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Vectors of Identity matrix are the only Solution for a Special Linear System of Equations and Linear Programming Problems

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Abstract: This article is going to prove that the vectors of identity matrix are the only solutions for special kind of linear system of equations with n equations n unknown problems and for the Linear Programming Problems (LPP) problems of n variables if the objective function is Maximize and also we are also going to prove if it is given in LPPtoMinimize objective function the optimal solution is trivial solution (zero solution) and therefore MinZ=0.

Keywords:C-Matrix, LPP, System of Liner Equations, normal form

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I. Introduction

Definition 1.1: A square matrix is said to be C -Matrix if all the principal diagonal elements are same, in which rest of the elements are different form Princiapal diagonal element and at least two elements are different.

Examples:

1. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ is C -Matrix

2. $A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 3 & 2 & 6 & 8 \\ 5 & 6 & 2 & 8 \\ 6 & 6 & 7 & 2 \end{bmatrix}$ is C -Matrix

3. $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ is **not** C –Matrix because 1 is the principal diagonal element that should not present in other than principal diagonal element

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4. $A=\begin{bmatrix} 2 & 2 & 5 & 4 \\ 3 & 2 & 6 & 8 \\ 5 & 6 & 2 & 8 \\ 6 & 6 & 2 & 2 \end{bmatrix}$ is **not** C –Matrix because 2 is the principal diagonal element that should not present in other than principal diagonal element

Definition 1.2.A general system of *n* linear equations with *n* unknowns can be written as

AX=B

Where A= $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{nxn}$, X= $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and B= $\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

and A is called Coefficient Matrix with constants, X is a matrix with unknowns and B is a constant matrix

Definition 1.3: Linearprogrammingproblem(LPP) that can be expressed in

Minimize/Max Zc^Tx

Subject to $Ax \leq b$

and $x \geq 0$

where **x** represents the vector of variables (to be determined), **c** and **b** are vectors of (known) coefficients, *A* is a (known) matrix of coefficients,

Definition 1.4: An n-tuple (x1 ,x2….. xn) of numbers which satisfies the constraints given by (b) of G.L.L.P.is called a solution to G.L.P.P.

Definition 1.5: Any solution to a G.L.P.P. which satisfies the non-negativity restrictions of the problem is called a feasible solution to a general L.P.P.

Definition 1.6: AnyFeasible solution to a G.L.P.P. which optimizes (maximizes/ minimizes) the objective function of G.L.P.P. is called an optimum solution to the G.L.L.P.

Definition 1.7:A basic solution to the system is called degenerate if one or more of the basic variables vanish i.e. if any of the basic variable has zero value then it is called degenerate basic solution.

Definition 1.8: A basic solution the system is called non-degenerate if all the basic variables are non-zero (either positive or negative)

Definition 1.9: A feasible solution to L.P.P. which is also a basic solution to the problem is called a basic feasible solution (B.F.S.) to the L.P.P.

Problem 1.1: If A is C matrix and AX=B is the linear system of Equations (1) Then

i) If B is replaced with 1^{st} column of A in AX=B then the solution is $(1,0,0,..0)^T$

ii) If B is replaced with 2^{nd} column of A in AX=B then the solution is $(0,1,0,..0)^T$

iii) If B is replaced with 3^{rd} column of A in AX=B then the solution is $(0,0,1,..0)^T$

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iii) If B is replaced with nth column of A in AX=B then the solution is $(0,0,0,..1)^T$

Solution

Case i) Let a general system of *3* linear equations with *3* unknowns can be written as

1x+2y+3z=1

2x+1y+4z=2

3x+4y+1z=3

If we observe the coefficient matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 3 & 4 & 1\end{bmatrix}$ is a C matrix and last and first column are same in AX=B

Claim: The solution is x=1,y=0,z=0

The solution by Normal form method

The Augmented matrix

$$[AB]=\begin{bmatrix}1 & 2 & 3 & 1\\ 2 & 1 & 4 & 2\\ 3 & 4 & 1 & 3\end{bmatrix}$$

R_2->R_2-2R_1; R_3->R_3-3R_1;

$$\approx\begin{bmatrix}1 & 2 & 3 & 1\\ 0 & -3 & -2 & 0\\ 0 & -2 & -8 & 0\end{bmatrix}$$

$$\approx\begin{bmatrix}1 & 2 & 3 & 1\\ 0 & -3 & -2 & 0\\ 0 & -2 & -8 & 0\end{bmatrix}$$

