Y *and each semi-open set G of X containing* $\mathbf{f}^{-1}(\mathbf{K})$, *there exists* $\mathbf{I}_{g\delta s}$*-open set V in Y containing* $\mathbf{K}$ *such that* $f^{-1}(V) \subset U$.

Proof: Similar to the proof of theorem 3.10.

Theorem 4.8. *If* $\mathbf{f}$ *is δ-continuous and* $\mathbf{I}_{pg\delta s}$*-closed, then* $\mathbf{f}(H)$ *is* $\mathbf{I}_{g\delta s}$*-closed in Y for each* $\mathbf{I}_{g\delta s}$*-closed H in* $\mathbf{X}$ *if Y is* δ-$T_{1/2}$ *space.*

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Proof: Suppose H is any $\mathbf{I}_{g\delta s}$-closed set of $\mathbf{X}$ and V is a δ-open set of Y containing $f(H)$. This implies $H \subset f^{-1}(V)$. Since f is δ-continuous, $f^{-1}(V)$ is a δ-open set containing H, therefore, $\mathbf{I}_{scl}(H) \subset f^{-1}(V)$ and hence $f(\mathbf{I}_{scl}(H)) \subset V$. Since $\mathbf{I}_{scl}(H)$ is semiclosed and f is $\mathbf{I}_{pg\delta s}$-closed, implies $f(\mathbf{I}_{scl}(H))$ is $\mathbf{I}_{g\delta s}$-closed set contained in V over Y, implies $\mathbf{I}_{g\delta s}\text{-cl}(f(\mathbf{I}_{scl}(H))) \subset V$. Thus, $\mathbf{I}_{g\delta s}\text{-cl}(f(H)) \subset \mathbf{I}_{g\delta s}\text{-cl}(f(\mathbf{I}_{scl}(H))) \subset V$. That is, $\mathbf{I}_{g\delta s}\text{-cl}(f(H)) \subset V$. Since $\mathbf{Y}$ is δ-$T_{1/2}$ space, therefore $\mathbf{I}_{scl}(f(H)) \subset \mathbf{I}_{g\delta s}\text{-cl}(f(H)) \subset V$. This shows that $f(H)$ is $\mathbf{I}_{g\delta s}$-closed in Y.

Theorem 4.9. *Let* $f : (X, \tau) \to (Y, \sigma, I)$ *and* $g : (Y, \sigma, I) \to (Z, \mu, \mathbf{J})$ *be any two functions.Then* $(g \circ f) : \mathbf{X} \to \mathbf{Z}$ *is* $\mathbf{I}_{pg\delta s}$*-closed function if* f *and* g *satisfy one of the following conditions:*

(1) f, g *are* $\mathbf{I}_{pg\delta s}$*-closed functions if* Y *is* $\mathbf{I}_{g\delta s}$-$T_{1/2}$ *space.*

(2) f *is pre-closed and* g *is* $\mathbf{I}_{g\delta s}$*-closed function.*

(3) f *is pre semi-closed and* g *is* $\mathbf{I}_{pg\delta s}$*-closed function.*

(4) f *is* $\mathbf{I}_{pg\delta s}$*-closed function and* g *is* δ*-continuous,* $\mathbf{I}_{pg\delta s}$*-closed function if* Y *is* δ-$T_{1/2}$ *space.*

Proof :

(1) Suppose F is semi-closed set in $\mathbf{X}$. Since f is $\mathbf{I}_{pg\delta s}$-closed function, $f(F)$ is $\mathbf{I}_{pg\delta s}$-closed set in Y. Now Y is δ-$T_{1/2}$ space, therefore $f(F)$ is semiclosed set in Y. Also g is $\mathbf{I}_{pg\delta s}$-closed function, implies $g(f(F)) = (g \circ f)(F)$ is $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{Z}$. Hence $(g \circ f)$ is $\mathbf{I}_{pg\delta s}$-closed function.

