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Effect of Apos Instructional Approach with Geogebra on Pre-Service Teachers' Performance in Linear Programming

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Abstract

Background: The Pedagogical process of using Technology is growing at a phenomenal rate and has been proven useful as a tool in supporting and transforming teaching and learning, especially in the Mathematics classroom. As a result, educationists see the urgent need for integrating technology into students' mathematical activities.

Objective: the purpose of this quasi-experimental study was to investigate students' understanding of learning linear programming using GeoGebra.

Method: One hundred (100) pre-service teachers from Abetifi Presbyterian College of Education, College Algebra students participated in this study with one group assigned as the experimental and the other as the control group respectively. The control group was taught linear Programming using the lecture method while the experimental group underwent learning using the Geogebra approach. The Pre-Service teachers' mathematics achievement was measured using post- tests at the end of the intervention, and a questionnaire was also used to ascertain the impact of GeoGebra on their understanding. The test format was based on College Algebra EBS102 Course Outline.

Results: Independent samples t-test results showed that there was a significant difference in mean mathematical achievement between the GeoGebra Experimental group ($M = 78.14, SD = 8.57$) and the Control group($M = 65.38, SD = 7.34$); $t(49) = 21.21, 0.000 < 0.05$. This study also found that, there was statistically significant difference in the pre-test control group ($M = 46.12, SD = 16.036$) and the Post-test Control group($M = 65.38, SD = 7.35$); $t(49) = 6.10, 0.034 < 0.05$. The findings also showed that there was a statistically significant difference in the scores for the pre-test Experimental group ($M = 43.28, SD = 26.040$) and the scores of the Post-test experimental group ($M = 78.140, SD = 8.571$); $t(49) = 9.46, 0.000 < 0.05$.

Conclusions: These findings showed that the use of GeoGebra enhanced the students' performance in learning linear programming. It was also statistically inferred from questionnaires through percentage testing, that students instructed with GeoGebra were more motivated to learn linear programming than those instructed without the software, hence it was recommended that teachers employed GeoGebra software in teaching and learning Linear Programming and any other mathematics topics.

Keywords: APOS Instructional, GeoGebra, Pre-Service Teachers' Performance, Linear programming

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1. Introduction

Pre-service Teachers with weak mathematics backgrounds often have problems when formulating and interpreting Linear Programming questions. This mostly comes from the weak understanding they have in interpreting word problems into symbolic form. They read the word problem quickly and then grope for the appropriate algebraic expressions. Even though the examples are irrelevant to the current topic, these expressions frequently mimic those found in text or in-class examples. When the learner is finished, he or she may be unable to describe what the variables in the program represent, the meaning of the constraints, or the importance or logic of any offered solutions. Even among otherwise good students, this haphazard approach is

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widespread, and overcoming it necessitates a more intuitive approach to formulation.Some of these issues can be addressed with the GeoGebra application, which is now frequently used in teaching Linear Programming (LP) formulation and solutions.

The teaching materials utilized by teachers, classroom management, teacher content expertise, and personality, as well as linking themes to real-life problems, are all elements that influence students' views about Mathematics (YilmazAltun&Olkun, 2010), and teaching methods (Papanastasiou, 2000). Mathematics can be regarded as a challenging subject. Understanding the ideas and formulas used to describe anything is an important part of learning mathematics. In a typical classroom, students are challenged to investigate complex issues. Learning challenges can be solved thanks to developments in multimedia technologies. Teaching and studying Mathematics presents a more complex problem, as teachers must blend mental, stationery, and digital methods for teaching and learning abstract mathematical topics that are difficult for children to grasp (Prieto, SordoJuanena& Star, 2013). Technology plays a pertinent role in the overall development of the educational process (Gursul and Keser, 2009). Existing technology equipment such as GeoGebra, Geometer's Sketchpad and Mathematica should be used to the maximum by the educators. The use of technology is important because it serves as an object of education, which affects the learning content and objectives, and as a medium to improve the teaching and learning process (Voogt, 2008).

The major problem that this study seeks to address is poor achievement in Linear Equations and Inequalities including linear programming that has its origin in an inadequate background in Algebra and poor motivation to learn it. The emphasis was to discover whether the method of instruction (computer assisted instruction using GeoGebra) motivates students, enhances their problem-solving techniques and ultimately improves their achievement in algebra. This study seeks to investigate the effect of integrating GeoGebra into the teaching of Linear Programming in Algebra on students' achievement through the use of the APOS theorem. The key variables affecting students' achievement focused on in this study are students' Actions, Processes, Objects, and Schemas, (APOS). The researchers believe there are wide knowledge gaps in the effective teaching and learning (instructional techniques and strategies) of Linear Programming.