R_2->R_2/-3

$$\approx\begin{bmatrix}1 & 2 & 3 & 1\\ 0 & 0 & 2/3 & 0\\ 0 & -2 & -8 & 0\end{bmatrix}$$

R_1->R_1-2R_2

$$\approx\begin{bmatrix}1 & 0 & 5/3 & 1\\ 0 & 1 & 2/3 & 0\\ 0 & -2 & -8 & 0\end{bmatrix}$$

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$R_3 \to R_3+2R_2$

$$\approx\begin{bmatrix} 1 & 0 & 5/3 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & -20/3 & 0 \end{bmatrix}$$

$R_3 \to (-3/20)R_3$

$$\begin{bmatrix} 1 & 0 & 5/3 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 \to R_1-(5/3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 \to R_2-(2/3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

It is in normal form Therefore the solution is x=1,y=0,z=0

Case ii) Let a general system of *3* linear equations with *3* unknowns can be written as

1x+2y+3z=2

2x+1y+4z=1

3x+4y+1z=4

If we observe the coefficient matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C matrix and secondand last column are same in AX=B

Claim: The solution is x=0,y=1,z=0

The solution by Normal form method

The Augmented matrix

$$[AB]=\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 1 & 4 & 1 \\ 3 & 4 & 1 & 4 \end{bmatrix}$$

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R_2->R_2-2R_1; R_3->R_3-3R_1;

$$\approx\begin{bmatrix}1 & 2 & 3 & 2\\ 0 & -3 & -2 & -3\\ 0 & -2 & -8 & -2\end{bmatrix}$$

R_2->R_2/-3

$$\approx\begin{bmatrix}1 & 2 & 3 & 2\\ 0 & 1 & 2/3 & 1\\ 0 & -2 & -8 & -2\end{bmatrix}$$

R_1->R_1-2R_2

$$\approx\begin{bmatrix}1 & 0 & 5/3 & 0\\ 0 & 1 & 2/3 & 1\\ 0 & -2 & -8 & -2\end{bmatrix}$$

R_3->R_3+2R_2

$$\approx\begin{bmatrix}1 & 0 & 5/3 & 0\\ 0 & 1 & 2/3 & 1\\ 0 & 0 & -20/3 & 0\end{bmatrix}$$

R_3->(-3/20)R_3

$$\begin{bmatrix}1 & 0 & 5/3 & 1\\ 0 & 1 & 2/3 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

R_1->R_1-(5/3)R_3

$$\begin{bmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 2/3 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

R_2->R_2-(2/3)R_3

$$\begin{bmatrix}1 & 0 & 0 & 1\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

It is in normal form Therefore the solution is x=0,y=1,z=0

Case iii) Let a general system of *3* linear equations with *3* unknowns can be written as

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1x+2y+3z=3

2x+1y+4z=4

3x+4y+1z=1

Claim: The solution is x=0,y=0,z=1

If we observe the coefficient matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C matrix and 3rd and last column are same in AX=B

The solution by Normal form method

The Augmented matrix

$$[AB] = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 1 & 4 & 4 \\ 3 & 4 & 1 & 1 \end{bmatrix}$$

R_2->R_2-2R_1; R_3->R_3-3R_1;

$$\approx \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & -3 & -2 & -2 \\ 0 & -2 & -8 & -8 \end{bmatrix}$$

R_2->R_2/-3

$$\approx \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & -2 & -8 & -8 \end{bmatrix}$$

R_1->R_1-2R_2

$$\approx \begin{bmatrix} 1 & 0 & 5/3 & 5/3 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & -2 & -8 & -8 \end{bmatrix}$$

R_3->(-3/20)R_3

$$\begin{bmatrix} 1 & 0 & 5/3 & 5/3 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

R_1->R_1-(5/3)R_3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

R_2->R_2-(2/3)R_3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

It is in normal form Therefore the solution is x=0, y=0, z=1

$\therefore$ If the coefficient matrix is C matrix and B is changing column 1, 2, 3 in AX=B then the solution is

$X_1=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, X_2=\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, X_3=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Problem1.2: Solve the linear Programming Problem using Simplex (Big M) method when the coefficient matrix is C matrix when objective function is Maximum and subjective to $AX\leq B, X\geq 0$

i) If B is replaced with 1st column of A in AX=B then the solution is $(1,0,0,..0)^T$

ii) If B is replaced with 2nd column of A in AX=B then the solution is $(0,1,0,..0)^T$

iii) If B is replaced with 3rd column of A in AX=B then the solution is $(0,0,1,..0)^T$

iii) If B is replaced with nth column of A in AX=B then the solution is $(0,0,0,..1)^T$