(2) Suppose F is semi-closed set in X. Since f is pre-closed, f(F) is closed set in Y. Now g is $\mathbf{I}_{g\delta s}$-closed function, implies $g(f(F)) = (g \circ f)(F)$ is $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{Z}$. Hence $(g \circ f)$ is $\mathbf{I}_{pg\delta s}$-closed function.

(3) Suppose F is semi-closed set in X. Since f is pre semi-closed function, f(F) is semi-closed set in Y. Now g is $\mathbf{I}_{pg\delta s}$-closed function, implies $g(f(F)) = (g \circ f)(F)$ is $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{Z}$. Hence $(g \circ f)$ is $\mathbf{I}_{pg\delta s}$-closed function.

(4) Suppose H is a semi-closed set in $\mathbf{X}$. Since f is $\mathbf{I}_{pg\delta s}$-closed function $f(H)$ is $\mathbf{I}_{g\delta s}$-closed set in Y. Since g is δ-continuous, $\mathbf{I}_{pg\delta s}$-closed function and $\mathbf{Y}$ is δ-$T_{1/2}$ space, by theorem 4.8, $g(f(H)) = (g \circ f)(H)$ is $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{Z}$. Hence $(g \circ f)$ is $\mathbf{I}_{pg\delta s}$-closed function.

5. STRONGLY $\mathbf{I}_{g\delta s}$-CLOSED AND QUASI $\mathbf{I}_{g\delta s}$-CLOSED FUNCTIONS

Definition 5.1. *A function* $f : (\mathbf{X}, \tau) \to (Y, \sigma, , \mathbf{I})$ *is said to be strongly* $\mathbf{I}_{g\delta s}$*-closed (resp.* $\mathbf{I}_{g\delta s}$*- open), if* $f(A)$ *is* $\mathbf{I}_{g\delta s}$*-closed (resp.* $\mathbf{I}_{g\delta s}$*-open) set in* Y *for every* $\mathbf{I}_{g\delta s}$*-closed (resp.* $\mathbf{I}_{g\delta s}$*-open) set* A *in* X.

Remark 5.2. *Every strongly* $\mathbf{I}_{g\delta s}$ *closed function is* $\mathbf{I}_{g\delta s}$*-closed function.*

The proof follows from the fact that every $\mathbf{I}_{g\delta s}$-closed set is closed.

Theorem 5.3. *A surjective function* $f : (\mathbf{X}, \tau) \to (Y, \sigma, , \mathbf{I})$ *is strongly* $\mathbf{I}_{g\delta s}$*-closed (resp. strongly* $\mathbf{I}_{g\delta s}$*-open), if and only if for any subset* B *of* V *and each* $\mathbf{I}_{g\delta s}$*-open (resp.* $\mathbf{I}_{g\delta s}$*-closed) set* U *of* $\mathbf{X}$ *containing* $f^{-1}(B)$*, there exists a* $\mathbf{I}_{g\delta s}$*-open (resp.* $\mathbf{I}_{g\delta s}$*-closed) set* V *of* Y *containing* B *and* $f^{-1}(V) \subset U$.

Proof: Similar to the proof of theorem 3.9.

Theorem 5.4. *If a function* $f : (\mathbf{X}, \tau) \to (Y, \sigma, , \mathbf{I})$ *is a strongly* $\mathbf{I}_{g\delta s}$ *closed function, then for each* $\mathbf{I}_{g\delta s}$*-closed set* $\mathbf{K}$ *of* Y *and each* $\mathbf{I}_{g\delta s}$*-open set* U *of* $\mathbf{X}$ *containing* $f^{-1}(\mathbf{K})$*, there exists* $\mathbf{I}_{g\delta s}$*-open set V in Y containing* $\mathbf{K}$ *such that* $f^{-1}(V) \subset U$.

Proof: Similar to the proof of theorem 3.10.

Theorem 5.5. *A function* $f : (\mathbf{X}, \tau) \to (Y, \sigma, \mathbf{I})$ *is strongly* $\mathbf{I}_{g\delta s}$*-closed if and only if* $\mathbf{I}_{g\delta s}\text{-cl}(f(A)) \subset f(\mathbf{I}_{g\delta s} - \text{cl}(A))$ *for every subset A of* $\mathbf{X}$.