2. Methodology

Research Design

The study adopted the quasi-experimental research design because 'it provided the best approach to investigating cause and effect relationships' (McMillan, 2000, p. 207). A quasi-experiment is an empirical study that determines the causal effects of an intervention on the population it is intended to benefit, according to Dinardo (2008). This view is also supported by Fraenkel and Wallen (2010), who argued that quasi-experimental research is a way to establish cause-and-effect relationships. Gribbons and Herman (1997) concur that quasi-experimental research shares similarities with the traditional experimental design or randomized controlled trial, but quasi-experiments lack the element of random assignment to treatment or control. This study was a quasi-experimental of non-equivalent comparison group design. The reason for this decision was that practically, it was not possible to assign the students randomly into groups because of the different timetables that the classes followed

Population

Vanderstoep and Johnston (2009), the population is the universe of people to which the study could be generalized. There are two types of population in any educational research study, the target population, and the accessible population. According to Amedahe (2002), the target population of a study is the aggregate of cases about which the researcher would like to generalize and it is the units from which the information is required and studied. In addition, Amedahe (2002) explains the accessible population as the designated criteria that are accessible to the researcher as a pool of subjects for a study. The population for the research is level 100 College Geometry Students of Abetifi Presbyterian College of Education. The research focused on those pursuing Mathematics ICT as their elective course. A total population of about two hundred (200) students was considered for this research work, with the main focus being on the level 100 College Algebra students.

Sampling Procedure and Sample

The sample size for this study was considered according to the effect size, power, significance level, and the number of variables used in this study (Teijlingen& Hundley, 2001). According

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to the literature, small effects are difficult to detect and in practice, researchers would generally not invest in studies where small effects exist. Medium-sized effects are worth the effort of researchers where a sample size between 80 and 200 could establish differences depending on the power of the test. Burn & Grove (1993) also defines a sample to involve the examination of a carefully selected proportion of the units of a phenomenon to help extend knowledge gained from the study of the part to the whole from which the part was selected. Therefore, a sample size of one hundred (100) students was selected from a population of 200 students for the study. Thus Fifty(50) students for the experimental group and another fifty (50) students for the control group respectively, making a total of one hundred (100) respondents; this represents 50% of the population being used. Coincidentally, in statistical analysis a sample size of 50 or more is classified as a large sample, as a result, any relevant discrete statistical and inferential statistical analysis can be done (Castillo, 2010).

The National Education Association Research Bulletin (1960) published the formula below for determining the sample size for the known population size.

S = $X^\wedge 2\ NP\ (1-P) \div d^\wedge 2\ (N-1) + X^\wedge 2\ P(1-P)$

S = required sample size

X^2 = the table value of chi-square for 1 degree of freedom at the desired confident level (3.841)

N = the population size

P= the population proportion (assumed to be 0.50 since this would provide the maximum sample size

d= the degree of accuracy expressed as a proportion (0.05)

Sampling procedure

In this study, a non-Probabilistic sampling procedure was used. Also, Convenience sampling was used because it is inexpensive and participants are readily available (Castillo, 2010). In addition, Ferrance (2000), argued that research studies conducted by educators themselves, in a familiar school setting, with their students, would help solve real problems experienced in schools and thus contribute towards improving teaching and student achievement.

Research Instruments

The instrument used in this study is the performance tests; pre-performance test and post-performance test and also a questionnaire. The performance tests were used to compare what they knew before in a pre-performance test and what they experienced in the post-performance test. This was categorized into four phases. The performance phase was a lecture approach that was done to assess the rate of students' understanding of the concept of linear programming. The first phase was the pre-achievement tests phase which consists of one question and is carried out simultaneously on both the experimental group and the control group. In this phase, both groups were given the same question after a tutorial was conducted to assess the rate of understanding during the lectures and how they perceive the topic in lecture form. This allowed the researcher to categorize the students into control and experimental groups respectively. The experimental group was the students whom the researcher saw that was finding a bit of difficulty in understanding the concept, whilst the control group was those who were able to solve the question but needs extra tutorials.

The experimental group's second phase involves intervention utilizing GeoGebra, whereas the control group was instructed using conventional teaching techniques (without GeoGebra). The third phase is the post-performance test for both groups after two weeks. After they have gone through the three phases, the test results were evaluated to determine whether GeoGebra Affects students' achievement test results for the topic of linear programming. The fourth phase was where the researcher used questionnaires to assess students' interest and understanding of the use of the GeoGebra software.

Treatments (Control and experimental groups)

The treatment groups were identified by the researcher based on an oral interview with the students during class sessions. Students' concepts and understanding of linear programming were assessed. About 70% of the students declared that they were taught in Senior High School (SHS) though with little understanding of the concept and in addition could not even finish with the topic. About 30% were emphatic that it was treated thoroughly but through the talk and chalk approach. This prompted the researchers to conduct a short written test to assess their level of understanding. The result gave the researchers an informed decision to group them into two.