Case i)The last and first columns are same in AX≤B

and A is C matrix in LPP

Max Z=5x1+10x2+8x3

Subject to,

x1+5x2+2x3≤1;

4x1+1x2+4x3≤4;

2x1+4x2+x3≤2

Where x1,x2,x3≥0

Sol. Given Linear Programming Problem

Max Z=5x1+10x2+8x3

Subject to,x1+5x2+2x3≤1;

4x1+1x2+4x3≤4;

2x1+4x2+x3≤2

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Where x1,x2,x3≥0

If we observe the coefficient matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C matrix and last and first column are same in AX=B

Claim: The solution is x=1,y=0,z=0

The Problem is converted to canonical form by adding slack,surplus and artificial variables

1.As the constraint -1 of the type ≤ we should add slack variable S1

1.As the constraint -2 of the type ≤ we should add slack variable S2

1.As the constraint -1 of the type ≤ we should add slack variable S3

After introducing slack variables

Max Z=5x1+10x2+8x3+0S1+0S2+0S3

Subject to

x1+5x2+2x3+S1 =1

4x1+1x2+4x3 + S2 =4

2x1+4x2+x3 +S3 =2

Where x1,x2,x3.S1,S2,S3≥0

Iteration 1		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	1	1	**(5)**	2	1	0	0	1/5←
S2	0	4	4	1	4	0	1	0	4/1
S3	0	2	2	4	1	0	0	1	2/4
Z=0		Zj	0	0	0	0	0	0	
		Zj-Cj	-5	**-10** ↑	-8	0	0	0	

R1(new)=>R1(old)/5

R2(new)=>R2(old)-R1(new)

R3(new)=>R3(old)-4R1(new)

Negative minimum *Zj- Cj*is - 10 and its column index is 2. So, the entering variable is *x*2.

Minimum ratio is 0.2 and its row index is 1. So, the leaving basis variable is *S*1.

∴The pivot element is 5.

Entering = *x*2, Departing = *S*1, Key Element = 5

Iteration 2		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
X2	10	1/5	1/5	1	**2/5**	1/5	0	0	1/2 ←
S2	0	19/5	19/5	0	18/5	-1/5	1	0	19/18
S3	0	6/5	6/5	0	-3/5	-4/5	0	1	2/4
Z=2		Zj	2	10	4	2	0	0	
		Zj-Cj	-3	0	**-4** ↑	2	0	0	

R1(new)=>R1(old)*5/2

R2(new)=>R2(old)-(18/5)R1(new)

R3(new)=>R3(old)+(3/5)R1(new)

Negative minimum *Zj- Cj*is - 4 and its column index is 3. So, the entering variable is $x3$.

Minimum ratio is 0.5 and its row index is 1. So, the leaving basis variable is $x2$.

∴The pivot element is 2/5

Entering = $x3$, Departing = $x2$, Key Element =2/5

Iteration 3		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
X3	8	1/2	1/2	5/2	**1**	1/2	0	0	1
S2	0	2	2	-9	0	-2	1	0	1
S3	0	3/2	**3/2**	**3/2**	0	-1/2	0	1	1 ←
Z=4		Zj	4	20	8	4	0	0	
		Zj-Cj	-1	10 ↑	**0**	4	0	0	

R3(new)=>R3(old)*2/3

R1(new)=>R1(old)-(1/2)R3(new)

R2(new)=>R2(old)-2R3(new)

Negative minimum *Zj- Cj*is - 1 and its column index is 1. So, the entering variable is $x1$.

Minimum ratio is 1 and its row index is 3. So, the leaving basis variable is $S3$.

∴ The pivot element is 3/2 .