Proof: Let f be strongly $\mathbf{I}_{g\delta s}$-closed function and $A \subset \mathbf{X}$. Then $f(\mathbf{I}_{g\delta s} - \text{cl}(A))$ is $\mathbf{I}_{g\delta s}$-closed in Y. Since $f(A) \subset f(\mathbf{I}_{g\delta s} - \text{cl}(A))$, implies $\mathbf{I}_{g\delta s} - \text{cl}(f(A)) \subset \mathbf{I}_{g\delta s} - \text{cl}(f(\mathbf{I}_{g\delta s} - \text{cl}(A))) = f(\mathbf{I}_{g\delta s} - \text{cl}(A))$. Therefore, $\mathbf{I}_{g\delta s} - \text{cl}(f(A)) \subset f(\mathbf{I}_{g\delta s} - \text{cl}(A))$.
Conversely, A is any $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{X}$. Then $\mathbf{I}_{g\delta s} - \text{cl}(A) = A$, implies, $f(A) = f(\mathbf{I}_{g\delta s} - \text{cl}(A))$. By hypothesis, $\mathbf{I}_{g\delta s} - \text{cl}(f(A)) \subset f(\mathbf{I}_{g\delta s} - \text{cl}(A)) = f(A)$. Hence $\mathbf{I}_{g\delta s} - \text{cl}(f(A)) \subset f(A)$. But $f(A) \subset \mathbf{I}_{g\delta s} - \text{cl}(f(A))$ is always true. This shows, $f(A) = \mathbf{I}_{g\delta s} - \text{cl}(f(A))$. Therefore, $f(A)$ is $\mathbf{I}_{g\delta s}$-closed set in Y. Hence f is strongly $\mathbf{I}_{g\delta s}$-closed-closed function.

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Theorem 5.6. *Let* $f : (X, \tau) \to (Y, \sigma, I)$ *and* $g : (Y, \sigma, I) \to (Z, \mu, J)$ *be two functions, such that* $(g \circ f) : X \to Z$ *is strongly* $I_{g\delta s}$*-closed function. Then*

(1) f *is* $I_{g\delta s}$*-irresolute and surjective implies* g *is strongly* $I_{g\delta s}$*-closed.*

(2) g *is* $I_{g\delta s}$*-irresolute and injective implies* f *is strongly* $I_{g\delta s}$*-closed.*

Proof :

(1) Let A be $I_{g\delta s}$-closed set of Y. Since f is $I_{g\delta s}$ irresolute and surjective, $f^{-1}(A)$ is $I_{g\delta s}$-closed set in X. Also since $(g \circ f)$ is strongly $I_{g\delta s}$-closed function, implies $(g \circ f)(f^{-1}(A)) = g(A)$ is $I_{g\delta s}$-closed in Z. Therefore g is strongly $I_{g\delta s}$-closed.

(2) Let A be $I_{g\delta s}$-closed set of X. Since $(g \circ f)$ is strongly $I_{g\delta s}$-closed function $(g \circ f)(A)$ is $I_{g\delta s}$-closed in Z. Also since g is $I_{g\delta s}$-irresolute and injective, $g^{-1}(g \circ f)(A) = f(A)$ is $I_{g\delta s}$-closed set in Y. Therefore f is strongly $I_{g\delta s}$closed.

Theorem 5.7. *For any bijection,* $f : (X, \tau) \to (Y, \sigma, I)$ *the following statements are equivalent.*

(1) f^{-1} *is* $I_{g\delta s}$*-irresolute.*

(2) f *is a strongly* $I_{g\delta s}$*-open function.*

(3) f *is a strongly* $I_{g\delta s}$*-closed function.*

Proof: Similar to the proof of theorem 3.15.

Definition 5.8. *A function* $f : (X, \tau) \to (Y, \sigma, I)$ *is said to be quasi* $I_{g\delta s}$*-closed (resp. quasi* $I_{g\delta s}$*-open), if for each* $I_{g\delta s}$*-closed (resp.* $I_{g\delta s}$*-open) set* F *of* X*, f(F) is closed (resp. open) set in* Y.