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Control Group

The control group was taught by the researcher using the traditional talk-and-chalk teaching method. Three content developments with worksheets and graphs, similar in content to the experimental groups' worksheets and graphs, were used. All the questions and tasks were the same for the two groups. The difference was how students carried out their tasks. In the control group, the talk-and-chalk teaching method was used; students learn primarily by listening to the teacher and reading whatever the teacher writes on the chalkboard (auditory and visually). In the experimental group students learn in three different ways: visually, auditory and kinaesthetically. Each lesson was two-hour long, and teaching was done for four days within the week.

Fifty (50) students were in the control group and another 50 students in the experimental group respectively. The researcher decided to write on a sheet of paper numbers from 1 – 50 for the students to select from. The chosen numbers were used to identify the participants. The researchers chose to use numbers to hide the identity of the participants. These numbers were used in both the Pre-test and the Post-test respectively. A pre-test was conducted on linear programming for both the control and experimental group to assess the students' knowledge and abilities. A total of nine (9) questions each carrying fifteen (15) marks, were given to students during the extra contact hours to solve individually. Thus three (3) pre-test questions for both experimental and control groups for assessment, three Post-test questions for the Control group, and another three for the experimental group (see Appendix C). Each student was given a printed question paper and answer booklet which he or she was supposed to use.

The duration of both tests was forty-five (45) minutes. The answers to the Pre- Test were marked using a prepared marking scheme made by the researchers. The pre-test marks were recorded out of thirty (30) and later converted to one hundred percent (100%), and the Post-test was also marked out of forty-five (45) and was later converted to one hundred percent (100%). The researchers used four credit hours for the exercise. Thus, two hours for each session. The meeting days were Mondays and Tuesdays with two hours duration each for the control group. For two weeks, teaching took place during the additional contact hours on Wednesdays and Thursdays for the experimental group.

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The experimental group

The experimental group was taught using GeoGebra. Each of the students had a laptop with GeoGebra software installed on it. With a laptop attached to an overhead projector, the teacher gave lessons and gave examples. After two days of GeoGebra and computer introductory lessons and one day of topic introduction (2- hour lessons per day), content development worksheets were used during lesson delivery. A total of nine lessons were delivered to each group (control and experimental). Each lesson was two hours long. The worksheets had 'open-ended' questions to allow students to explore different solution strategies and/or skills of answering Linear programming questions. Despite the diverse teaching and learning strategies, the substance of the content development worksheets was the same for the experimental group and the control group.

The intervention activity using the GeoGebra method (with computer) using APOS phases of instruction

Here the researchers introduced the usage of GeoGebra through the use of the APOS Phases presentation. Thus to help students progress from one level to the next, Dubinsky proposed a sequence of four phases of learning,

i. Phase 1: Action: The teacher engages the students in conversation about the topic of study, evaluates their responses, learns how they interpret the words used and gives them some awareness of why they are studying the topic, to set the stage for further study, thus mental manipulation to transforms abstracts into objects

ii. Phase 2: Process: students can see the process as a whole, can use multiple representations, can reverse the process, compose with other processes, etc. that are to say students actively explore the topic of study by doing short (often one-step) tasks designed to elicit specific responses. These steps help students acquaint themselves with the objects from which algebraic ideas are abstracted.

iii. Phase 3: Objects. Students can distinguish compositions (Algebraic functions applied consecutively) from transforms (reality). In this phase, students learn to express their opinions about the structures observed during class discussions. The teacher leads students' discussion of the objects of study in their own words so that students become explicitly aware of the objects of study.

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iv. Phase 4: Schema is what allows one to decide if actions, processes, objects, and other schemas which are linked by some general principles to form a framework can be used in a particular mathematical situation here the teacher challenges students with more complex tasks that can be completed in different ways. The teacher encourages students to solve and elaborate on these problems and their solution strategies.

Dubinsky has proven that by replacing the lecture method with constructive, interactive methods involving computer/mathematical activities and cooperative learning the amount of meaningful learning that takes place, can radically improve, and that "Experience, theory, and research all point to the fact that verbal explanations that do not relate to the students' prior experience are quite ineffective" (Dubinsky, 1989).

Data Collection Procedure

Quantitative and qualitative data were employed in this study. Quantitative in the sense that the research seeks to explain, predict and control phenomena of interest. The division of the participants of the study was done by the school administration before the study. The students were picked randomly as an experimental group and a control group by the researchers. In this manner, the assignment of participants into the groups was not manipulated by the researchers. This was done to ensure transparency and also figure out the true performance of the students. To correct for any possible difference in their ability and knowledge before the intervention, both groups were administered the linear programing test (LPT) by lecture method along with the mathematics and technology attitude scale (MTAS) with the use of the GeoGebra.