Entering = $x1$, Departing = $S3$, Key Element =3/2

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Iteration 4		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
X3	8	0	0	2	**1**	2/3	0	-1/3	
S2	0	0	0	-11	0	-4/3	1	-4/3	
X1	5	1	1	1	0	-1/3	0	2/3	
Z=5		Zj	5	21	8	11/3	0	2/3	
		Zj-Cj	0	11	**0**	11/3	0	2/3	

Since all Zj-Cj≥0
Hence, optimal solution is arrived with value of variables as : x1=1,x2=0,x3=0
Max Z=5

Caseii) The last and second columns are same in AX≤B,and A is C matrix in LPP

Solve the linear Programming Problem using Simplex (Big M) Method

Max Z=5x1+10x2+8x3

Subject to,

x1+5x2+2x3≤5;

4x1+1x2+4x3≤1;

2x1+4x2+x3≤4

Where x1,x2,x3≥0

Sol. Given Linear Programming Problem

Max Z=5x1+10x2+8x3

Subject to,x1+5x2+2x3≤5;

4x1+1x2+4x3≤1;

2x1+4x2+x3≤4

Where x1,x2,x3≥0

If we observe the coefficient matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C matrix and last and second column are same in AX=B

Claim: The solution is x=1,y=0,z=0

The Problem is converted to canonical form by adding slack,surplus and artificial variables

1.As the constraint -1 of the type ≤ we should add slack variable S1

1.As the constraint -2 of the type ≤ we should add slack variable S2

1.As the constraint -1 of the type ≤ we should add slack variable S3

After introducing slack variables

Max Z=5x1+10x2+8x3+0S1+0S2+0S3

Subject to

x1+5x2+2x3+S1 =5

4x1+1x2+4x3 + S2 =1

2x1+4x2+x3 +S3 =4

Where x1,x2,x3.S1,S2,S3≥0

Iteration 1		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	5	1	5	2	1	0	0	5/5
S2	0	1	4	1	4	0	1	0	1/1
S3	0	4	2	**(4)**	1	0	0	1	4/4 ←
Z=0		Zj	0	0	0	0	0	0	
		Zj-Cj	-5	**-10** ↑	-8	0	0	0	

Negative minimum *Zj- Cj*is - 10 and its column index is 2. So, the entering variable is *x*2.

Minimum ratio is 1 and its row index is 3. So, the leaving basis variable is *S*3.

∴The pivot element is 4.

Entering = *x*2, Departing = *S*3, Key Element =4

R3(new)=>R3(old)/4

R1(new)=>R1(old)-5R3(new)

R2(new)=>R2(old)-R3(new)

Iteration 2		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	0	-3/2	0	¾	1	0	-5/4	0
S2	0	0	7/2	0	**(15/4)**	0	1	-1/4	0 ⟶
X2	10	1	1/2	1	¼	0	0	1/4	4
Z=10		Zj	5	10	5/2	0	0	5/2	
		Zj-Cj	0	0	**-11/2** ↑	0	0	5/2	

Negative minimum *Zj- Cj*is -11/2 and its column index is 3. So, the entering variable is *x*3.

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is s2.

∴The pivot element is 15/4

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Entering = *x*3, Departing = s2, Key Element =15/4

R2(new)=>R2(old)*4/15

R1(new)=>R1(old)-(3/4)R2(new)

R3(new)=>R3(old)-(1/4)R2(new)

Iteration 3		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	0	-11/5	0	**0**	1	-1/5	-6/5	
X3	8	0	14/5	0	1	0	4/5	-1/15	
X2	10	1	**4/5**	**1**	0	0	-1/15	4/15	
Z=10		Zj	152/15	10	8	0	22/15	32/15	
		Zj-Cj	77/15	8	**0**	0	22/15	32/15	

Since all Zj-Cj≥0
Hence, optimal solution is arrived with value of variables as :

x1=0,x2=1,x3=0
Max Z=10

Caseiii*) The last and third columns are same in AX≤B, and A is C matrix in LPP*

the linear Programming Problem using Simplex (Big M) Method

Max Z=5x1+10x2+8x3

Subject to,

x1+5x2+2x3≤2;

4x1+1x2+4x3≤4;

2x1+4x2+x3≤1;

Where x1,x2,x3≥0

Sol. Given Linear Programming Problem

Max Z=5x1+10x2+8x3

Subject to,x1+5x2+2x3≤2;

4x1+1x2+4x3≤4;

2x1+4x2+x3≤1;

Where x1,x2,x3≥0

If we observe the coefficient matrix A= $\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 3 & 4 & 1\end{bmatrix}$ is a C matrix and last and third column are same in AX=B

Claim: The solution is x=1,y=0,z=0

The Problem is converted to canonical form by adding slack,surplus and artificial variables