Remark 5.9. *Every quasi* $I_{g\delta s}$*-closed function is closed, strongly* $I_{g\delta s}$*closed and* $I_{g\delta s}$*-closed function.*

Remark 5.10. *Every quasi* $I_{g\delta s}$*-closed function is* $I_{pg\delta s}$*-closed.*

Remark 5.11. *Following diagram is obtained from the Definitions.*

$I_{pg\delta s}$*-closed*
↗ ↑ ↘
quasi $I_{g\delta s}$*-closed* → *Strongly* $I_{g\delta s}$*-closed* → $I_{g\delta s}$*-losed*
↘ ↗
closed

Theorem 5.12. *A surjective function* $f : (X, \tau, I) \to (Y, \sigma)$ *is quasi* $I_{g\delta s}$*-closed (resp. quasi* $I_{g\delta s}$*-open), if and only if for any subset* B *of* Y *and each* $I_{g\delta s}$*-open (resp.* $I_{g\delta s}$*-closed) set* U *of* X *containing* $f^{-1}(B)$*, there exists an open (resp. closed) set* V *of* Y *containing* B *and* $f^{-1}(V) \subset U$.

Proof ; Similar to the proof of theorem 3.9.

Theorem 5.13. *A function* $f : (X, \tau, I) \to (Y, \sigma)$ *is quasi* $I_{g\delta s}$*-closed if and only if* $cl(f(A)) \subset f(I_{g\delta s} - cl(A))$ *for every subset* A *of* X.

Proof: Suppose that f is quasi $I_{g\delta s}$-closed function and $A \subset X$. Then $I_{g\delta s} - cl(A)$ is $I_{g\delta s}$- closed set in X. Therefore $f(I_{g\delta s} - cl(A))$ is closed in Y. Since $f(A) \subset f(I_{g\delta s} - cl(A))$, implies $cl(f(A)) \subset cl(f(I_{g\delta s} - cl(A))) = f(I_{g\delta s} - cl(A))$. This implies, $cl(f(A)) \subset f(I_{g\delta s} - cl(A))$.
Conversely, A is any $I_{g\delta s}$-closed set in X. Then $I_{g\delta s} - cl(A) = A$. Therefore, $f(A) = f(I_{g\delta s} - cl(A))$. By hypothesis, $cl(f(A)) \subset f(I_{g\delta s} - cl(A)) = f(A)$. Hence $cl(f(A)) \subset f(A)$. But $f(A) \subset cl(f(A))$ is always true. This shows, $f(A) = Cl(f(A))$. This implies f(A) is closed set in$Y$. Therefore, f is quasi $I_{g\delta s}$-closed function.

Theorem 5.14. *Let* $f : (X, \tau, I) \to (Y, \sigma, J)$ *be a function from a space* X *to a* $TI_{g\delta s}$*-space* Y.
Then following are equivalent

(1) f *is strongly* $I_{g\delta s}$*-closed function.*

(2) f *is quasi* $I_{g\delta s}$*-closed function.*

Proof :

(1) $\Rightarrow$ (2) Suppose (1) holds. Let $\mathbf{F}$ be a $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{X}$. Then $\mathbf{f(F)}$ is $\mathbf{I}_{g\delta s}$-closed in Y. Since Y is $T\mathbf{I}_{g\delta s}$-space, $\mathbf{f(F)}$ is closed in Y. Therefore $\mathbf{f}$ is quasi $\mathbf{I}_{g\delta s}$-closed function.

(2) $\Rightarrow$ (1) Suppose (2) holds. Let $\mathbf{F}$ be a $\mathbf{I}_{g\delta s}$-closed set in $\mathbf{X}$. Then $\mathbf{f(F)}$ is closed and hence $\mathbf{I}_{g\delta s}$-closed in Y. Therefore $\mathbf{f}$ is strongly $\mathbf{I}_{g\delta s}$-closed function.

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