Data analysis

This study generated mainly quantitative data from tests (pre-test and post-test) and questionnaires. Data was jointly analysed using APOS levels of geometric understanding and traditional descriptive statistical methods and inferential statistical methods. APOS levels of algebraic understanding were analysed for both the control group and the treatment group after the treatment (teaching) to show whether there was a difference in the achievements of the students in the two groups. Descriptive statistics such as t-test was used to describe and compare sets of data from the study. The statistical package for social sciences (SPSS) version 20 was used for the inferential analysis

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of the data. Inferential statistics are concerned with making predictions or inferences about a population from observations and analyses of a sample. The results of the analysis of the sample can be used to generalize information about the population that the sample represents. Descriptive and inferential statistics were calculated and analysed, such as measures of central tendency and significance testing, (t-test). The objective of this study was to reveal whether there is a significant relationship between the independent variable (learning with GeoGebra) and the three dependent variables of this study (problem-solving, achievement and motivation, and/or motivation).

Ethical Considerations

When proposing any project, it is essential to pre-empt any issues that may arise during the data collection and analysis phases to act on such issues and put solutions in place. The data collection method for this project does provide a unique insight into students' performance in the area of Linear Programming an aspect of College Algebra, the software that was used for the study is very simple, low cost, and low maintenance. However, the main ethical concern that needs addressing with this method is that of privacy. By this, students did not write their names on any of the tests (Pre-test and Post-Tests) and questionnaires that were provided to ensure confidentiality. The potentially invasive nature of such a method means it may be difficult to find participants who agree to the recording as people may feel uncomfortable with their conversations and interactions being recorded and consequently analysed (Tang, Liu, Muller, Lin &Drews, 2006).

Thus the college's ethical guideline was followed so that any feelings of discomfort can be avoided. Firstly, 'informed consent was adhered to. Participants were made to be fully aware of what the researchers involved and how the research was used. Secondly, the anonymity of participants was protected and all the data collected was used only for the study and destroyed after completion. Thirdly, all participants have the right to withdraw from the study at any time and inform researchers if there is any data they are uncomfortable with being used as part of the study.

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3. Results

Demographic Characteristics of the Respondents

Table 1 presents the demographics of the 100 pre-service teachers comprising $\mathbf{50}\,(\mathbf{50}\%)$ Control groups and $\mathbf{50}$ Experimental groups $\mathbf{50}\,(\mathbf{50}\%)$, who took part in the research. The control group was those taught with the normal lecture approach, while the Experimental group was those taught with both the lecture approach and the GeoGebra Approach. Table 1 shows that the pre-service teachers under the control group were made up of $\mathbf{32}$ males representing $\mathbf{64}\%$ and $\mathbf{18}$ females representing $\mathbf{36}\%$. Pre-service teachers under an experimental group, out of the $\mathbf{50}$ sampled for the study, $\mathbf{30}$ were males representing $\mathbf{60}\%$ and $\mathbf{20}$ females representing $\mathbf{40}\%$. **Table 1 Demographics of respondents (N= 100).**

Gender	*Control Group*		*Experimental Group*		*Total*	
	N	*%*	*N*	*%*	*N*	*%*
Male	*32*	*64*	*30*	*60*	*62*	*62*
Female	*18*	*36*	*20*	*40*	*38*	*38*
Total	*50*	*100*	*50*	*100*	***100***	***100***

Table 2 shows that out of the total of 100 students who participated in the study, thus 50 Control students and 50 Experimental students, the mean or average age of both groups were 21.46 which represents25% in both groups.

Table 2 Ages of Respondents

Group	N	Average Age	Percentage (%)
Control	50	21.46	25
Experimental	50	21.14	25
Total	100	42.60	50

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Errors Pre Service Teachers CommitWhen Solving Linear Programming Questions

The first research question investigated the common errors that pre service teachers commitwhen solving linear programming questions. Error analysis was done with. The

APOS steps were used in activity form to analyse the students' misunderstandings on the *linear programming* tasks. The most common errors identified by the researchersare classified under the following two sub-categories:

i. Errors in solving linear programming tasks (interpretation);
ii. Errors in plotting inequalities graphically.

Errors in solving linear programming tasks (interpretation)

The most common error committed by over 75% of the pre-service teachers in solving the linear programming tasks was 'division of negative or positive variable involving linear inequalities in two variables. That is, the pre-service teachers' difficulties were errors related to the change ofsymbols or notations pertaining to inequalities with two variables which might seem trivial but not. Most of the students found it difficult in solving an inequality question like $3x - 2y \leq 6$; especiallya situation by which there is a division of by a negative integer or variable to obtain the solution. Exhibits 1 and 2 show examples of the errors related to changing symbols or notations pertaining to inequalities with two variables and by a negative integer or variable.