1.As the constraint -1 of the type ≤ we should add slack variable S1

1.As the constraint -2 of the type ≤ we should add slack variable S2

1.As the constraint -1 of the type ≤ we should add slack variable S3

After introducing slack variables

Max Z=5x1+10x2+8x3+0S1+0S2+0S3

Subject to

x1+5x2+2x3+S1 =2

4x1+1x2+4x3 + S2 =4

2x1+4x2+x3 +S3 =1

Where x1,x2,x3.S1,S2,S3≥0

Iteration 1		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	2	1	5	2	1	0	0	0.4
S2	0	4	4	1	4	0	1	0	4
S3	0	1	2	**(4)**	1	0	0	1	0.25→
Z=0		Zj	0	0	0	0	0	0	
		Zj-Cj	-5	**-10** ↑	-8	0	0	0	

Negative minimum *Zj- Cj*is - 10 and its column index is 2. So, the entering variable is *x*2.

Minimum ratio is 0.25 and its row index is 3. So, the leaving basis variable is *S*3.

∴The pivot element is 4.

Entering = *x*2, Departing = *S*3, Key Element =4

R3(new)=>R3(old)/4

R1(new)=>R1(old)-5R3(new)

R2(new)=>R2(old)-R3(new)

Iteration 2		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio

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									$\frac{XB}{x2}$
S1	0	¾	-3/2	0	¾	1	0	-5/4	1
S2	0	**(15/4)**	7/2	0	**(15/4)**	0	1	-1/4	1 ⟶
X2	10	¼	1/2	1	¼	0	0	1/4	1
Z=5/2		Zj	5	10	5/2	0	0	5/2	
		Zj-Cj	0	0	**-11/2** ↑	0	0	5/2	

Negative minimum *Zj- Cj*is -11/2 and its column index is 3. So, the entering variable is *x*3.

Minimum ratio is 0 and its row index is 2. So, the leaving basis variable is s2.

∴The pivot element is 15/4

Entering = *x*3, Departing = s2, Key Element =15/4

R2(new)=>R2(old)*4/15

R1(new)=>R1(old)-(3/4)R2(new)

R3(new)=>R3(old)-(1/4)R2(new)

Iteration 3		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	0	-11/5	0	**0**	1	-1/5	-6/5	
X3	8	0	14/5	0	1	0	4/5	-1/15	
X2	10	1	**4/5**	**1**	0	0	-1/15	4/15	
Z=8		Zj	152/15	10	8	0	22/15	32/15	
		Zj-Cj	77/15	0	**0**	0	22/15	32/15	

Since all Zj-Cj≥0
Hence, optimal solution is arrived with value of variables as :

x1=0,x2=0,x3=1
Max Z=8

∴ The vectors of Identity matrix are the solutions of LPPspacial problems

Problem1.3: Solve the linear Programming Problem using Simplex (Big M) method when the coefficient matrix is C matrix when objective function is Minimum and subjective to AX≤B,X≥0,the solution of is trivial solution and MinZ=0 for each case given below

i) If B is replaced with 1st column of A in AX=B then the solution is $(0,0,0,..0)^T$

ii) If B is replaced with 2nd column of A in AX=B then the solution is $(0,0,0,..0)^T$

iii) If B is replaced with 3rd column of A in AX=B then the solution is $(0,0,0,..0)^T$

iii) If B is replaced with nth column of A in AX=B then the solution is $(0,0,0,..0)^T$

Case i)*The last and first columns are same in AX≤B*

and A is C matrix in LPP

Min Z=5x1+10x2+8x3

Subject to,

x1+5x2+2x3≤1;

4x1+1x2+4x3≤4;

2x1+4x2+x3≤2

Where x1,x2,x3≥0

Sol. Given Linear Programming Problem

Min Z=5x1+10x2+8x3

Subject to,x1+5x2+2x3≤1;

4x1+1x2+4x3≤4;

2x1+4x2+x3≤2;

Where x1,x2,x3≥0

If we observe the coefficient matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C matrix and last and first column are same in AX=B

Claim: The solution is x=0,y=0,z=0

The Problem is converted to canonical form by adding slack,surplus and artificial variables

1.As the constraint -1 of the type ≤ we should add slack variable S1

1.As the constraint -2 of the type ≤ we should add slack variable S2

1.As the constraint -1 of the type ≤ we should add slack variable S3

After introducing slack variables

Max Z=5x1+10x2+8x3+0S1+0S2+0S3

Subject to

x1+5x2+2x3+S1 =1

4x1+1x2+4x3 + S2 =4

2x1+4x2+x3 +S3 =2

Where x1,x2,x3.S1,S2,S3≥0

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Iteration 1		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	1	1	**(5)**	2	1	0	0	
S2	0	4	4	1	4	0	1	0	
S3	0	2	2	4	1	0	0	1	
Z=0		Zj	0	0	0	0	0	0	
		Zj-Cj	-5	**-10**	-8	0	0	0	