Clearly from Exhibit 1, one would realize that these students were unable to identify whether the greater than sign will change or remain the same after division by negative number. What the students forgot to identify was that, after subtracting 3x from both sides of the inequality, they should have divided both sides of the inequality by -2 to obtain the y variable, which will in turn change the inequality sign from less than to greater than hence making the inequality or the final answer to be $y \geq \frac{6-3x}{-2}$.

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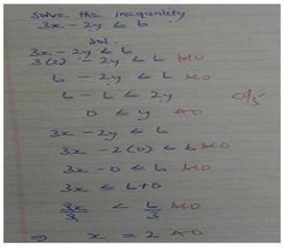

Exhibits 1

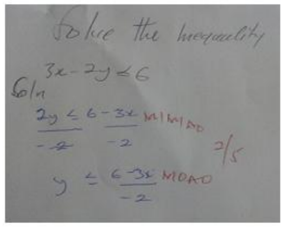

Exhibit 2

Secondly, from Exhibit 2, the student substituted 2 in place of x at $3x$ and then solved to make y the subject and again substituted 0 in place of y at $2y$ and then solved for the x variable, which was a total error. Instead of adding both sides of the inequality by $2y$ and then subtracting both sides of the inequality by 6 to get $3x - 6 \leq 2y$ and then divide both sides by 2 to get the y variable thus $(3x - 6)/2 \leq y$. These students were unable to identify which variable was to be made the subject of the inequality and even what sign to use at the final stage of the answer. These clearly show the misunderstanding of students about the procedures for solving the inequalities. This buttress the Action and Process stage of the APOS theorem, which states that;

Action is an external conception in the sense that each step of the transformation needs to be performed explicitly and instructed by external guidance; additionally, each step operates the next, that is, the steps of the action cannot be imagined and none can be skipped (Arnon et al., 2014, p. 19).

Errors in graphical plotting of constraints (feasible region)

The second type of error that the researcher identified was confusion with respect to the direction in terms of the graphical shading. Because of the misunderstanding they have concerning how the signs in the inequality is used (thus $3x - 2y \geq 6, 3x + 4y > 12, x \geq 0, y \geq 0$), plotting the graph becomes very difficult. Exhibit 3 and 4show how some students graphically represented the inequalities below.

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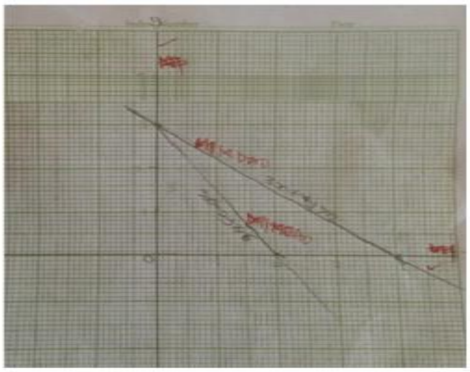

Exhibits 3

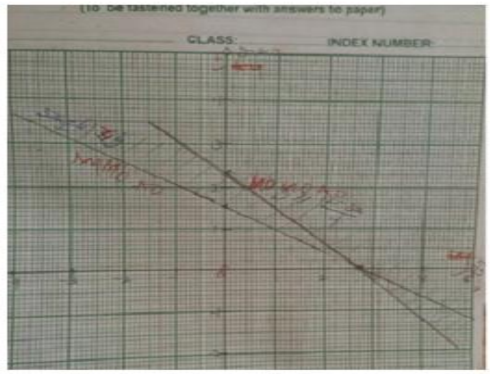

Exhibit 4

From Exhibit 4, the students were supposed to use the intercept rule to find the coordinates or points for the various inequalities to enable them to draw the lines. After that make y, the subject identifies the direction of the line in terms of shading. Thus $3x - 2y \geq 6 \;\approx\; 2y \geq 3x - 6$ dividing both sides by 2 gives $y \leq 3x/3 - 6/2 \;\approx y \leq 3x/3 - 3$this means that the variables of y are less than or equal to $3x/3 - 3$, hence, shading the lower part of the graph. For the inequality$3x + 4y > 12$, following the same procedure by making y the subject, $4y > 12 - 3x$ dividing both sides by 4, we have $y > 12/4 - 3x/4$ which gives $y > 3 - 3x/4$. This means the direction of y is greater than $3 - 3x/4$, hence shading the upper part of the graph. This will in turn meet the lower part at a point that is considered the feasible region at the point where they all intersect. It could be deduced that students' inability to identify the less than ($<$) and the greater than ($>$) etc. symbols and their direction (interpretation) on the graph in terms of shading cause them to make all those errors.

From Exhibit 4, though it is the same question, the students should have followed the same process as explained in Exhibit 4, the students were supposed to shade to identify the feasible region but did nothing on that. This means the process they have to follow to interiorize into objects was not clearly understood by students. These were some of the major causes of the errors in both the linear graph and the shading, hence affecting the location of the feasible region. The distribution of pre-service teachers making errors in solving the linear programming tasks is presented in Table 3.

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Table 3Distribution of pre-service teachers making errors in solving linear programming tasks

		Control		Experimental	
ITEM	Total in Sample	Total Correct with (%)	Total wrong with (%)	Total Correct with (%)	Total wrong with (%)
Calculation Error	50	20(40)	30(60)	19(38)	31(62)
Graphical Error	50	9(18)	41(81)	10(20)	40(80)
Shading Error	50	10(20)	40(80)	10(20)	40(80)
Identification of Feasible Region	50	11(22)	39(78)	11(22)	39(78)

Source: field survey 2020

From Table 3, it was indicated that the major problem of students was the calculation error, it was revealed that a total of 20(40%) and 19(38%) in both the Control and experimental group respectively had the calculation correct, and a total of 30(60%) and 31(62%) had error calculation. Graphically, a total of 9(18%) and 10(20%) had their graphs correctly drawn while 41(81%) and 40(80%) had their graphs wrongly drawn by groups.

In addition, in terms of directional shading, 10(20%) and another 10(20%) had it correct while 40(80%) and another 40(80%) had it wrong. Finally, considering the identification of feasible region, 11(22%) and another 11(22%) had it right, 39 (78%) and an additional 39(78) for both control and experimental groups had it wrongly drawn and this could be attributed to poor concept involved in the calculation. This indicates that the errors in calculation turned out to bring a lot of errors in the work of the students. The pretest and marks of both experimental and control groups show the effects of these errors on the students' scores in both experimental and control groups.

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Application of APOS Theorem of instruction using GeoGebra to change students' Performance in the Learning of Linear Programming

Table $\mathbf{4}$ compares the pre-test and post-test results of the students in the two groups. In the experimental group, the results showed an improvement in students' performance (that is, mean score) in carrying out linear programming tasks increased from $\mathbf{43}$%to $\mathbf{78}$%. There was also an improvement in the control group's performance, with the mean score increasing from $\mathbf{46}$% to $\mathbf{65}$%.But the increase in the former was higher. The minimum scores the groups obtained in the pretest were $\mathbf{9}$ and $\mathbf{10}$ for the control and experimental groups respectively, while the maximum scores were $\mathbf{70}$ and $\mathbf{68}$ respectively. However, in the post-test, the minimum scores increased substantially to $\mathbf{50}$ and $\mathbf{60}$ for the control and experimental groups respectively, while the maximum score for the experimental group nearly reached $\mathbf{100}$%. These are indications that in the post-test, every student's performance slightly increased in the control group.

To ascertain whether or not the difference observed in the means are statistically different, a paired samples t-test was conducted to test the null hypothesis that there is no significant difference between the pre-test and post-test scores of students in the experimental and control groups. Table $\mathbf{5}$ presents the results of the paired samples t-test on the pre-test and post-test performance of students taught with APOS theorem of instruction using the GeoGebra approach.

Table 4Descriptive statistics of the students' performance

		N	Min	Max	Mean	Std. Deviation
Pretest	Control	50	9	70	46.12	16.04
	Experimental	50	10	68	43.28	18.27
Posttest	Control	50	50	78	65.38	7.35
	Experimental	50	60	97	78.14	8.57

Source: Field Survey, 2020

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Table 5 presents the results of the paired samples t–test on the pre-test and post-test performance of students in the experimental and control groups**.** With regards to the experimental group, the paired sample t-test results showed the post-test mean score $(M = 78.14, SD = 8.57)$ and the pretest score $(M = 43.28, SD = 18.27)$ is not the same but are statistically significantly different at a 5% significant level. Similarly, to the control group paired sample t-test results showed the post-test mean score $(M = 65.38, SD = 7.35)$ and the pretest score $(M = 46.12, SD = 16.04)$ are not equal and are statistically significantly different at a 5% significant level. Therefore, by the conclusion, since the p-value is less than the significance level (i.e., $0.000 < 0.05$) the null hypothesis is rejected, and accept the alternative hypothesis. This concludes that there is a statistically significant difference between the mean scores of the pretest and posttest.

The effect size of the experiment on each group was calculated using Cohen's d and the results are shown in the last column of Table 5. The effect size for the treatment group (2.44) was found to be larger than that of the control group (1.54) implying the APOS theorem of instruction using GeoGebra led to better performance on linear programming than the traditional approach. But this outcome is an indication that a well-structured traditional approach to teaching can also improve students' performance in learning linear programming.