Since all Zj-Cj≤0
Hence, optimal solution is arrived with value of variables as :

x1=0,x2=0,x3=0;
Min Z=0

Case ii) The last and second columns are same in AX≤B

and A is C matrix in LPP

Min Z=5x1+10x2+8x3

Subject to,

x1+5x2+2x3≤5;

4x1+1x2+4x3≤1;

2x1+4x2+x3≤4;

Where x1,x2,x3≥0

Sol. Given Linear Programming Problem

Min Z=5x1+10x2+8x3

Subject to,x1+5x2+2x3≤5;

4x1+1x2+4x3≤1;

2x1+4x2+x3≤4;

Where x1,x2,x3≥0

If we observe the coefficient matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a C matrix and last and first column are same in AX=B

Claim: The solution is x=0,y=0,z=0

The Problem is converted to canonical form by adding slack,surplus and artificial variables

1.As the constraint -1 of the type ≤ we should add slack variable S1

1.As the constraint -2 of the type ≤ we should add slack variable S2

1.As the constraint -1 of the type ≤ we should add slack variable S3

After introducing slack variables

Max Z=5x1+10x2+8x3+0S1+0S2+0S3

Subject to

x1+5x2+2x3+S1 =1

4x1+1x2+4x3 + S2 =4

2x1+4x2+x3 +S3 =2

Where x1,x2,x3.S1,S2,S3≥0

Iteration 1		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	5	1	**(5)**	2	1	0	0	
S2	0	1	4	1	4	0	1	0	
S3	0	4	2	4	1	0	0	1	
Z=0		Zj	0	0	0	0	0	0	
		Zj-Cj	-5	**-10**	-8	0	0	0	

Since all Zj-Cj≤0
Hence, optimal solution is arrived with value of variables as :

x1=0,x2=0,x3=0;
Min Z=0

Case iii) The last and third columns are same in AX≤B

and A is C matrix in LPP

Min Z=5x1+10x2+8x3

Subject to,

x1+5x2+2x3≤2;

4x1+1x2+4x3≤4;

2x1+4x2+x3≤1;

Where x1,x2,x3≥0

Sol. Given Linear Programming Problem

Min Z=5x1+10x2+8x3

Subject to,x1+5x2+2x3≤2;

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4x1+1x2+4x3≤4;

2x1+4x2+x3≤1;

Where x1,x2,x3≥0

If we observe the coefficient matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 3 & 4 & 1\end{bmatrix}$ is a C matrix and last and first column are same in AX=B

Claim: The solution is x=0,y=0,z=0

The Problem is converted to canonical form by adding slack,surplus and artificial variables

1.As the constraint -1 of the type ≤ we should add slack variable S1

1.As the constraint -2 of the type ≤ we should add slack variable S2

1.As the constraint -1 of the type ≤ we should add slack variable S3

After introducing slack variables

Max Z=5x1+10x2+8x3+0S1+0S2+0S3

Subject to

x1+5x2+2x3+S1 =2

4x1+1x2+4x3 + S2 =4

2x1+4x2+x3 +S3 =1

Where x1,x2,x3.S1,S2,S3≥0

Iteration 1		C_J	5	10	8	0	0	0	
B	CB	XB	X1	X2	X3	S1	S2	S3	Min Ratio $\frac{XB}{x2}$
S1	0	5	1	**(5)**	2	1	0	0	
S2	0	1	4	1	4	0	1	0	
S3	0	4	2	4	1	0	0	1	
Z=0		Zj	0	0	0	0	0	0	
		Zj-Cj	-5	**-10** ↑	-8	0	0	0	

Since all Zj-Cj≤0
Hence, optimal solution is arrived with value of variables as : x1=0,x2=0,x3=0;Min Z=0

II.Conclusions

Therefore we have proved the vectors of identity matrix are the only solutions for special kind of linear system of equations with n equations n unknown problems and for the Linear Programming Problems (LPP) problems of n variables if the objective function is Maximize and also we have proved if it is given in LPP to Minimize objective function the optimal solution is trivial solution (zero solution) and therefore MinZ=0.

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