Table 5: Results of the paired samples t–test on the pre-test and post-test performance of students in the experimental and control groups

		N	Mean	Std. Dev.	T	Df	Sig. (2-tailed)	Effect size
Control	Pretest	50	46.12	16.04	-6.100	49	.000	1.54
	Posttest	50	65.38	7.35				
Experimental	Pretest	50	43.28	18.27	-9.466	49	.000	2.44
	Posttest	50	78.14	8.57				

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Table 6 shows how Students' perceive the usefulness of APOS Theorem of instruction using GeoGebra in Teaching and Learning Linear Programming. The findings reveal that nearly all the students agreed with the statements. None disagreed with the statement that "the use of APOS theorem of instruction makes the learning of GeoGebra interesting" or the statement that "finding the feasible regions with their coordinates under GeoGebra is very simple and easy". The results show that the students' in the experimental group were not only able to visualize the questions so as to understand the abstract content through visualization but also felt confident using the GeoGebra software and were engaged throughout the learning process. The experimental group was also made to rate their engagement and motivation to learn under the APOS instruction with GeoGebra, and the results are presented in Table 7.

Table 6Experimental group students' rating of statements about the effectiveness of the APOS theorem of instruction using GeoGebra approach

S/N	ITEM	Agree	Strongly Agree	Disagree	Strongly Disagree
a)	The use of APOS theorem of instruction makes learning of GeoGebra interesting	16 (33)	67 (67)	0	0
b)	I felt confident using the GeoGebra software during the activities	13 (26)	74 (74)	0	0
c)	I was very engaged in the learning process using GeoGebra	24 (48)	52 (52)	0	0
d)	I was able to visualize and answer the questions after each activity	32 (64)	36 (36)	0	0
e)	APOS theorem, toppled with GeoGebra motivates me to understand the abstract	31 (62)	38 (38)	0	0

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	content through visualization				
f)	Finding the feasible regions with its coordinates under GeoGebra is very simple and easy.	16 (32)	68 (68)	0	0

Field survey, 2020

From Table 7, it is noticed that 80% of the students disagreed with the statement that "using GeoGebra is extremely hard so it takes the enjoyment of my learning linear programming" corroborating the statement that learning linear Programming using GeoGebra makes them think critically which 76% agreed with. All (100%) of the students in the experimental group agree the approach made them think creatively and critically in discussion during the question-and-answer session and enhanced their learning ability in linear programming. The responses of the experimental group reported in Tables 6 and 7 were not surprising because it was observed during the lessons that the student's attitude toward making presentations and asking questions in class was boosted due to the opportunity they had to experience the APOS instruction with GeoGebra. From the interaction the researcher had with the experimental group, it was revealed that the students loved to use technology. The majority of the students in this group complimented the use of technology because they had not had the opportunity to use the computer lab since they came to the college.

Table 7 Experimental group students' ratings of their engagement and perceived effectiveness of the APOS theorem of instruction using GeoGebra approach

S/N	ITEM	Agree	Strongly Agree	Disagree	Strongly Disagree
a)	Using GeoGebra is extremely hard so it takes the enjoyment of my learning	5 (10)	5 (10)	31 (62)	9 (18)

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	linear programming.				
b)	I was able to think creatively and critically in the discussion and during the question-and-answer session	31 (62)	19 (38)	(0)	(0)
c)	Learning linear Programming using GeoGebra makes me do critical thinking a lot.	29 (58)	9 (18)	8 (16)	4 (8)
d)	GeoGebra instruction approach has enhanced learning ability in linear programming.	12 (24)	76 (76)	(0)	(0)
e)	I learnt a lot with clearer understanding using GeoGebra	13 (26)	74 (74)	(0)	(0)

Field survey, 2020

4. Discussion

The results of the analysis of the t-test on the performance of students taught using GeoGebra and those taught using the conventional method of instruction (talk-and chalk) indicated a significant difference in achievement in favour of the students taught with GeoGebra thus experimental group post-test mean score $(M = 78.14, SD = 8.57)$ and the pretest score $(M = 43.28, SD = 18.27)$ and the control group paired sample t-test results showed the posttest mean score $(M = 65.38, SD = 7.35)$ and the pretest score $(M = 46.12, SD = 16.04)$. The students exposed to GeoGebra achieved a higher average score compared to the control group of students.

The possible reasons for this finding could be that GeoGebra enabled students in the experimental group to check the correctness of their methods and the accuracy of their work. Being able to check one's work goes a long way in determining achievement levels. Because GeoGebra is dynamic, students in the experimental group had opportunities of re-examining their work, while those in the control group could not do the same. In the control group, teaching was limited to a few examples,

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because drawing many diagrams on the chalkboard consumed both time and space. In addition, the production of good-quality sketches requires competence in technical drawing skills, which not all teachers possess. GeoGebra-generated sketches are neat and accurate. GeoGebra allowed students in the experimental group real-time exploration opportunities. Consequently, this improved the learning process in terms of speed and quality (Ljajko&Ibro, 2013). When students learn using GeoGebra they spend less time drawing diagrams (sketches) and making calculations; this allows them more time to explore the characteristics of different circle theorems. All these factors could have contributed to the superior achievement of the experimental group.

Conclusion

The study has determined that the use of GeoGebra improves students' achievement, improves students' geometric thinking with the use of APOS instruction and motivates students to learn linear programming. Based on the findings of the study, the researcher recommends GeoGebra assisted instruction as has been instituted by University of Cape Coast in the teaching and learning of Algebra. Motivation is the key determinant of student achievement; hence any teaching and learning method that motivates learners to learn will go a long way in solving the Colleges of Education (Pre-Service teachers) students' problem of poor achievement in Algebra in particular and poor achievement in mathematics in general.

The results show that the use of GeoGebra in the teaching and learning of linear programming resulted in significant improvement of achievement of students' geometric thinking. Thus it was noticed that 80% of the students disagreed with the statement that "using GeoGebra is extremely hard so it takes the enjoyment, again corroborating the statement that learning linear Programming using GeoGebra makes them think critically, over76% of the students agreed. This proves that GeoGebra offers countless opportunities for pre-service teachers to progress towards mathematical explanations which provide a foundation for further deductive reasoning in mathematics. This study has revealed that using GeoGebra in teaching and learning not only increases students' achievement in general, but also increases achievement in quality learning.

References

[1]. Amedahe, F. (2002). *Fundamentals of Educational Research Methods.* Cape Coast: Mimeograph,University of Cape Coast.

[2]. Brijlall, D., &Maharaj, A. (2013). Pre-service teachers' understanding of continuity functions in the context of assessment items. South African journal of Higher Education, 27(40, 797-814.

[3]. Burns, N. & Grove, S. K. (2003). Understanding nursing research, 3rd Edition. Philadelphia. W.B Saunders Company Chiu, M.M. (2000) Group problem solving processes: Social interactions and individual actions. Journal for the Theory of Social Behaviour, 10 (1), 27 – 50 and 600 – 631.

[4]. Castillo, J. (2010, June 10). *Experimental Resources*. Retrieved April 4, 2018, from Retrieved from www.experiment-resources.com: http://www.experimentresources.com/sample-random.html.

[5]. Dubinsky, E. (1991a). Constructive Aspects of Reflective Abstraction in Advanced Mathematical thinking. New York: Springer Verlag.

[6]. Dubinsky, E. (1991b). Constructive Aspects of Reflective Abstraction in Advanced Mathematical thinking. New York: Springer Verlag.

[7]. Ferrance, E. (2000). Themes in education: Action research. *Providence*, 17(1), 144-159.

[8]. Gürsul, F. and H. Keser (2009). The effects of online and face to face problem based learning environments in mathematics education on student's academic achievement. *Procedia-Social and Behavioral Sciences* 1(1): 2817-2824.

[9]. JoglarPrieto, N., et al. (2013). Designing Geometry 2.0 learning environments: a preliminary study with primary school students. *International Journal of Mathematical Education in Science and Technology (ahead-of-print)*: 1-21.

[10]. Maharaj, A. (2010). An APOS analysis of students' understanding of the concept of a limit of a function. Durban: School of Mathematical Sciences: University of KwaZulu-Natal.

JOURNAL OF ALGEBRAIC STATISTICS
Volume 13, No. 3, 2022, p. 4046-4068
https://publishoa.com
ISSN: 1309-3452

[11]. Papanastasiou, C. (2002). Effects of background and school factors on the mathematics achievement. *Educational Research and Evaluation* 8(1): 55-70.

[12]. Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as two sides of the same coin. Educational Studies in Mathematics, 22, 1-36.

[13]. Teijlingen, V., & Hundley, V. (2001). The importance of conducting and reporting pilot studies:the example of the scottish births survey. *Journal of AdvanceNursing*, 34, 289- 295.

[14]. Vanderstoep, S. W., & Johnston, D. D. (2009). *Research Methods for Everyday Life: Blending Qualitative and Quantitative Approaches.* San Francisco: JosseyBass.

[15]. Yılmaz, Ç.,Altun, S. A., &Olkun, S. (2010). Factors affecting students' attitude towards Math: ABC theory and its reflection on practice. *Procedia-Social and Behavioral Sciences*, *2*(2), 4502-4506.